WHAT IS MY CARBON FOOTPRINT?

A classroom-tested activity uses authentic, diet-related data to focus on sustainability.

Benjamin J. Galluzzo, Jean McGivney-Burelle, and Rikki B. Wagstrom


GBH007/THINKSTOCK

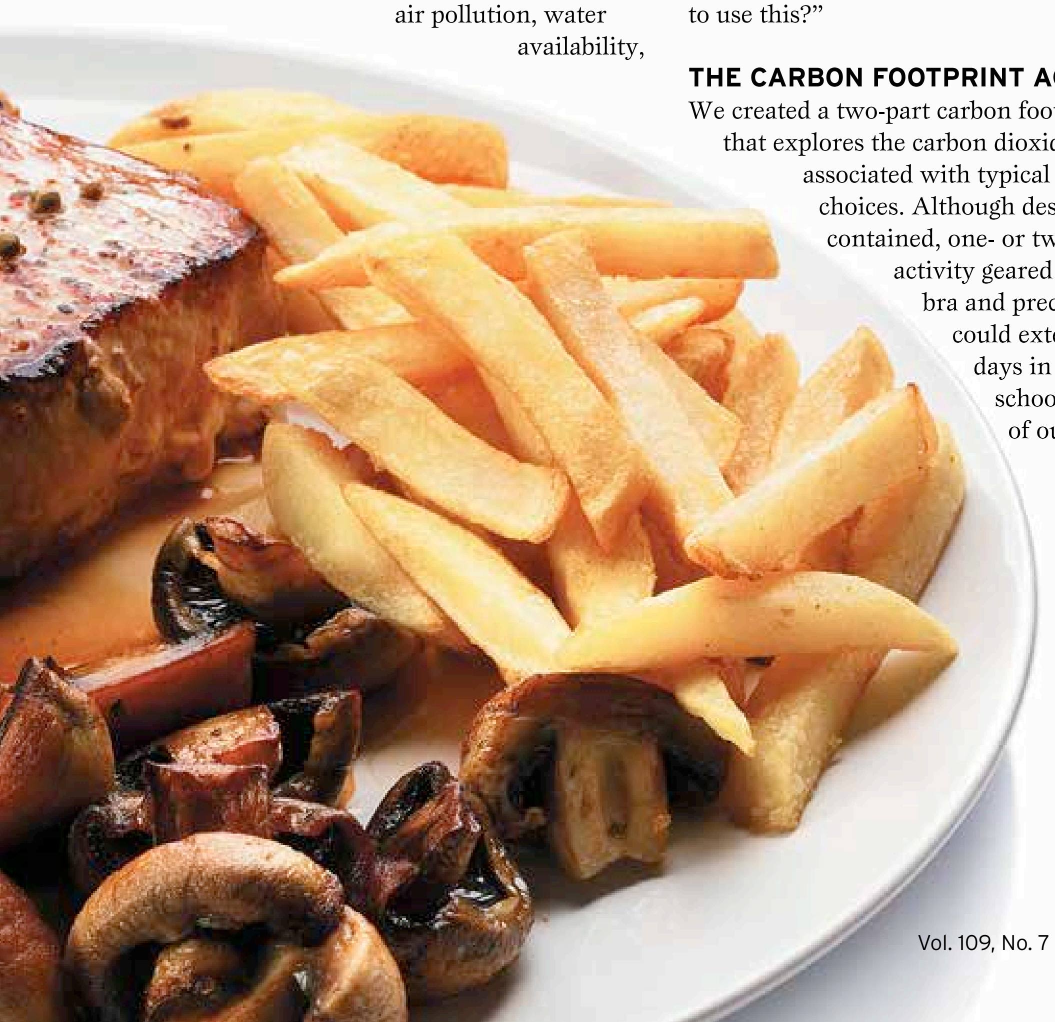
Human beings are having a profound impact on the environment. The opportunity to investigate this timely issue during one or two class periods gives algebra and precalculus students insight into a sustainability topic of great international concern—carbon footprints. Students use mathematical thinking in matters that are pertinent to their everyday lives. Further, they use key quantitative reasoning and mathematical modeling skills: interpreting units, computing with large values, using assumptions to derive an appropriate mathematical model (in this case, a linear model), using models to make predictions, and understanding when a model is appropriate for making predictions—practices aligned with *Principles and Standards for School Mathematics* (NCTM 2000) and the Common Core Standards (CCSSI 2010).

