Alexis Stevens and John Stevens

Using Mathematics to Elect the U.S. President

How is the president of the United States elected? Why is this the method used? Is this the best and most efficient way of electing the president of the United States? Questions such as these are well suited for a mathematics discussion that promotes numeracy, because, “notwithstanding the immense value of numeracy for education and vocation, its most profound value to society may be the role it plays in supporting informed citizenship and democratic government” (Steen 1999, p. 11).

These questions can help high school students, who are just about to reach voting age, grapple with challenging mathematics that help build numeracy and political issues like voting. They are also important because “school mathematics experiences at all levels should include opportunities to learn about mathematics by working on problems arising in contexts outside of mathematics” (NCTM 2000, pp. 65–66). In this activity, students will be guided through a discussion that helps them make sense of the U.S. Electoral College and how its role in the election process relies on an important mathematical topic: proportional reasoning.

This activity is an excellent opportunity for mathematics, particularly the understanding of proportionality, to “emerge through problem solving and reasoning, [as] it is important [to connect] . . . mathematics and other domains such as science and art” (NCTM 2000, p. 212). Students explore the development of the Electoral College, which is essential to the process of electing the president of the United States. The Electoral College emerged as a compromise between using a direct popular vote or a vote of Congress (in which representatives vote on behalf of their states) to elect the president. Both of these approaches are modeled in the class activity. Each option poses difficulties. A direct popular vote that allows each voter to choose a candidate might lead to an overwhelming number of candidates, for example. In the second option, an elected official who is expected to cast a vote on behalf of the people in his or her state might instead act from a personal agenda. Because neither method was ideal, the Electoral College was devised. (See U.S. National Archives and Records Administration 2016 for further information about the Electoral College.)

The Electoral College has 538 members, and each state plus the District of Columbia sends electors to participate in the process. The number of electors allotted to each state equals the number of members it has in its congressional delegation—the

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The often misunderstood
Electoral College is based
on the simple, yet powerful,
mathematical idea of
proportional reasoning.

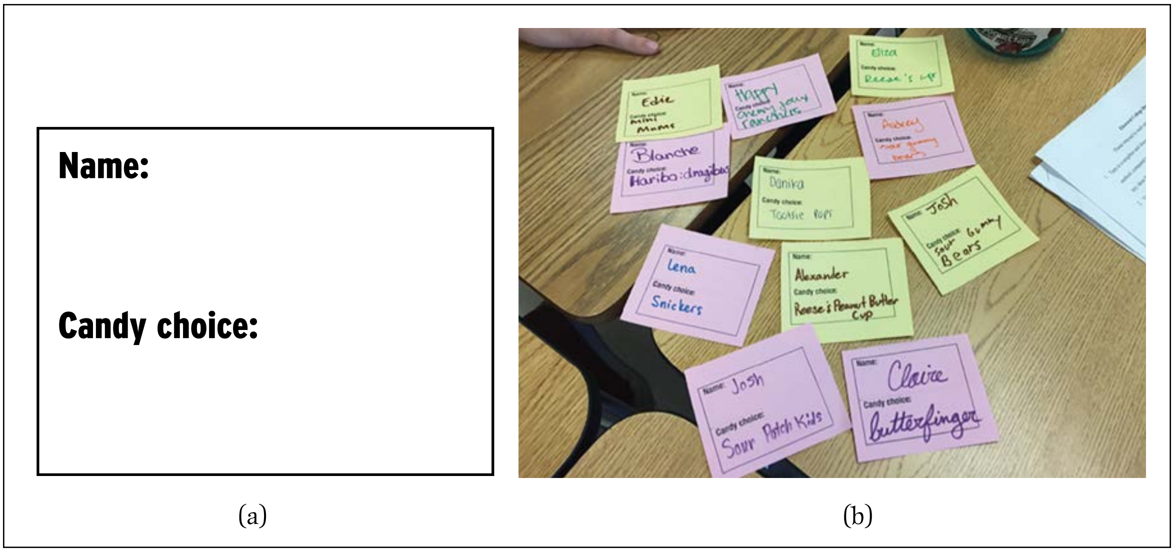


Fig. 1 Ballots like the blank example on the left were used in the direct popular vote for candy. Filled-in ballots (b) indicate a variety of candy "candidates."

number of its members in the House of Representatives plus two for its senators. So, for example, a state with a congressional delegation of 11 House members and 2 senators will have 13 electors. The number of House members a state has is based on its population. (That is, the 435 members of the House of Representatives are apportioned throughout the 50 states based on population.) The District of Columbia has three electors.

