Using Mathematics to Elect the U.S. President

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Electoral College Discussion Guide

INTRODUCING THE ACTIVITY

The mathematical focus of the activity is the necessity of proportional reasoning to make the Electoral College a fair and equitable process for electing the president of the United States. Because students may not be familiar with the Electoral College, begin with a discussion about what the process is and why it exists. Potential questions:

- "Do you know how the president of the United States is elected?"
- "Do you know how leaders in other countries are elected?"
- "Why do you think we use an Electoral College to elect the president?"

Introduce students to the development of the Electoral College using electronic or print resources (e.g., http://www.archives.gov/federal-register/electoral-college/about.html). After students explore multiple methods of voting systems as part of the activity, invite them to summarize why the United States uses the Electoral College process (DQ6). Some key ideas:

- Holding a direct popular vote was difficult at the time of the nation's founding. The absence of political parties meant that a staggering number of candidates could run for president. Also, there was no way to easily communicate throughout the whole country for everyone to know about all the candidates.
- Relying on elected officials in Congress (a vote by representative) was difficult because the elected officials might not actually act on behalf of their represented state.
- The Electoral College was developed as a compromise between these two options.

CLASS CANDY ELECTIONS

To model the reasons why the Electoral College was developed by the nation's Founding Fathers, hold a class candy election, implementing multiple methods and rounds of voting to represent various voting strategies. Students will vote on their favorite type of candy. Candy types should vary (with at least one chocolate option and one fruity option) to encourage lively discussion about the "best" candy being offered. Even if their absolute favorite type of candy is not offered, students still will have to select the best option from the given choices. The three different voting methods, and possible discussion topics, are described (DQ1–DQ3).

Direct Popular Vote

To demonstrate a direct popular vote, allow students to make choices without telling them what candies are being offered and prohibiting them from discussing their choices with their classmates when voting. This lack of information models the absence of political parties and the difficulty of communicating across the nation. Even though this direct popular vote method is not very effective, students may believe it is fair because each person's vote is counted equally; one vote is not weighted more than another. Although this method may seem fair, and arguments against it (no identified political parties and limited communication) are no longer significant in the context of a national election, the U.S. president is not elected using this process. To motivate the need for alternative voting methods, the teacher may want to comment on the lengthy process of counting individual votes or on how tedious it is to keep track of them.

Representative Vote

To demonstrate a second voting method, limit the candy choices offered and arrange students into groups of varying sizes. These groups represent states. One spokesperson for each group (acting as a state representative) casts his or her vote, without consulting the rest of the group. The spokesperson's vote counts as the vote for the entire group. Students will likely think this method is not fair because the spokesperson voted for the entire group without consulting the members of the group. The spokesperson simply chose his or her favorite type of candy while the other members of the group did not get to voice an opinion.

Modified Electoral College Vote

Modify the process, still informing students of the types of candy being offered and arranging them into groups of different sizes. Each group selects its choice cooperatively. Each group gets one vote and the candy with the most votes is declared the winner. Students will recognize that this method does not seem fair, since groups with only a few people will have the same influence as groups with many people. The example outcome presented in the article shows that it is possible for a winning candy to be selected even though the majority of students did not vote for it. If the class vote does not point to such an inequity, teachers can use the given example (or have students invent a hypothetical election result where this concern arises) or they can alter the student groupings and repeat this round of voting.

INTRODUCING PROPORTIONAL REASONING

Students will likely think that the large groups' votes should count more than the small groups' votes. A natural follow-up question might be "How much more?" Make the allocation of votes more concrete by having the total number of votes equal to the number of students in the class (DQ4). Assign the number of votes to each group in correspondence to the number of students in the group. For example, in a class of 20 students, a group of 7 students should receive 7 of the total 20 of votes. This ratio, the number of students in the group compared to the total number of students in the class, will be used in a proportion—an equation of two equal ratios.