TOPICS THAT MATTER

Integrating sustainability content into mathematics curricula has gained attention and traction in recent years (see the sidebar **Curriculum Resources**). An early example of a sustainability-focused curriculum is Earth Algebra (and later Earth Math) (Schaufele and Zumoff 1993), which illuminates how algebra can be used for solving problems and analyzing phenomena related to the environment, including population growth, air pollution, water availability, food supply, and oil and coal consumption.

In a randomized study, treatment and control groups showed the effects of the Earth Algebra course on students' attitudes and achievement. Students in the experimental section dramatically improved their attitudes about mathematics, became more skilled at data analysis and mathematics modeling, and performed as well as their control group counterparts on assessment of algebraic skills (Schaufele and Zumoff 1993). The students in Earth Algebra gained as much mathematical knowledge as students in the traditional algebra course but grew much more aware and appreciative of the conceptual power and utility of mathematics.

Wagstrom (2010) found similar positive results in a sustainability-focused, precollege algebra course that she developed and tested. Students in the experimental group performed as well as students in the traditional algebra group with respect to algebraic skills. Moreover, students in the experimental group showed increased interest in and confidence in learning and applying mathematics. Finally, as noted by Sustainability Improves Student Learning (SISL), using sustainability to motivate the study of mathematics can produce more engaged, civic-minded students, allow for interdisciplinary collaborations, leave a legacy for future generations, and ultimately answer the common student question, "When are we ever going to need to use this?"



THE CARBON FOOTPRINT ACTIVITY

We created a two-part carbon footprint activity that explores the carbon dioxide emissions associated with typical American food choices. Although designed to be a self-contained, one- or two-day, in-class activity geared for college algebra and precalculus classes, it could extend over several days in a typical high school classroom. One of our overriding goals was to create an authentic experience for students, in which the use of mathematics follows naturally and necessarily from an understanding of the real-world context and the information we want to know. In

CURRICULUM RESOURCES

A growing community of mathematics and science educators around the country has been integrating sustainability content into their math and sciences classes and making their resources freely available to other teachers. Since its inception in 2013, more than 100 scientific societies, universities, research institutes, and organizations all over the world, including NCTM, have joined the Mathematics of Planet Earth project (MPE). A central mission of MPE is to encourage K–16 educators to communicate issues related to planet Earth and highlight the fundamental role of mathematics in responding to the challenges of our planet. For more information about MPE, visit http://mpe.dimacs.rutgers.edu/.

At the K–12 level, Facing the Future offers hands-on, standards-based global sustainability curriculum. One community initiative within higher education is Sustainability Improves Student Learning (SISL), which seeks to increase student learning in undergraduate mathematics and science and to better prepare students for the real-world challenges of the twenty-first century including energy, air and water quality, and climate change. SISL has created an online resource (https://serc.carleton.edu/sisl/sustain_in_math.html) for educators to implement sustainability-integrated math and science curriculum. Much of the curriculum available through SISL is written for introductory-level college courses and would be appropriate for high school classrooms as well.

short, we wanted to provide students with a true-to-life mathematical modeling experience.

In practice, such experiences are inherently open-ended and require individuals to make assumptions, gather relevant data and interpret it correctly, determine appropriate mathematics to use, perform messy computations, and interpret results correctly in a real-world context. We have designed this activity for students who are new to such experiences, and we did not assume that students would be familiar with carbon footprints before working on this activity. As such, we have added scaffolding and structure to enable students to engage the problem in small, sequential steps that prompt them to make useful connections along the way. (The entire two-part activity is available as a .docx file online with this article at **www.nctm.mt/org**.)

