

Generations and Generations

... [Priscilla Steiner's] day-old son, Ethan John Curtis, became the newest member of a very large family, which currently has six generations living on the planet at the same time. ...

"They never believe it," said Steiner, 19. ...

Stephanie Steiner is the mother of Priscilla and the newly minted grandmother of Ethan. Just 34 years old herself, Stephanie Steiner knows what it is like to be a very young mom, just like her own mother, grandmother and great-grandmother, who all had the chance to meet Ethan after he was born. ...

Amazingly, Ethan's great-grandmother, Brenda Steiner, is just 51 years old.

His great-great-grandmother, Marilyn Cross, is 68, while his great-great-great-grandmother, Doreen Byers, is 86 years young. ...

Source: "6 Generations of Same Family Living in Ontario," <http://www.cbc.ca/news/canada/toronto/6-generations-of-same-family-living-in-ontario-1.1325128>

- Using the article, complete the chart for this family (see **fig. 1 ["Generations and Generations"]**).
- Create a graph for these data and produce a line of best fit. How well does the line fit these data?
- What does the slope of this line mean in this situation?
- Using your model, estimate the age of someone in the seventh generation of this family. Does this estimate seem practical? Justify your answer.
- The oldest woman to have lived in Canada is Merle Barwis, who lived to be 113. If Merle were the oldest member of a family discussed in the clip, how many generations would there be?
- Collect the data for the living mothers and grandmothers in your family line. How do these data compare with those of the family in the clip? Is the relationship linear? Could it be represented by another model?
- The average age of motherhood in a Canadian family is 29 (see *Financial Post* [2012], <http://business.financialpost.com/2012/07/17/baby-boom-or-bust/>).
- In 2014, CBC reported the average life expectancy of Canadian women as 84 (see <http://www.cbc.ca/news/health/life-expectancy-in-canada-hits-80-for-men-84-for-women-1.2644355>). If the average age of motherhood is 29 and the average life expectancy is 84, how many generations would a typical Canadian family have?
- Thus far, we have considered a linear model.
 - Now create a "family" for whom the data are quadratic.
 - What does a quadratic model imply about the family's ages? That is, how many living generations are plausible?
 - A student claims that families whose ages can be represented as linear models will always have more generations than families represented as quadratic models. The reason given is that a parabola curves upward, causing values to increase at a more rapid rate. Do you agree or disagree? Explain.
 - What other kind of nonlinear models might make sense for families? Justify your thinking.

Generation	Age (Years)
0 (Baby)	0
1 (Mother)	
2 (Grandmother)	
3	
4	
5	

Fig. 1 ("Generations and Generations")

Media Clips appears in every issue of *Mathematics Teacher*, offering readers contemporary, authentic applications of quantitative reasoning based on print or electronic media. All submissions should be sent to the editors. For information on the department and guidelines for submitting a clip, visit <http://www.nctm.org/mtcalls>.

Department editors

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Mid-Parental Height

To determine a child's expected height, experts suggest calculating the "mid-parental height." For boys, add 13 centimetres to the father's and mother's combined height (in centimetres) and divide the result by two. For girls, subtract 13 centimetres from the parent's combined height and divide by 2. A child who is growing normally will eventually fall within 10 centimetres of the mid-parental height.

Source: "Personal Health," *New York Times*, July 19, 1995, <http://www.nytimes.com/1995/07/19/us/personal-health-283395.html>

- Describe precisely what each of the variables in the formulas in **figure 1 ("Mid-Parental Height")** represents.
- Match each item in the left column of **figure 1 ("Mid-Parental Height")** with the correct algebraic formula for estimating the child's expected height in the right column. Indicate your matches by putting the appropriate letter beside each formula.
- At one time, Fisher-Price®, the toy company, provided medical information on child rearing on its website (http://www.fisher-price.com/en_US/playtime/parenting/index.html), such as these guidelines regarding a child's height:
 - Find the average of the parents' height by adding the mother's and father's height (either in inches or centimeters) and dividing by 2.
 - To calculate the height of a boy, add 2.5 inches (6.5 centimeters) to the average of the parents' height. To calculate the height of a girl, subtract 2.5 inches (6.5 centimeters) to the average of the parents' height.
 - The resulting number is the "mid-parental height" for girls or boys. The child's adult height can be expected to fall within a range of 4 inches (10 centimeters) less or more than the mid-parental height.
- Jack (height 74 in.) and Mary (height 60 in.) are expecting a baby boy.
 - Use the Fisher-Price guidelines in inches to find the maximum predicted height of their child in inches.
 - Convert the answer in part (a) to centimeters.
 - If, instead, Jack and Mary convert their heights initially to centimeters and then use the centimeter guidelines, will they get the same maximum height? Why is this so?
- The Canadian Paediatric Society also suggests that doctors use parental heights to determine mid-parental height (go to <http://www.cps.ca/documents/position/toddler-falling-off-the-growth-chart>), using these formulas:

Boys: $(\text{father's height} + \text{mother's height})/2 + 6.5 \text{ cm} \pm 8.5 \text{ cm}$

Girls: $(\text{father's height} + \text{mother's height})/2 - 6.5 \text{ cm} \pm 8.5 \text{ cm}$

 - How do the recent Canadian recommendations for physicians differ from the *New York Times* and Fisher-Price recommendations?
 - Using the variables described in question 2 with both the Fisher-Price and Canadian recommendations, write the predicted ranges of heights of a girl as absolute value inequalities.
- Using the two nonequivalent recommendations, give an example of a boy's height that would be considered typical by one formula but not by the other.
- What would be some practical problems for doctors using mid-parental height?
- In 1999, United Kingdom researchers Charlotte Wright and Tim Cheetham found that mid-parental height does not predict well for parents who are "unusually tall or short."
 - What could be the reason?
 - Suggest your own formulas for estimating the height of a boy and a girl.

A	Minimum expected height for girls	$\frac{F + M - 13}{2} + 10$
B	Maximum expected height for girls	$\frac{F + M + 13}{2} + 10$
C	Minimum expected height for boys	$\frac{F + M + 13}{2} - 10$
D	Maximum expected height for boys	$\frac{F + M - 13}{2} - 10$

Fig. 1 ("Mid-Parental Height")

Is Fisher-Price's medical advice the same as that of the *New York Times*? (Work in centimeters.) Justify your reasoning algebraically.



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"Generations and Generations" answers

1. The complete chart for this family is shown in **figure 2** ("Generations and Generations").
2. We can use a graphing calculator to create a graph for these data and a line of best fit. Although the data are discrete, we can use the line of best fit to predict between and beyond the points given. The regression line is a very good fit because the correlation coefficient is very close to 1 (see **fig. 3** ["Generations and Generations"]). We could consider the slope to be approximately 17 and the y -intercept approximately 0. Thus, the equation of the line is approximately $y = 17x + 0$, where x represents the number of generations and y represents the age in years.
3. The slope, approximately 17, represents the average age of the mother when her baby was born. Further, this means that a new generation of the family begins approximately every 17 years.
4. The person in the seventh generation would be 102 years old: $y = 17x + 0 = 17(6) + 0 = 102$. This age is definitely possible; some people have lived to be 120 years old.
5. Merle would also most likely be in the

Generation	Age
0 (Baby Ethan)	0
1 (Mother Priscilla)	19
2 (Grandmother Stephanie)	34
3 (Great-grandmother Brenda)	51
4 (Great-great-grandmother Marilyn)	68
5 (Great-great-great-grandmother Doreen)	86

Fig. 2 ("Generations and Generations")

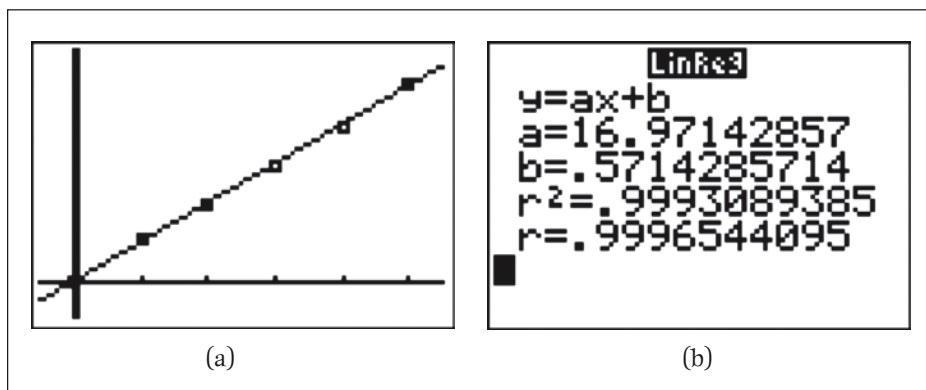


Fig. 3 ("Generations and Generations")