Ask students to imagine the situation that a total of 100 votes (DQ5) be distributed. For example, suppose there are 7 students in the group and 20 students in the class. If 100 votes are to be counted, and the number of votes assigned to the group is designated as x, then the following proportion holds:

$$\frac{7}{20} = \frac{x}{100}$$

Students may struggle to create a proportion or to be able to articulate why the two ratios are equal to each other. The solution process should be explored in class discussion. A method of cross-multiplication or other solution strategies, such as using algebraic manipulation, can be used in this example and to answer DQ8 and DQ9. This specific example results in a whole number, 35, for the number of votes given to the group. However, depending on the size of the group and the size of the class, the result may not be a whole number. Suppose there are 6 students in the group and 19 students in the class; the result would be $x = 31.5789474$. Should the group receive 31 votes or 32 votes? The class discussion surrounding the issue of rounding will be relevant again later in the activity, specifically for DQ8, DQ9, and DQ10.

THE ELECTORAL COLLEGE

At this point, prepare students to transition from the class candy election to the presidential election. Three important pieces of information make the voting fair: (1) the total number of votes, (2) the number of students in each group, and (3) the number of students in the whole class. In terms of the Electoral College, the same three pieces of information are essential. Congress has declared the total number of electoral votes to be 538, and the remaining pieces of information come from the U.S. Census (DQ7). Census data provides key pieces of information: the population of each state (which is like the size of the group from the class candy election) and the population of the entire country (which is like the size of the class from the class candy election). Students should use U.S. Census data to calculate the number of electoral votes for their state (DQ8) and the number of electoral votes a populous state, like California, should be allocated (DQ9). If students are from California, they should calculate the number of electoral votes a small (less populous) state should get.

Here, we provide solutions to DQ8 and DQ9, assuming the students are from North Carolina.

- According to the 2010 U.S. Census, North Carolina had a population of 9,535,483 and the United States had a population of 308,745,538. To calculate the number of electoral votes for North Carolina, create a proportion of two equal ratios:

$$\frac{\text{population of North Carolina}}{\text{population of the United States}} = \frac{\text{electoral votes for North Carolina}}{\text{total electoral votes based on population}}$$

$$\frac{9,535,483}{308,745,538} = \frac{x}{435}$$

$$0.0308846 = \frac{x}{435}$$

$$13.4348018 = x$$

  Based on these calculations, a student might conclude that the number of electoral votes allocated to North Carolina would be 15: The votes based on population would be 13 (since 13.4348018 might be rounded down to 13) plus the additional 2 votes allocated to each state because of the state's U.S. senators. However, some students might round up to 14 votes based on population and reach a conclusion that North Carolina is awarded 16 electoral votes.

- According to the 2010 U.S. Census, California had a population of 37,253,956 and the United States had a population of 308,745,538. To calculate the number of electoral votes for California, create a proportion of two equal ratios:

$$\frac{\text{population of California}}{\text{population of the United States}} = \frac{\text{electoral votes for California}}{\text{total electoral votes based on population}}$$

$$\frac{37,253,956}{308,745,538} = \frac{x}{435}$$

$$0.1206623 = \frac{x}{435}$$

$$52.4881136 = x$$

  Based on these calculations, a student might conclude that the number of electoral votes allocated to California would be 55 or 54.

Calculating the number of votes for a particular state will likely bring up a number of mathematical issues in class discussion. Explore some of these issues (DQ10). One possible reason for students to not get an accurate number of electoral votes is because students may forget the 2 additional electoral votes allotted for a state's U.S. senators (two per state). For example, the calculations led to a conclusion that North Carolina should receive 13 electoral votes, but North Carolina actually receives 15 electoral votes.

Students need to decide whether to use 435 (the number of Electoral College votes that are awarded based on population) or 538 (the total number of electoral votes available) in their

calculations. Since the ratios are based on population, 435 is the appropriate total to use. Using 538 as the total would lead to 19 electoral votes for North Carolina (assuming the students round up):

$$\begin{aligned}
&\frac{\text{population of North Carolina}}{\text{population of the United States}} \\
&\qquad = \frac{\text{electoral votes for North Carolina}}{\text{total electoral votes based on population}} \\
&\frac{9{,}535{,}483}{308{,}745{,}538} = \frac{x}{538} \\
&0.0308846 = \frac{x}{538} \\
&16.6159158 = x
\end{aligned}$$

If students proceeded to calculate the number of electoral votes for all fifty states this way, the total would be much greater than 538.