ACTIVITY DESCRIPTION

Setting Up

This activity can be done with students as young as early algebra learners, but it is certainly not restricted to them. Older students, especially those who are closer to voting age, can also enjoy engaging in this kind of thinking, as this lesson fits nicely into a data analysis or mathematical reasoning course. Furthermore, the political and historical undertones make this activity an ideal addition to a larger, cross-disciplinary study of elections and voting systems. Because of the emphasis on both the historical discussion and the mathematical exploration of the power of numeracy, this activity will likely last 75 to 90 minutes. If the challenge activities are used for a second day of instruction, they will take between 60 and 75 minutes. A handout that accompanies the activity (see sidebar, "Electoral College Discussion Questions," on p. 196) is referenced at the appropriate places through the analysis of the activity: "DQ4," for example, refers to Discussion Question 4 on the handout. An editable version of the handout and a discussion guide for teachers, including sample calculations and details of potential student errors, are available from the more4U section at **www.nctm.org/mt**.

Let's Vote

Begin the activity by asking students if they know how the president of the United States is elected or if they know how elections are held in other countries. To model the U.S. system of electing the president, students will hold a class election to establish their favorite kind of candy. The winning candy will be shared with the class at the end of the lesson. Be prepared with three or four types of candy to offer to students, but do not identify these until after the first round of voting. Choosing favorites can evoke strong feelings, so the activity works best with a variety of candy choices, such as chocolate, chewy, peanut butter, and fruit options (e.g., M&M's®, gummy bears, Reese's® Peanut Butter Cups, and Skittles®). Multiple rounds of voting will explore the "fairness" of different methods as students try to elect the best type of candy.

Direct Popular Vote. The first round of voting will model the direct popular vote, in which each person gets one vote and each vote is weighted equally. Without being told what types of candy are being offered and without discussing their candy choice with others in the classroom, students individually vote for one and only one type of candy—any candy of their choice—on a secret ballot (**fig. 1a**). The ballots are collected and tallied to determine the winning candy. The candy choices on the ballots will be highly varied, as seen in the example class candy election (**fig. 1b**). This voting method replicates some of the challenges of the direct popular vote that the Founding Fathers anticipated: no choice emerging with a sufficient majority, and the difficulty of gathering information about candidates who were not nearby or locally known (i.e., the "unknown candy candidates").

Table 1 Class Candy Elections						
Group	A	B	C	D	E	F
Candy Choice	M&M's	Skittles	M&M's	Reese's Peanut Butter Cups	M&M's	Skittles
Size of Group	3	4	3	9	2	6