So what is a carbon footprint, anyway? According to the U.S. Energy Information Administration, approximately 82 percent of the energy currently demanded by the United States is created from the combustion of fossil fuels (petroleum, natural gas, and coal). When fossil fuels are burned, they emit carbon dioxide into the atmosphere. Almost every component of an average American's life depends on fossil fuel energy—for example, powering household appliances and technology, fueling cars and other modes of transportation that we require, and manufacturing and distributing products that we need and enjoy, such as food, clothing, and recreational items. Consequently, in the course of a typical day, each of us is responsible for the emission of a certain amount of carbon dioxide. A carbon footprint is a metric that estimates the amount of carbon dioxide a person, an entire nation, or the world emits over the course of a day or year. Scientists and policy makers are interested in carbon footprints because of the role that atmospheric carbon dioxide plays in the warming of Earth's surface and resulting effects on its climate system.

What do students know about their carbon footprint? Before teaching the lesson, we gauged our students' understanding of the term *carbon footprint* and found that the majority of our students were already familiar with the term and recognized it as a quantifiable value. The fact that students were already able to connect this real-world concept to mathematics on their own removed the familiar motivational barrier that frequently hampers confidence and interest in mathematical problem solving. As a result, we were able to more easily help students identify how math can be used to calculate values and make predictions about matters that are relevant to our everyday lives.

PART 1: REFLECTING ON OUR DIET

The initial activity sets the context of carbon footprints by introducing students to the role that dietary choices play in determining food-related carbon dioxide emissions. Teachers can initiate this activity by allowing students to study the table in **figure 1.**

Students can discuss the data in small groups and then share their ideas with the entire class. Following this general brainstorming session, students can consider the two discussion questions:

1. How would you characterize the food groups that have the higher-intensity values? Why do you think that these food groups might have higher carbon dioxide (CO_2) values?

2. The USDA estimates the average daily food availability at 3750 calories per person. In other words, the food available to people living in the United States averages 3750 calories per person per day. On the other hand, an average American consumes approximately 2,590 calories of food per day. Discuss the significance of the difference between these two numbers as it relates to the average American's carbon footprint.

Real-world problems such as this are typically data intensive. For students to work on such a

problem, they must interpret data correctly, recognize trends and characteristics, and ultimately integrate the data into an informed summary of the significant contributing factors in the problem that would be of interest to study.

Discussion question 1 prompts students to examine the data table, locate the row corresponding to carbon dioxide intensity values, and look for a commonality among corresponding food groups. Students recognized that the largest values are associated with animal products, and the discussion quickly focused on possible reasons for this—energy-intensive processing and transportation as well as the large food and water requirements of agricultural animals. Some students noted that the data table does not specify which aspects of the food production process are included.

Discussion question 2 prompts students to consider the implications of food waste in the United States in the context of carbon dioxide (CO_2) emissions. Students may identify the paradox of hunger in America despite the overabundance of food, perhaps leading to a valuable discussion of loss of (food) energy. For example, if we assume that the "surplus" food is allocated across all food types, how many extra people can the U.S. food supply accommodate?

PART 2: MATHEMATICS IN CONTEXT

This part of the activity consists of a mathematical modeling exploration of how individual dietary choices, when coupled with population growth, impact national-level carbon dioxide emissions over time. Specifically, students consider the following problem:

> Given that the annual food-related carbon dioxide emissions for an average person living in the United States is 2.32 tons per year, create a mathematical model that will estimate total food-related carbon dioxide emissions for the United States over time. Use your model to predict the national food-related carbon dioxide emissions in the year 2020. Be sure to state all assumptions that you make.

Tackling this problem requires an array of quantitative reasoning and mathematical modeling skills: determining the relevant data to gather, studying data for trends over time, making an assumption about the annual growth of national food-related carbon dioxide emission, deriving an appropriate model equation, and knowing how and when the model can be used for predictions. Developing each skill requires practice. Here we focus on development of the three latter skills; moreover, we have broken up the problem into steps (see **fig. 2**) that prompt students to consider the ideas that are central to the problem. We supply students with relevant data—namely, U.S. population values in 2015 and 2028 (projected)—and we have them assume that the growth of the U.S. population is constant. Although long-term population growth in the United States is not uniform, data from the period from 1950 to the present, for example, are well approximated by a linear model. Then we prompt students to consider what this means for the

Part 1: The Major Food Groups

The data come from multiple U.S. Department of Agriculture reports and data sets and from the 2014 study by Heller and Keoleian (2014). The carbon dioxide intensity, reported as g CO_2 equivalents per calorie, is calculated using calories per food pattern unit for each food type and average kg CO_2 equivalents per food pattern unit.

