



# Narrowing Participation Gaps



*By noticing students' contributions as individuals or part of the team, teachers can focus on engagement.*

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**S**hrinking the achievement gap in mathematics is a tall order. One way to approach this challenge is to think about how the achievement gap manifests itself in your classroom and take concrete action. For example, opportunities to participate in activities that involve mathematical reasoning and argumentation in a safe and supportive manner are central to the development of mathematical competence and identity. It follows that one way to think about equity in your class-

room is to consider the kinds of opportunities you create for different groups of students to participate in these activities (NCTM 2000).

We describe three strategies that can help narrow classroom participation gaps—that is, differences in the types and rates of participation among groups of students from dominant versus nondominant ethnic, racial, socioeconomic, or linguistic backgrounds in mathematics classrooms. These strategies are the result of ten years of researching successful mathematics teachers, our experience



teaching middle school and high school mathematics in demographically diverse classrooms, and new theories of mathematics learning that take into account identity, culture, and power in the mathematics classroom. We end each section by noting tensions that we and other teachers have faced in attempting to broaden participation in our classrooms.

### **STRATEGY 1: ORGANIZE MATHEMATICAL “CONTRIBUTIONS”**

When students make mathematical statements in the classroom, teachers often treat these as “responses,” evaluate these responses for accuracy and completeness, and use them to make judgments

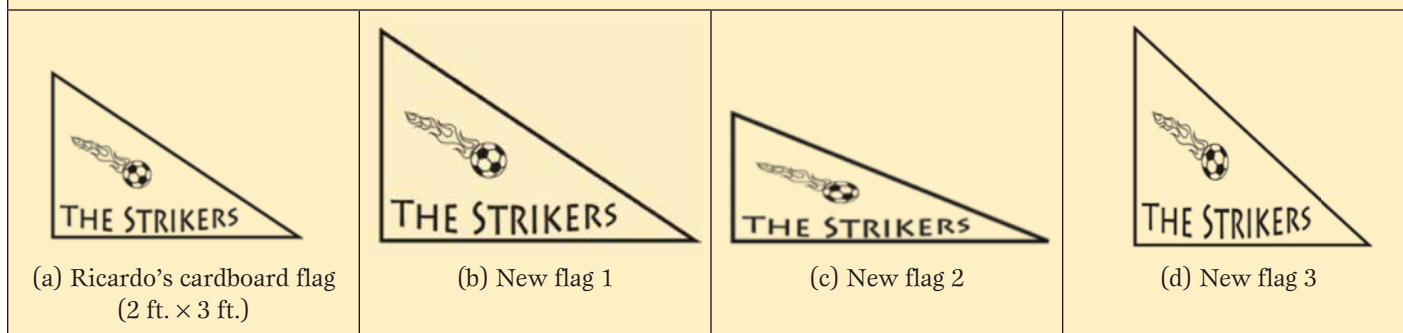
about the students’ understanding of the mathematics. Patterns of classroom interaction like this are common in mathematics classrooms. However, they tend to reinforce deficit perspectives of certain groups of students. Looking only for “responses” and “answers” in classroom mathematics activity inclines teachers toward judging and evaluating what students are saying and doing instead of noticing what or how students are attempting to contribute to the class’s learning. This orientation also prioritizes correct uses of academic language over students’ sense making. Mathematical ideas embedded in speech or expressions that do not fit norms that teachers are familiar with from their families, communities, and preparation programs can be overlooked, discounted, or, worse, viewed as detrimental to mathematics learning (Moschkovich and Nelson-Barber 2009). When teachers expect mathematical contributions to take certain forms, they can overlook the mathematical potential of some of their learners.

A different orientation toward students’ mathematical thinking involves noticing, drawing out,



Ricardo wants to have an even larger version of his school's pennant that he can take to the games. Ricardo takes his cardboard flag to a flag maker and asks her to make a larger flag. A few days later, she calls Ricardo to tell him that she has made three flags for him to look at. When he arrives at the shop, Ricardo says, "Wait! Two of the flags don't look correct!"

*Question:* Which flag is a correct enlargement of Ricardo's cardboard flag? Share your reasoning.



