MY FAVORITE lesson

Michael Todd Edwards and James Quinlan

Repeated Square Roots

The concept of limit lies at the foundation of modern mathematics (Fernández 2004). Too often, however, students' initial encounter with the concept occurs in calculus, where the idea can seem "abstract and forbidding" (Beigie 2002, p. 438). This is unfortunate, because technology provides opportunities to explore such concepts earlier, making the transition to calculus less threatening and more familiar. Here we describe how algebra students can explore limits with graphing calculators.

Students begin by entering a nonzero value and using the **ANS** feature to calculate its square root. Pressing **ENTER** repeatedly calculates the square root of the previous answer. Students do so and observe the results (see **fig. 1**). Ultimately, they reach the same result—values repeating at 1.

This exploration generates significant interest and questions: Does this process work for all numbers? Why does it work? Is the final number really 1?

Because most students begin with whole numbers, we ask them to repeat the process with different initial values. Again, students find that the process appears to yield 1.

What is going on here? Exponent properties provide an explanation for the calculator's behavior. The first few button pushes in **figure 1** may be represented as

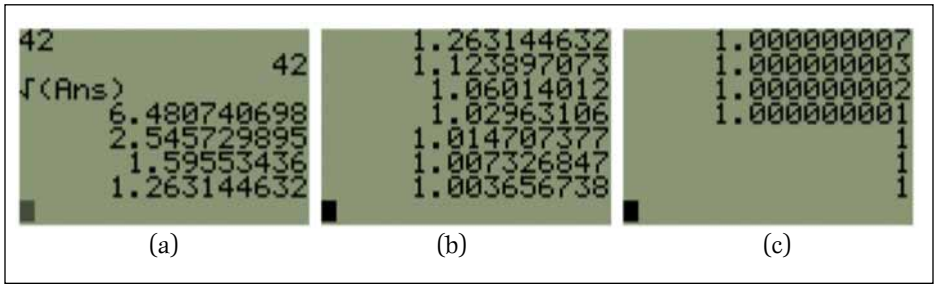

Fig. 1 Repeatedly calculating the square root—here, with initial value 42—results in a value of 1.

$$\sqrt{\sqrt{\sqrt{42}}} = \left(\left(\left(42\right)^{0.5}\right)^{0.5}\right)^{0.5}.$$

Each time students press **ENTER**, their calculators compute the square root again. Using the exponent property $(n^a)^b = n^{(ab)}$, students see that calculating three consecutive square roots is equivalent to raising the initial value to the $(0.5)^3 = 0.125$ power.

Repeated button presses raise the initial value to the 1/2 power many times. For instance, calculating the square root 10 times with initial value 42 is equivalent to raising 42 to 1/1024 (i.e., $42^{0.0009765625}$). Pressing **ENTER** repeatedly raises the initial value to a power approaching 0. The result approaches 1, because any nonzero value raised to the 0 power is 1, an important concept that emerges in beginning algebra courses.

Do we ever reach 1? Each button press multiplies the previous exponent by 1/2. Because half of a nonzero value is never zero (consider Zeno's paradox), at no step in the process do students raise the initial value to precisely 0, so they never reach 1. They can, however, get so close to 1 that the calculator is no longer able to present exact values.

Are there initial values that do not work? Because 0 raised to any positive power equals 0, starting the process with 0 yields 0. Zero is the only initial value that does not yield repeated 1s using a graphing calculator. Not convinced of that? Allow your calculated roots to be complex numbers and watch the results.

Are there other functions that work? If so, what are they? If not, why not?

Calculating repeated square roots provides students with opportunities to explore the concept of limit before calculus. As students attempt to make sense of output generated by their calculators, they consider exponent rules within a fun problem-solving contex

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Beigie, Darin. 2002. "Investigating Limits in Number Patterns." *Mathematics Teaching in the Middle School* 7 (8): 438–43.

Fernández, Eileen. 2004. "The Students' Take on the Epsilon-Delta Definition of a Limit." *Problems, Resources, and Issues in Mathematics Undergraduate Studies* 14 (1): 43–54.

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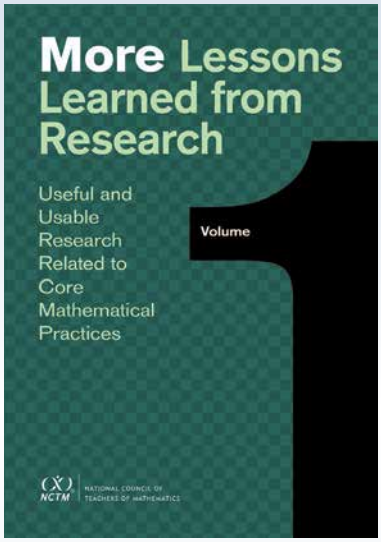

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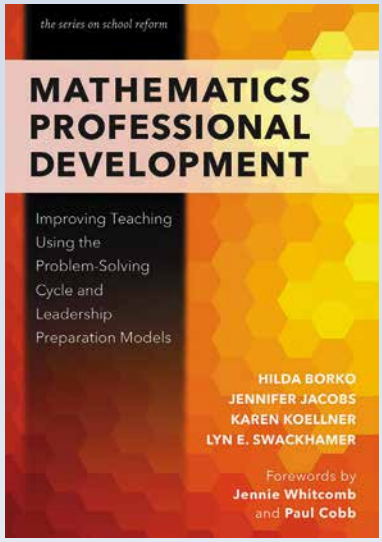

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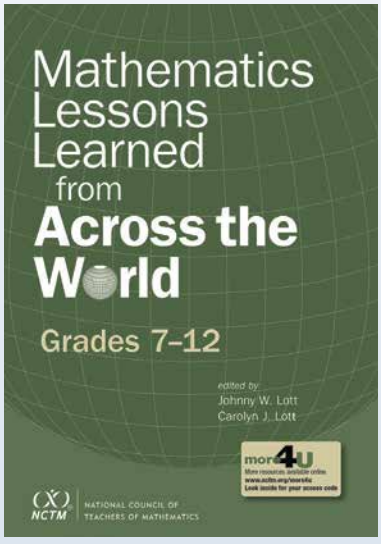

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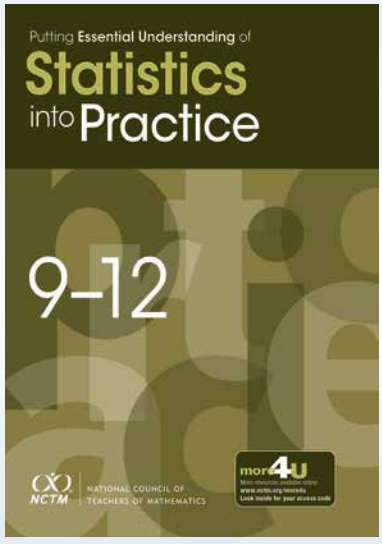

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