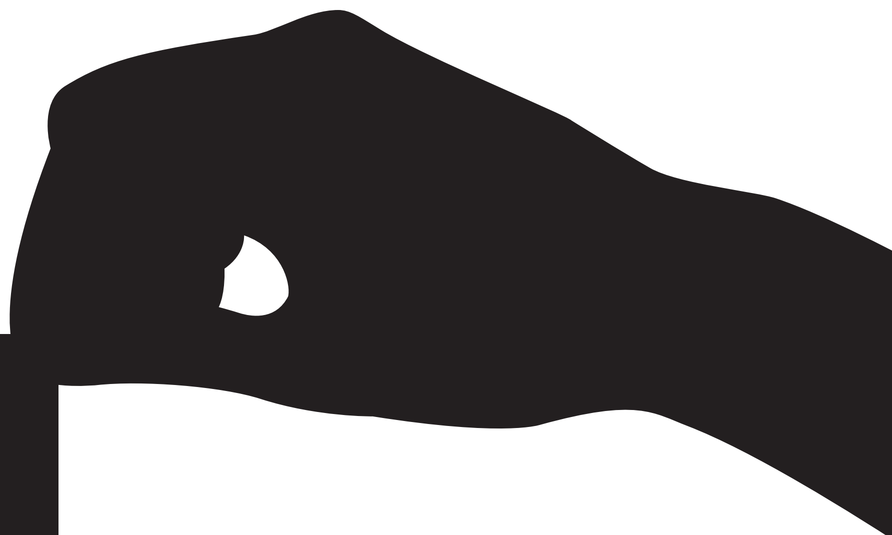
Direct students to reflect on the "fairness" of this process (DQ1). The direct voting method may seem to be fair because each person has his or her voice heard during the vote and each person's vote is counted equally; one person's vote is not worth more than another's. However fair the process may seem, teachers can guide students to see that this is an ineffective method of voting. A majority of students may prefer anything chocolate, but when given free rein to write in a specific kind of chocolate treat, their nuanced choice of chocolate candy loses to the more consistent categorical choice of "gummy worms." Invite students to think of simple hypothetical examples to share with the class: Can the generally preferred category of candy lose overall because of the variability within that category? Use this dialogue to establish the role of political candidates in a voting system. In the candy context, "political candidates" would be M&M's, gummy bears, Reese's Peanut Butter Cups, and Skittles (or whatever other candies you have selected).

Representative Vote. The second round of voting will model representative voting, in which an elected representative casts a vote on behalf of an entire state. At this point, announce the "political candidates" of candy types that the class will vote on. For this voting method, arrange students into groups. In order to expose one of the weaknesses of this method, groups should vary in size. Have each group select one of its members to act as the group's spokesperson, who then immediately casts a vote for his or her favorite type of candy (from among those "candidates" provided) *without consulting the rest of the group*. This counts as the entire group's vote. The candy that receives the most votes from the spokespersons is the winning candy for the whole class.

Encourage students to reflect on the "fairness" of this process (DQ2). Students, particularly those who were not the spokesperson for their group, will likely be outraged at this method of voting. The obvious inequity of the vote by representatives who do not consult their groups further motivates the need to find a more suitable method of voting.

Modified Electoral College Vote. The activity's third voting method, as a compromise between the previous two, will model the development of the Electoral College. Using the groups formed in the last round, each spokesperson, acting as an "elector," should interview the other group members and establish the group's top choice of candy. The elector will announce the group's choice to the class, which will count as the entire group's vote. If a tie arises in any group, the group itself will decide how to determine the group's choice. This is consistent with the Electoral College policy of letting state law determine how to handle a tie within the state's popular vote (http://www.archives.gov/federal-register/electoral-college/faq.html#statetie).

The candy that receives the most votes from the electors is the winning candy. For example, consider the possible outcome in the election between M&M's, Skittles, and Reese's Peanut Butter Cups (see **table 1**). Because M&M's received the most



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Three important pieces of information are used to determine how many electors each state sends to the Electoral College.

votes from the electors (three groups voted for M&M's; two for Skittles; one for Reese's), it is the winning candy.

Again, students reflect on the "fairness" of process (DQ3). Students will likely respond that this method does not seem fair because the groups are different sizes. The small groups, with just a few students in them (e.g., Group E), were allotted one elector, the same as the large groups, with many students in them (e.g., Group D). To further illustrate the inequity of this voting method, multiple iterations of this voting method could be implemented. For example, the first voting round of this method could consist of a mix of small, medium, and large groups. The second voting round could consist of mostly small groups and just one large group. The third could consist of mostly medium groups with just one small group and one large group. This type of voting moves in the direction of the Electoral College but lacks the essential foundation of proportional reasoning.