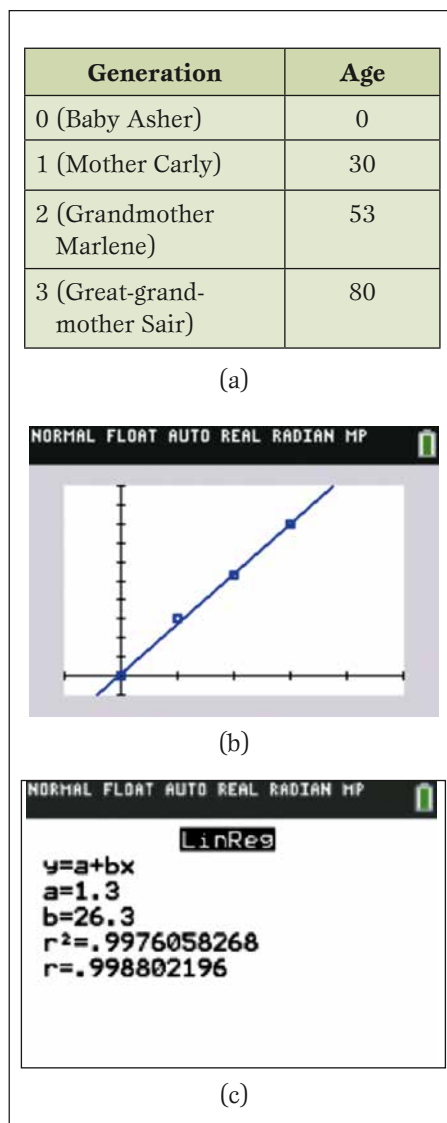


Fig. 4 ("Generations and Generations")

seventh generation: $113 = y = 17x + 0$
yields $x = 113/17 \approx 6.6$.

6. Answers will vary. See the chart in **figure 4** ("Generations and Generations") for the ages of the author's family. The relationship appears linear. The most recent generation (shown above the line) is older, relative to its following generation, than the others.
7. (a) A linear model for this typical Canadian family can be expressed as an equation: $y = 29x + 0$.
(b) Using the model in (a), we find that a centenarian would be in the third generation: $100 = y = 29x + 0$ yields $x = 100/29 \approx 3.4$.
8. The typical family would have either three or four generations (see **fig. 5** ["Generations and Generations"]).
9. (a) Answers will vary. Note that the second differences of a quadratic model are constant. An example,

Generation	Age
0 (Baby)	0
1 (Mother)	29
2 (Grandmother)	58
3 (Great-Grandmother)	87

Fig. 5 ("Generations and Generations")

Generation	Age	First Differences	Second Differences
0	0	—	—
1	25	25	—
2	51	26	1
3	78	27	1
4	106	28	1

Fig. 6 (“Generations and Generations”)

shown in **figure 6 (“Generations and Generations”)**, has second differences of 1. Using a graphing calculator, we see that the equation is $y = 0.5x^2 + 24.5x + 0$.

(b) This example indicates that five generations seem possible. Can you create a quadratic example with more than five generations?

(c) The student is incorrect. The quadratic given in answer 9 has five generations, whereas the linear model in answer 8 has only four generations.

(d) Using the same reasoning, you could make the third differences constant for a cubic. Other models might work, such as a square root function (e.g., $y = 49\sqrt{x}$), whose graph is concave down. Regardless of the model, the graph would be strictly increasing because people in later generations are necessarily older.

