Masha Albrecht

The Handshake Problem and Preassessing Practice Standard Skills

ADDITIONAL SAMPLE STUDENT RESPONSES

Maria's Response

Maria showed a solid understanding of the recursive method in solving the problem for 10 people (see **fig. 4**). She shows the recursive structure in her table and also in a sample case.

She correctly uses a closed formula in her answer for 100 people, although her explanation for the closed formula mistakenly uses the partial sum model. Her responses show confidence and fluency.

HANDSHAKE PROBLEM WRITE-UP

1. How many handshakes does it take for 10 people to meet? Explain using complete sentences, diagrams, and anything else you can think of. Please show off your math skills!
2. How many handshakes does it take for 100 people to meet? Show your work. If you get stuck, explain what got in your way.
3. How many handshakes does it take for n people to meet? Show your work. If you get stuck, explain what got in your way.
4. What strategies did you use in your attempt to solve the problem?
5. How did you feel about your math class today?

Maria's Work

Answer to question 1

It takes 45 handshakes for 10 people to meet.

# of people		# of handshakes
1	+	0
2	+	1
3	+	3
4	+	6
5	+	10
6	+	15
7	+	21
8	+	28
9	+	36
10	+	45

After starting with no people and one handshake you could just add the number of people to the number of handshakes to get your next answer. For example adding two to one to gets three as the number of handshakes for three people.

Answer to question 2

It would take 4950 handshakes for 100 people to meet. I got this using

$$\frac{(n-1)n}{2} = \text{number of handshakes.}$$

I got this because the last person left to shake hands has already met everyone leaving no one for them to meet, making $n - 1$. Then the next person shakes hands, making $n - 2$ people, and so on. For $n = 100$,

$$\frac{(n-1)n}{2}$$

becomes

$$\frac{(100-1)100}{2} = \frac{(99)(100)}{2} = \frac{9900}{2} = 4950.$$

Answer to question 5

It was good.

Fig. 4 Throughout the year, I reminded Maria of her good mathematics on the Handshake problem at the beginning of the year.

Orman's Response

We can see deep confusion right away in Orman's work (see **fig. 5**). He does not seem to understand the problem itself. In the first answer, he presents conflicting solutions (both 5 and 20) and seems to be unclear about the difference between people meeting and people shaking hands. He does have some structure to his work: In two cases, the handshakes are half the number of people. He is glad to have the help of his classmates and teacher.

Orman's Work

Answer to question 1

20. I got that answer by timsing.
(10 • 2 = 20)
If 10 people meet, it will be 5 handshakes.

Answer to question 2

If it's 100 people, you will meet 50 people because 50 + 50 = 100.

Answer to question 5

It was a little difficult, but with help I was able to get through.

Fig. 5 Orman's struggle was apparent from early in the year. He was transferred to a special education math class.

Tasha's Response

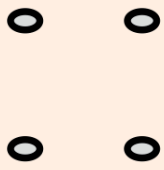
Tasha has a strong understanding of the partial sum method (see **fig. 6**). Her understanding is so sound that she is able to connect the counting pattern back to the actions of people shaking hands. She makes use of the structure of the problem and is able to reason abstractly using the model she developed with her teammates. For the case of 100 people, she perseveres with the partial sum model but uses the associative and commutative properties of addition to reorder her sum into a more useful pattern. Her final solving method of 49 • 100 + 50 is not too far from the formula

$$\left(\frac{n}{2}-1\right)n+\frac{n}{2},$$

although she was not able to generalize to a formula yet. Her method shows creative thinking and excellent uses of mathematical structures.

Tasha's Work

Answer to question 1



4 people

One person shakes 3 hands, the next person shakes 2 hands, and the one left shakes 1 hand.
For 10 people:

1 person	shakes 9 hands
2	8
3	7
4	6
5	5
6	4
7	3
8	2
9	1

Add all the handshakes together to get 45

Answer to question 2

4950
So what I did was wrote out all the combinations there were to make 100 going from 99 all the way down to 51. And those were all the 49 combinations in total. So I multiplied 49 • 100 and got 4900, and then I had 50 left over from the combinations, so I added 50 to 4900 and got 4950.

99 + 1 = 100
98 + 2 = 100
97 + 3 = 100
96 + 4 = 100
95 + 5 +100
94 + 6 = 100
93 + 7 = 100
… + … = 100
53 + 47 = 100
52 + 48 = 100
51 + 49 = 100
50 is left over.

Answer to question 5

I really liked math class today because I really like doing group work. It helps me learn a lot.

Fig. 6 Tasha continued to grow mathematically throughout the year and kept her team quietly focused.