An important mathematical discussion would explore the differences between finding a portion of 435 and then adding 2 versus finding a portion of 437 (435 plus 2).

A third possible reason for inaccuracy is rounding. Students can explore and justify the decisions they made about rounding up or rounding down. Students may think that the rounding is unfair—especially if they are from a state like North Carolina, for which the number of electoral votes is rounded down, because part of their share of the electoral votes is being taken away. Likewise, a state like California is getting "extra" votes based on rounding up at the end of the calculations.

Miscalculation is another possible reason for inaccuracy. Cross-multiplication results in very large numbers, and many students may not know how to appropriately record and make calculations with such large numbers. These calculations lead to an opportunity to discuss large numbers and scientific notation.

Examine the differences in results when comparing state populations. For example, the ratio comparing North Carolina's electoral votes based on population to North Carolina's population could be expected to be equal to the ratio comparing California's electoral votes based on population to California's population. However, this is not the case.

Based on these calculations, a student might conclude that the number of electoral votes allocated to California based on population would be 51:

$$\begin{aligned}
&\frac{\text{electoral votes for North Carolina}}{\text{population of North Carolina}} \\
&\qquad = \frac{\text{electoral votes for California}}{\text{population of California}} \\
&\frac{13}{9{,}535{,}483} = \frac{x}{37{,}253{,}956} \\
&484{,}301{,}428 = 9{,}535{,}483x \\
&50.7893966 = x
\end{aligned}$$

However, the number of electoral votes allocated to California based on population is 53. This outcome leads to an interesting discussion of why these two ratios are not actually equal to each other: that is, they are not directly proportional. One key element of this discussion would be the issue of rounding, especially when working with large numbers.

WRAP UP

After this mathematical discussion, reflect on the entire process: why the United States uses an Electoral College, what the Electoral College is and how it works, and the "fairness" of voting for the president using the Electoral College (DQ11). Students' responses will vary. Many students may champion the Electoral College because it seems to result in equitable partitioning of the votes given to each state. Other students may declare a preference for using a direct popular vote, also an equitable distribution, to elect the president of the United States because the issues that hindered its original use no longer seem to be significant factors. Either way, use the opportunity to show students just how mathematics plays a role important national decisions.

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Electoral College Discussion Questions

Please respond to each question as they come up in our discussion.

1. Direct popular vote: Is this voting method fair? Why or why not? If not, what can we do to make it better?
2. Representative vote: Is this voting method fair? Why or why not? If not, what can we do to make it better?
3. Modified Electoral College vote: Is this voting method fair? Why or why not? If not, what can we do to make it better?
4. If the total number of votes were equal to the number of students in this class, how should we distribute the votes to each group? Be as specific as possible and mathematically justify your reasoning.
5. If the total number of votes were 100, how should we distribute the votes to each group? Be as specific as possible and mathematically justify your reasoning.
6. Turn to a neighbor and discuss why the United States uses an Electoral College voting method, compared to the voting methods used in other countries. Capture one or two of the key ideas from your discussion.
7. The U.S. Census data provides information key to the Electoral College process. Turn to a neighbor and discuss why the U.S. Census data is so helpful. Capture one or two key ideas from your discussion.
8. Using what you know about the Electoral College and the U.S. Census data, calculate the number of electoral votes your state should get. Justify your reasoning mathematically.
9. Using what you know about the Electoral College and the U.S. Census data, calculate the number of electoral votes California should get. Justify your reasoning mathematically.
10. The mathematical process of calculating the number of votes a particular state should get will likely bring up a number of mathematical issues in class discussion. Explore some of these issues:
    a. Explain why you chose 435 or 538 (pick one) as the “total” vote number in your calculations.
    b. Was the number of electoral votes you calculated for your state and for California accurate? Why or why not?
    c. Did you have to round up or down in your calculations? Which way did you choose and why? Is this fair? Why or why not?
    d. Does this voting process favor bigger states, smaller states, or neither? Explain your reasoning.
    e. Are the total votes of California directly proportional to the total votes of your state? Why or why not?
11. Question for reflection: Do you think we should continue using the Electoral College to vote for the president? Why or why not?