Table 1 Carbon Dioxide (CO_2) Intensities and Daily Calorie Consumption

Food Group	Fruits	Vegetables	Dairy	Red Meat	Poultry	Fish	Eggs	Cereals and Grains	Nuts	Fats, Oils, and Sugars
CO_2 Intensity (g/Cal)	1.80	1.03	4.94	9.81	2.70	4.86	3.34	0.89	0.29	0.62
% of Daily Calories	3.13	4.82	10.57	10.33	6.08	0.55	1.30	23.54	2.76	36.93

Note also that the average American consumes approximately 2,590 calories of food per day. For comparison, the USDA Estimated Energy Requirements (EER) for a moderately active 16- to 18-year-old female is 2,000 calories per day; the EER for a moderately active male of the same age is 2,800 calories.

Fig. 1 Part 1 of the Carbon Footprint activity concerns dietary choices.

Part 2: Projecting Future National Carbon Dioxide Emissions

According to the U.S. Census Bureau, as of January 1, 2015, approximately 320 million people were living in the United States; the population is projected to reach 353 million people by January 2028. If we make two assumptions—(1) that annual population growth will be constant over the years 2015–28; and (2) that food production and consumption habits will stay the same in future years (an average of 2.32 tons of CO_2 per person will be emitted each year)—we can create a mathematical model to estimate national food-related CO_2 emissions.

1	Estimate total U.S. food-related CO_2 emissions in 2015 and 2028. Give your answers in millions of tons.
2	Calculate the average rate of change in total U.S. food-related emissions during the period 2015–28. Give your answer in millions of tons per year.
3	If we assume constant growth in the U.S. population and that each person's emissions are the same each year, what can you say about the growth of the *total* U.S. food-related emissions each year?
4	Use your answers to steps 1–3 to sketch a graph of the total U.S. food-related CO_2 emissions during the period 2015–28. Draw as carefully as you can.
5	What kind of mathematical equation (function) describes the growth in national CO_2 emissions over time? (e.g., linear, quadratic, exponential, etc.). Explain how you know.
6	Create an equation (a model) that describes the growth in national CO_2 emissions over time. (You drew a graph of this equation in step 4.) Note: Your equation should have— • time (in years since 2015) as the input variable; • CO_2 emissions (in millions of tons) as the output variable; and • a starting point of 2015 (that is, time = 0 represents the year 2015).
7	What are the projected total CO_2 emissions in the year 2020 according to your model in step 6?
8	According to your model, in what year will the total CO_2 emissions be 15% larger than the 2015 level?

Fig. 2 Part 2 of the Carbon Footprint activity uses six steps to help students develop a model.

growth rate of national emissions (see step 3) and, consequently, which type of model is appropriate to use in describing national emissions (see step 5).

With the assumptions of constant population growth and constant per capita emissions, students recognized that a linear function was appropriate and that deriving the mathematical model for national carbon dioxide emissions was essentially a problem of the form "find the equation of the line with slope m and y-intercept b." The challenge was to correctly translate the information from the problem.

In steps 1 and 2, students calculated the average rate of change in national carbon dioxide emissions—approximately 5.9 million tons per year. Most students recognized that this was the slope of the line in steps 4 and 6. Obtaining the correct y-intercept was a greater challenge. Some students did not perceive the significance of the phrase "time = 0 represents the year 2015" and used the calendar year 0 as the initial time, which resulted in a different y-intercept.

Another common mistake centered around unit consistency. Because the function in this problem calculates the national carbon footprint in units of millions of tons, students needed to recognize that the y-intercept also required these units—namely, 742 million tons. For example, a student might give the equation $y = 5.9x + 742{,}000{,}000$ instead of the correct formulation $y = 5.9x + 742$. To aid students in putting all the pieces together, we prompted them to sketch a graph of the national emissions over time (question 4) before asking them to write down the model equation (question 6).

