

MY FAVORITE
lesson

Melike Yiğit Koyunkaya, Signe Kastberg, James Quinlan,
Michael Todd Edwards, and Jane Keiser

Dynamic Right Triangles

Right triangles play a significant role in mathematics. In our favorite lesson, we help students understand variant and invariant properties by considering relationships among angle measures and side lengths in right triangles. Students explore these relationships using interactive mathematics software, changing one angle and observing the effect on the side lengths and other angles. Students observe that while some properties of the triangle vary, others remain constant, or *invariant*.

Making predictions. We give students a 30-60-90° right triangle (see **fig. 1a**). They predict changes and invariants to attributes of right triangle ABC if the 30° angle A increases to 60° while angle C remains 90°. Students discuss and share their predictions with classmates. Students identify some invariant properties, such as that the sum of the interior angle measures remains 180°. Students may advance other correct predictions: “The side length opposite the increased angle always increases,” or “The side length opposite the decreased angle always decreases.” Incorrect predictions also emerge: “The hypotenuse stays the same length because the measure of the right angle does not change.” Some students also imagine altering the measure of the right angle. They may imagine the measure of angle A increasing to 60° and the measure of angle B kept at 60°, thereby

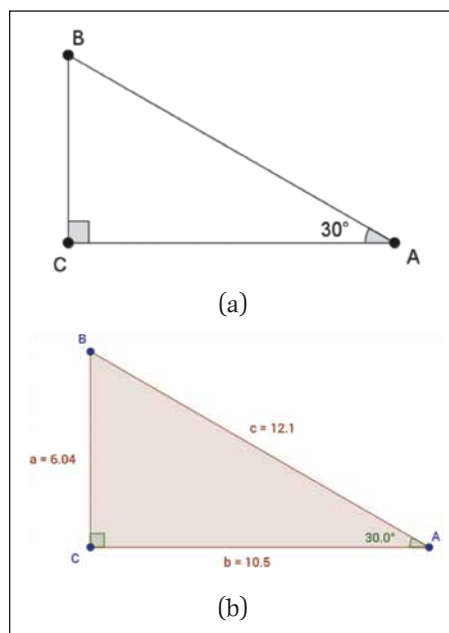


Fig. 1 Students imagine increasing angle A to predict and confirm properties of the resulting triangles.

altering the measure of angle C to 60°.

After this whole-class discussion, we ask students to predict changes in trigonometric ratios when an angle measure in a right triangle changes. We find that students who have difficulty identifying other properties of variance and invariance may also reason incorrectly about the changes in trigonometric ratios.

Testing predictions. Students then use the free software GeoGebra to explore an interactive version of the triangle (see **fig. 1b**) that preserves the measure of the right angle while altering the two acute angles. Students use the author’s activity (see resources) to test their predictions. Points A and B can be moved to resize the acute angles so that students can explore variations and invariance in the right-triangle attributes. Angle A can be altered by dragging vertex A or dragging vertex

B . Students then compare their predictions with their observations from this dynamic exploration. In dragging vertices, students may revise their predictions. The GeoGebra activity also allows students to observe the changing ratios of side lengths as the acute angles change; click on **Show ratio of sides**.

This activity encourages students to investigate relationships among right-triangle attributes as *variables*. Students begin to learn variant and invariant properties of right triangles by changing one or more attributes of the triangle. The exploration motivates students to think of right triangles as having dynamic variant and invariant properties rather than perceiving a right triangle as a static figure. Engaging in this activity, students observe how a change in one attribute affects other attributes. This flexible and dynamic perspective is productive in countless mathematical investigations.

ONLINE RESOURCES

International Geogebra Institute. 2015.

Geogebra. <https://www.geogebra.org/>. Koyunkaya, Melike Yiğit. 2015. “Dynamic Right Triangles.” <http://tube.geogebra.org/student/m1221143>.

MELIKE YİĞİT KOYUNKAYA, yigitmel@gmail.com, is a mathematics educator at Dokuz Eylül University in Turkey. **SIGNE KASTBERG**, skastber@purdue.edu, works with preservice teachers at Purdue University in West Lafayette, Indiana. **JAMES QUINLAN**, jquinlan@une.edu, is an assistant professor at the University of New England in Biddeford, Maine. **MICHAEL TODD EDWARDS**, edwardm2@miamioh.edu, and **JANE KEISER**, keiserjm@miamioh.edu, are mathematics educators at Miami University in Oxford, Ohio.

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Department editor

Roger Day, day@ilstu.edu, Illinois State University, Normal, IL

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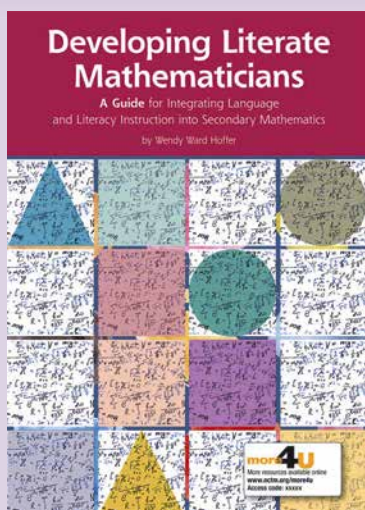
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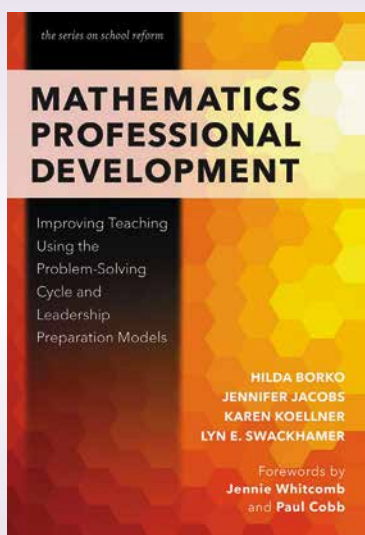
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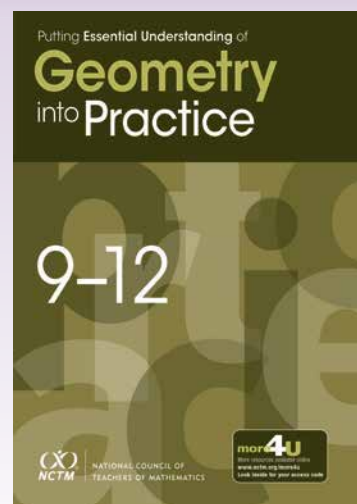
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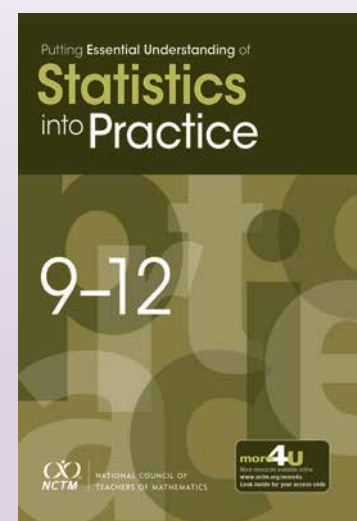


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