“Mid-Parental Height” answers

1. In the formulas, F and M are the heights, measured in centimeters, of the father and the mother, respectively.

2. The appropriate letters beside the formulas in **figure 1 (“Mid-Parental Height”)** are B, D, C, and A.

3. Using the formulas in the *New York Times* article, we have the following:

$$\text{Boys: } (F + M + 13)/2 = (F + M)/2 + 6.5$$

$$\text{Girls: } (F + M - 13)/2 = (F + M)/2 - 6.5$$

These formulas match those provided by Fisher-Price.

The Fisher-Price description also has a range of ± 10 cm. The corresponding minima and maxima in cm are shown in **figure 2 (“Mid-Parental Height”)**.

4. (a) The maximum expected height of Jack and Mary’s son is $(60 + 74)/2 + 2.5 + 4 = 73.5$ in.

(b) Since $2.54 \text{ cm} = 1 \text{ in.}$, the height in part (a) converts to approximately 186.7 cm.

(c) Jack and Mary are approximately 188.0 cm and 152.4 cm tall, respectively. The maximum height of their child is then $(188.0 + 152.4)/2 + 6.5 + 10 = 186.7$ cm. This answer closely matches that in part (b) because $2.5 \text{ in.} \approx 6.5 \text{ cm}$ and $4 \text{ in.} \approx 10 \text{ cm}$.

5. (a) According to the formula, the heights of Canadian children do not reach the extremes that the *New York Times* and Fisher-Price would suggest. Canadian recommendations add or subtract 8.5 cm, whereas Fisher-Price indicates ± 10 cm. This is almost a 1-in. difference in the maximum or minimum heights.

(b) The Fisher-Price model gives $|(F + M)/2 - 6.5| \leq 10$, whereas the Canadian formula gives $|(F + M)/2 - 6.5| \leq 8.5$.

6. Any child whose height is in the top 1.5 cm or bottom 1.5 cm of the heights in the Fisher-Price boundaries would be out of the expected range with respect to the Canadian guidelines.

I ♥ spherical analogs of truncated icosahedrons.

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Suppose the mother's height is 150 cm and the father's height is 200 cm. If their son were between 190 cm and 191.5 cm, he would be outside the typical range with respect to the Canadian guidelines but not according to Fisher-Price.

7. Answers will vary. The biological parents and their heights may be unknown. Another problem could be that the information given to the doctor was incorrect or incomplete.

8. (a) Statistically, we know that averages are responsive to outliers. Unusually tall or short parents would be statistical outliers and the resulting calculations would give mid-parental heights not necessarily good for predictions.

- (b) Answers will vary. Perhaps a different "average" is possible, such as the geometric mean rather than the arithmetic mean of the parents' heights.

A	Minimum expected height for girls	$\left(\frac{F+M}{2}\right) - 6.5 - 10 = \left(\frac{F+M}{2}\right) - 16.5$
B	Maximum expected height for girls	$\left(\frac{F+M}{2}\right) - 6.5 + 10 = \left(\frac{F+M}{2}\right) + 3.5$
C	Minimum expected height for boys	$\left(\frac{F+M}{2}\right) + 6.5 - 10 = \left(\frac{F+M}{2}\right) - 3.5$
D	Maximum expected height for boys	$\left(\frac{F+M}{2}\right) + 6.5 + 10 = \left(\frac{F+M}{2}\right) + 16.5$

Fig. 2 ("Mid-Parental Height")



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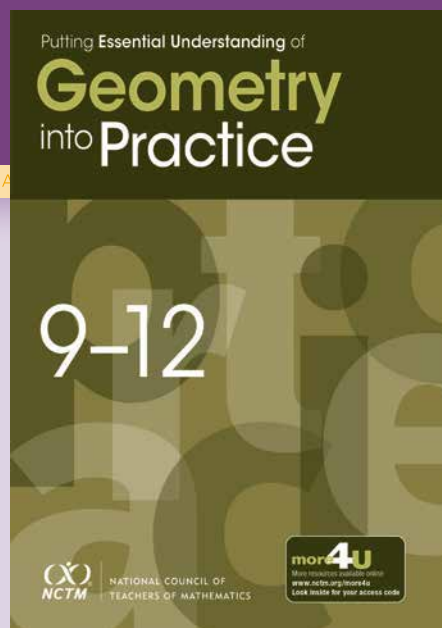
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