Although part 2 is organized around steps that build on one another and lead students toward the goal of the activity, this is not the only pedagogical approach for exploring the problem. An alternative approach would be to simply pose the question stated at the beginning of this section: "What are the projected total carbon dioxide emissions in the year 2020? Be sure to provide a mathematical model and a list of any assumptions you made."

Of course, pedagogical decisions of this nature should be based on the goals and objectives of the lesson, the students' mathematical preparation, the level of scaffolding required, and so on. Whereas

more open-ended questions and problems can often lead to more creative strategies and solutions, students need sufficient support to make progress on a problem.

In practice, once a person creates a mathematical model, the expectation is that it will be used to investigate questions of interest. Consequently, students need to understand what questions are appropriate for particular models, and they should practice using models to answer questions. In steps 7 and 8, respectively, we prompted students to answer two questions using their model.

Part 2 concludes with two discussion questions:

3. What assumptions underlie your answers in steps 7 and 8? How would altering your assumptions affect your answers?

4. To address the problem of overall increase in total food-related CO_2 emissions over time, suppose that the U.S. makes a focused effort in 2015 to freeze total food-related CO_2 emissions at the 2015 level—how could this goal be accomplished? Discuss possible strategies for achieving this goal. Refer back to part 1 of this activity to inform your discussion.

Discussion question 3 prompts students to reflect on the role of assumptions in shaping the information their model yields. Discussion question 4 prompts students to consider the implications of freezing the national food-related carbon footprint at the 2015 level. If students assume that the U.S. population continues to grow, then they are forced to consider options for reducing per capita emissions, such as shifting diets to include higher percentages of foods that are less carbon intensive, eliminating food waste, and reducing caloric intake. Each of these considerations can lead to a possible extension to this activity.

STUDENT LEARNING SURVEY

To get a sense of our students' experience with this lesson, we distributed an anonymous survey to one group of twenty-three students before and immediately following the Carbon Footprint activity. The survey included three opened-ended questions. These questions and a summary of responses follow:

Survey question 1. What factors influence the size of our national food-related carbon footprint?

For this question, the most common response (made by eleven students) before the activity was "unsure," whereas the number of "unsure" responses dropped sharply (three students) immediately following the lesson. Before the activity, responses to question 1 focused on population; calories consumed; and the farming, manufacturing, and transportation processes involved in food production. After the lesson, the variety of responses was much broader and included references to individuals' consumption of foods with a higher intensity of carbon dioxide, dietary choices, and the energy involved in processing foods.

Survey question 2. Describe how mathematical models can be used to study sustainability-related problems.

Again, students showed growth in their understanding of the content of the activity. Before the activity, thirteen students indicated that they were "unsure" how mathematical models could be used to study sustainability problems. After the activity, this number dropped to four. After the activity, students were able to articulate that by using models and incorporating assumptions in these models, they can make projections and use these projections to make decisions and influence sustainability policies.

Survey question 3. Do mathematical models yield reliable projections or estimates? Discuss.

Before the activity, there were nine students who either did not respond or indicated that they were unsure. After the activity, this number fell to four. In general, the most striking difference between pre- and post-activity survey responses is that before the activity no students mentioned the role of assumptions in creating and using mathematical models, whereas after the activity seven students mentioned the importance of assumptions.

SUSTAINABILITY MATTERS

When students ask, "Why does math matter?" we must be able to respond with effective answers. Without seeing relevant applications, many students lose motivation and without motivation, they struggle to learn and retain the concepts. In contrast, when students are first presented with big, authentic, real-world topics and they learn the math just in time to address those topics, this experience can be a very engaging and memorable one.

Sustainability—how we live our lives, the choices we make, and our obligations to other people and the natural world—is one such big topic. It addresses real-world issues including energy, water, food, air quality, health, quality of life, and climate change. It gets students to analyze their own personal choices and consider how individual decisions can help solve societal problems.