**Fig. 1** This problem requires students to describe concepts of size and enlargement.

and eventually organizing contributions with students. This approach is similar to Smith and Stein's (2011) practice of monitoring students' mathematical thinking while they work, but we note a tension between looking for a predetermined set of "responses" versus listening for mathematical sense making. Instead of categorizing students' mathematical ideas in terms of those that do or do not match an ideal, teachers taking this perspective understand that what students are doing and saying represents pieces of how they make sense of the mathematics in which they are engaged. The teacher's role, then, is to help bring students' sense making into the flow of classroom conversations. We find it useful to think of this process as organizing contributions with our students. The dialogue that follows illustrates how a teacher noticed and interpreted students' classroom participation related to the problem in **figure 1** in this more expansive way:

**Teacher:** Which one do you think it is? Flag 2, why?

**Student 1:** Because they have the same length.

**Teacher:** They have the same length. What do you mean—the same length?

**Student 1:** The sides are the same. The positioning.

**Teacher:** The positioning. What do you mean—the positioning?

**Student 1:** Like, flag 1 is stretched out.

[break in the transcript]

**Student 2:** I think, like, flag number 3.

**Teacher:** Flag number 3 what?

**Student 2:** Because if it's more deeper . . .

**Teacher:** It's deeper or steeper?

**Student 2:** Deeper.

**Teacher:** It's deeper. What do you mean by deeper?

**Student 2:** It's going—it's straight, and, like, you told me if you put a marble, it's going straight down faster.

**Teacher:** If you put a marble here or a ball, which one gets to the ground first? Which do you think is going to get to the ground first?

Notice in the excerpt how the teacher attended closely to the informal language that students used to describe different aspects of the flags. Instead of assuming what student 1 meant by "length," the teacher pressed the student to elaborate on this idea. The teacher did this several times, repeating the words and terms that students used to explain their ideas. Student 2 described the slope of flag 3 in terms of an idea from physics involving angle, velocity, and acceleration. The teacher maintained this new framing of the problem, in keeping with the student's understanding of it. When a teacher is confident that students' ideas are fully explained, he or she can weave academic language and specific terms into the conversation.

Teachers who are skilled at noticing students' mathematical activity in its various shapes and forms experience greater success in organizing contributions with their students (Jacobs and Philip 2004; van Es 2011). Effective teacher noticing entails attending to the details of the mathematics that students are working on, considering how the ideas that students generate can support instructional goals, and posing questions that clarify or push thinking even further (Smith and Stein 2011). Generally, teachers who notice their classrooms in this way come from a place of curiosity instead of expectation. This orientation fosters a sense of safety in the classroom for groups of students who are often misunderstood or perceived as incompetent to be able to share their thinking. In the sidebar **Notice and Organize Student Contributions**, we describe activities that, like Smith and Stein's (2011) five practices for orchestrating effective mathematics discussion,

encourage noticing and organizing contributions with students.

At first, teachers may find it difficult to play a backstage role in the development of mathematics in their classrooms. However, as they get better at organizing contributions with students, they will find that the class becomes much more interesting and manageable. Other tensions that we have faced with this strategy include grasping the mathematics in what students are doing and saying; weaving their disparate ideas into a coherent instructional trajectory; posing questions instead of providing answers; trusting the unpredictability of this process; and temporarily putting aside pacing guides to help students learn to communicate their ideas in concise, comprehensible, and compelling ways.

Whether or not a student decides to contribute something in class depends heavily on the conditions we create for her to do so. Students who have been consistently marginalized in their mathematics classrooms may need even more encouragement to share their thinking. These considerations lead us to the second principle.

## STRATEGY 2: EXPAND “SMARTNESS”

Research is unambiguous about the importance of providing multiple avenues for students to be competent in classroom mathematics learning (Boaler 2009). For the most part, it is often easy to tell who is and who is not “smart” in our mathematics classrooms, and smartness fits with social stereotypes about people who do mathematics (Horn 2007).

Complex Instruction, a research-based approach to broadening classroom participation, has been shown to expand perceptions of who is capable mathematically. This approach involves using group-worthy tasks that provide multiple and multidimensional paths to problem solving; instructional tools that involve student roles and norms for inclusive group work; and status treatments designed to interrupt the formation of classroom status hierarchies (Cohen and Lotan 1997). Using Complex Instruction in our classrooms has been a powerful way to shift patterns of participation among students. It is one of a number of pedagogical approaches that support structured and productive group work. (See Rachel Lotan, cofounder of Complex Instruction, speak at <http://www.youtube.com/watch?v=6b-WnJ3c2lo>).

One way that Complex Instruction is used to expand smartness is by assigning competence to students with low status due to social and cultural hierarchies around race, ethnicity, language, or socioeconomic status. Assigning competence involves noticing contributions made by low-status students in group work or whole-class discussions and describing these publicly to the class.

## Notice and Organize Student Contributions

- Quietly walk around and listen to how students are expressing their mathematical ideas. Be as unobtrusive as possible.
- Take note of what you are learning about your students and the mathematics that they are working on, with an eye toward what they understand (instead of what they have done correctly or incorrectly).
- Consider what you now know and still want to know about your students’ ideas and experiences.
- In the moment or during a break, generate questions that you might use to clarify or push mathematical thinking. Look for opportunities to pose them to the students (or groups).
- During classroom conversations with students, ask students to say more about language or ideas that need clarification or elaboration.
- Once you have a grasp on student work, decide how the ideas relate to one another in ways that extend class learning. Decide whether these ideas have a particular flow (e.g., the strategies used increasingly sophisticated mathematics; the solutions are more specific to the problem or can be generalized; etc.).
- Carefully orchestrate students’ sharing of explanations and statements in small groups or whole-class formats to fashion a productive flow of ideas.

We would add this approach to Smith and Stein’s (2011) practice of selecting students.

The following task was assigned to students in their Complex Instruction groups at the beginning of class: The sum of two consecutive odd numbers is 272; what are the numbers? The task is cognitively challenging in that it has multiple solution paths and promotes the development of an algebraic rule to describe a pattern. It is also group worthy because it encourages multiple approaches and because students—who had been working in Complex Instruction groups since the beginning of the term—engaged in group norms and roles that fostered rich discussion of these approaches. As the teacher walked around, she held groups accountable for these norms and roles and asked them to provide the basis for their next guess on the guess-and-check chart (see **fig. 2**):

**Teacher:** OK. So how would I write my sum then?

Can someone tell me? This is where people start to get messed up. [*Turns slightly to the left and points to Noelle.*] Go ahead, Noelle. You do this one.

**Noelle:**  $(x)$  plus  $(x + 2)$  equals 272.

**Teacher:** OK. So I actually really like the way Noelle set up this chart in the first place. She didn’t just write the sum. She also wrote herself a clue as to what she was adding. So then this translates really easily into this. [*Points to  $(x)$  in overhead chart with transparency marker.*] If number 1 is  $x$ , then she just writes that there, and number 2 then is whatever that is, which

**2. The sum of two consecutive odd numbers is 272? What are the numbers?**

Students' guesses	On the overhead			
	#1 (Consecutive odd)	#2 (Consecutive odd)	Sum (#1 + #2)	Check (272)
Noelle's guess	121	123	$121 + 123 = 244$	Too Low
Donald's guess	125	127	$125 + 127 = 252$	Too Low
Hui's guess	135	137	$135 + 137 = 272$	✓
Noelle's rule	(x)	(x + 2)	$(x) + (x + 2) = 272$	

**Fig. 2** Students' guesses, recorded and projected overhead, led to a general rule.

is plus 2. [Points to  $(x + 2)$  in the overhead chart with a transparency marker.] So she doesn't get messed up.

In the dialogue excerpt, the teacher detailed the way that Noelle set up her guess-and-check chart, describing how the structure that she created for herself supported effective problem solving. The teacher's language also frames Noelle as a strategic and efficient mathematical thinker. Assigning competence in this way serves to equalize status among students and encourages a range of students to participate.

Teachers can also assign competence to students around "social" and "emotional" aspects of their mathematical participation. For example, we validate students for encouraging their peers through difficulties, for using positive yet critical feedback, and for inviting members of their group to share their ideas.

We have touched on only one aspect of Complex Instruction pedagogy and its role in narrowing classroom participation gaps. Readers are encouraged to learn more about the entire Complex Instruction approach in Boaler (2009); Featherstone et al. (2011); Horn (2012); and Aguirre, Mayfield-Ingram, and Martin (2013). Teachers might read about Complex Instruction practices, group roles, and norms; then try out a group-worthy task and look for opportunities to assign competence to low-status students and notice how these students are participating.

Another step might be to make a list of each student's mathematical and interpersonal strengths (see the sidebar **Notice Student Strengths**).

Using Complex Instruction in your classroom will not fix classroom participation gaps overnight. However, it is vitally important to use it consistently so that students grow used to the norms and expectations. It is also important to work with tasks that are group worthy and cognitively challenging. When students cannot access the mathematics or contribute to their group, they often withdraw even further from the class. Teachers may also find Complex Instruction norms such as holding students accountable for listening to one another's ideas and

for staying together as a group challenging at first. Publicly validating groups early on for adhering to group roles and norms is helpful.

Complex Instruction works by leveling the playing field in the classroom so that a broader range of students can engage powerfully in mathematical practices. Powerful engagement is central to students' learning and is at the heart of the final strategy.

### STRATEGY 3: ENGAGE INSTEAD OF MOTIVATE

Teachers who organize contributions with students and use Complex Instruction pedagogy are focused on engaging their students rather than trying to motivate them. This orientation goes against the tide of improving student motivation, a concept we find to be problematic for two reasons. First, an orientation that focuses on motivation lays the blame on students for low participation and achievement. As a result, teachers try to change students instead of the conditions in their classrooms that lead to participation gaps (e.g., emphasis of solutions over reasoning, lack of awareness of the multiple resources that students bring to school learning, and stereotypical perspectives of smartness). The fact that participation gaps fall along racial, ethnic, and linguistic lines tells us that we must pay attention to structures, both local and distal to the classroom, and not just individuals. The second reason we find an orientation around improving student motivation problematic is that groups of students are repeatedly labeled "lazy" or "unmotivated." The positive and negative labels that we assign to students' motivation form the basis of our stereotypes about different ethnic and racial groups.