EXTENSION

One possible extension of this activity invites students to develop and explore a new model (a rational function) in which the per capita daily caloric intake is required to decrease over time. The pedagogical value of developing this new model is to demonstrate to students how models are built to address specific questions. To answer different kinds of questions, students sometimes need to change their assumptions, and doing so affects model equations. Instructor resources and student handouts for the complete activity are available from the SISL website (http://serc.carleton.edu/sisl/2012workshop/activities/99949.html).

BIBLIOGRAPHY

Common Core State Standards Initiative (CCSSI). 2010. Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf

"Dietary Guidelines for Americans." http://www.cnpp.usda.gov/dgas2010-policydocument.htm

"Facing the Future: Global Sustainability Curriculum and Teacher PD." February 2015. https://www.facingthefuture.org/

Heller, Martin, and Gregory Keoleian. 2014. "Greenhouse Gas Emission Estimates of U.S. Dietary Choices and Food Loss." *Journal of Industrial Ecology*.

"Key Components of Sustainability Assignments." http://serc.carleton.edu/sisl/pedagogies.html

"Mathematics of Planet Earth." http://mpe2013.org/

National Council of Teachers of Mathematics (NCTM). 2000. *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

Schaufele, Christopher, and Nancy Zumoff. 1993. Earth Algebra Executive Summary: project dates: September 1, 1991–August 31, 1993. Washington, DC: U.S. Dept. of Education, Office of Educational Research and Improvement, Educational Resources Information Center.

Schaufele, Christopher, Nancy Zumoff, Marlene Sims, and Stan Sims. 2003. *Earth Algebra: College Algebra with Applications to Environmental Issues*. Boston: Pearson Custom Publishing.

U.S. Census Bureau. "International Data Base." 2015. http://www.census.gov/population/international/data/idb/informationGateway.php.

———. "Projections of the Population and Components of Change for the United States: 2015 to 2060 (NP2014-T1)." http://www.census.gov/population/projections/data/national/2014/summarytables.html.

U.S. Department of Agriculture. "ERS Food Availability (Per Capita) Data System." http://www.ers.usda.gov/data-products/food-availability-%28per-capita%29-data-system/

U.S. Department of Agriculture, Agricultural Research Service. 2014. "USDA National Nutrient Database for Standard Reference, Release 27." http://www.ars.usda.gov/ba/bhnrc/ndl.

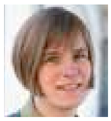
U.S. Department of Agriculture, Center for Nutrition Policy and Promotion. "Dietary Guidelines for Americans." http://www.cnpp.usda.gov/DietaryGuidelines

U.S. Department of Agriculture and U.S. Department of Health and Human Services. 2010. "Dietary Guidelines for Americans." Washington, DC: U.S. Government Printing Office.

"U.S. Energy Information Administration–EIA–Independent Statistics and Analysis." *Monthly Energy Review*. http://www.eia.gov/totalenergy/data/monthly/

Wagstrom, Rikki. 2010. "Teaching Pre-College Algebra Mathematics through Environmental Sustainability." *Science Education and Civic Engagement* 2 (2): 17–23. http://seceij.net/files/seceij/summer10/wagstrom_pdf.v2.pdf





BENJAMIN J. GALLUZZO, bjgalluzzo@ship.edu, is associate professor of mathematics at Shippensburg University in Shippensburg, Pennsylvania. As the principal investigator on the National Science Foundation funded grant "Undergraduate Sustainability Experiences in Mathematics" (USE Math), he has worked with mathematics educators across the country to develop and disseminate sustainability-focused classroom-ready activities. **JEAN McGIVNEY-BURELLE,** burelle@hartford.edu, is associate professor of mathematics and director of the Math Secondary Education Program at the University of Hartford in West Hartford, Connecticut. Her current research focuses on the effective use of technology to more fully engage students in constructing mathematical meaning. **RIKKI B. WAGSTROM,** rikki.wagstrom@metrostate.edu, is associate professor of mathematics at Metropolitan State University in St. Paul, Minnesota. Her teaching and research interests center around the integration of mathematical modeling and sustainability topics into lower-division and developmental college mathematics courses and impacts on mathematical learning.

more4U

For the entire two-part activity available as a .docx file, go to **www.nctm.org/mt**.