We know, however, that children's actions are always motivated by something. Their behavior often makes sense when placed in the broader contexts of their life experience. For example, consistently negative experiences in mathematics classrooms often lead students to seek ways to be valued or save face. They might try to gain status with peers through impertinent or comical behavior, or they might try to escape notice by remaining silent. Often when students engage in this behavior, teachers feel the need to control them in some way. However, exercising control over students' behavior tends to constrain students' mathematical sense making and deepen classroom inequities.

Teachers who understand this dynamic must walk a delicate tightrope; they must give students the freedom to show their personalities in the classroom, and they must also reinforce classroom structures that direct this energy toward mathematical activity. Balancing freedom with structure allows students to find ways of engaging in mathematics that support the people they want to become.

Learning mathematics, then, becomes an identity-affirming rather than invalidating experience. To us, this is powerful engagement in mathematics learning—when students are reasoning about and making sense of mathematics in ways that nurture all aspects of their identities.

**Figure 3** compares approaches to mathematics instruction that engage rather than motivate students.

Processes that perpetuate social injustice in society, and thus in mathematics classrooms, are constantly evolving. Techniques and strategies that serve teachers now may not be effective in the future, so it is important to continually monitor their efficacy in narrowing classroom participation gaps. The following questions are useful to consider:

1. Do differences in the classroom participation of my students fall along ethnic, racial, socioeconomic, or linguistic lines?
2. Have all my students had the opportunity to show what they can do mathematically?
3. Am I trying to notice how students are reasoning and use the information to organize learning?
4. What more can I do to invite my students to engage in mathematical practices?

### ORIENTATION TOWARD CHILDREN, MATHEMATICS TEACHING, AND SOCIETY

Mathematics teachers who use these strategies often hold particular dispositions toward children, their instruction, and social structures. They view mathematics classrooms as microcosms of the broader society, where mechanisms that create and structure inequities for communities of people are largely invisible. This framing galvanizes them to deliberately interrupt structural inequalities through their mathematics instruction because they know that failing to do so reinforces the status quo.

The strategies that we offer help teachers interrupt structural inequality in specific ways. First, the strategies involve blurring the line between what often gets categorized as mathematical as opposed to social activity. How we categorize someone’s behavior is dependent on our cultural perspectives; what looks like mathematics learning to one person may not to another. Second, the strategies refocus teachers’ instruction on inviting students to participate in mathematics, instead of expecting them to do so. Although teachers may feel that they do so already, they often send mixed signals. These strategies provide consistent signals. Finally, these strategies are aligned with what we know to be good mathematics instruction. They deepen students’ learning and affirm students’ identities as mathematics learners.

Notice Student Strengths		
<ul style="list-style-type: none"> <li>• Make a list of each student’s mathematical and interpersonal strengths.</li> <li>• Notice whether the students with blank spaces are predominantly from nondominant backgrounds. Observe these students and fill in their spaces over the next week.</li> <li>• How these students are participating? Are there any surprises?</li> </ul>		
Name	Mathematics Strength	Interpersonal Strength
Lucia	Graphing linear equations	
Tiana		
Danny	Making connections to every life	Enjoys leading and helping
Esmeralda	Building equations based on growth pattern.	
Aman		Makes students feel better when they do poorly
Blake	Connecting aspects of graphs to T-tables	Takes risks in front of the class

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Motivate Students	Engage Students
Wrap a basketball context around a problem involving percentages.	Use mathematics with students to research aspects of their extracurricular activities and the communities that they care about.
Require students to speak in English in class and to use academic terms when explaining their ideas.	Allow students to use their first language to make sense of and communicate mathematical ideas. Describe conventional English terms for these ideas and explain why using these terms is important.
Attempt to contain disruptive behavior and to isolate students who are being “resistant.”	Assign competence to students who engage in disruptive behavior by organizing mathematical contributions with them.

**Fig. 3** Common instructional practices might motivate rather than engage students.



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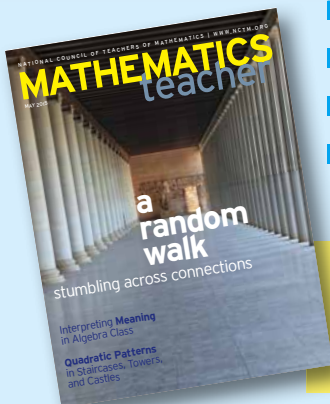
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