Assuming that students will claim that this method of voting is not fair because the small groups' votes weighed as much as the large groups' votes, follow up by asking, "If you think the large groups' votes should count more than the small groups' votes, how much more should they count?" To be more specific, suppose that the total number of votes is equal to the number of students in the class. Ask students, "How should we distribute the votes to each group in this case?" (DQ4). To help facilitate this discussion, focus on the bottom row of **table 1**, which reports the group size. For example, Group F has six people in it; therefore, Group F should get six of the total votes. Students will probably claim that this is fairer because it is proportional. Ask: "What does it mean to be proportional? What makes this proportional?" (Note that with this method, Skittles wins the election, based on the information in **table 1**, even though only two groups voted for Skittles and three groups voted for M&M's.)

Now suppose that exactly 100 votes for candy must be received. Ask students, "How should we distribute the votes to each group in this case?" (DQ5). In this case, students will actually need to

Electoral College Discussion Questions

Please respond to each question as they come up in our discussion.

1. Direct popular vote: Is this voting method fair? Why or why not? If not, what can we do to make it better?
2. Representative vote: Is this voting method fair? Why or why not? If not, what can we do to make it better?
3. Modified Electoral College vote: Is this voting method fair? Why or why not? If not, what can we do to make it better?
4. If the total number of votes were equal to the number of students in this class, how should we distribute the votes to each group? Be as specific as possible and mathematically justify your reasoning.
5. If the total number of votes were 100, how should we distribute the votes to each group? Be as specific as possible and mathematically justify your reasoning.
6. Turn to a neighbor and discuss why the United States uses an Electoral College voting method, compared to the voting methods used in other countries. Capture one or two of the key ideas from your discussion.
7. The U.S. Census data provides information key to the Electoral College process. Turn to a neighbor and discuss why the U.S. Census data is so helpful. Capture one or two key ideas from your discussion.
8. Using what you know about the Electoral College and the U.S. Census data, calculate the number of electoral votes your state should get. Justify your reasoning mathematically.
9. Using what you know about the Electoral College and the U.S. Census data, calculate the number of electoral votes California should get. Justify your reasoning mathematically.
10. The mathematical process of calculating the number of votes a particular state should get will likely bring up a number of mathematical issues in class discussion. Explore some of these issues:
    a. Explain why you chose 435 or 538 (pick one) as the "total" vote number in your calculations.
    b. Was the number of electoral votes you calculated for your state and for California accurate? Why or why not?
    c. Did you have to round up or down in your calculations? Which way did you choose and why? Is this fair? Why or why not?
    d. Does this voting process favor bigger states, smaller states, or neither? Explain your reasoning.
    e. Are the total votes of California directly proportional to the total votes of your state? Why or why not?
11. Question for reflection: Do you think we should continue using the Electoral College to vote for the president? Why or why not?

Another possible extension of this activity would be to investigate the difference in the results between the direct popular vote and the Electoral College vote. There has been a difference between them only four times in U.S. history.

start making sense of and calculating proportions. Students should share their process, procedures, and ways of thinking. They should elaborate on what they did and how they know it would result in something fair.

The students will have just demonstrated the way the Electoral College works. Each state's number of electoral votes is in proportion to its population relative to the population of the country—just as a group's number of electoral votes is in proportion to the size of the group relative to the whole class.

THE ELECTORAL COLLEGE

Now that students have seen voting methods that were either not efficient (direct popular vote) or not fair (representative vote) as well as modifications based on proportional reasoning to suggest the origin of the Electoral College voting system, summarize the key ideas of the historical development of the Electoral College (using references in the online teacher discussion guide, available at **www.nctm.org/mt**) and have students reflect on this process (DQ6).

In the class candy election, we needed to know three important pieces of information to make the voting fair: the total number of votes; the number of students in each group; and the number of students in the whole class. These are the same three key pieces of information used in determining how many electors each state sends to the Electoral College.

Give students a chance to examine the U.S. Census data, either using the U.S. Census website (U.S. Census Bureau 2016) or a printed handout with relevant information and graphics (DQ7). Then, challenge students to calculate the number of electoral votes their state should get (DQ8). Next, have students calculate the number of electoral votes of California, a state with a large population. If the students are from California, have them choose a state with a small population for comparison (DQ9).

These questions will likely bring up important mathematical issues related to proportionality. Key mathematical ideas that arise from Discussion Question 10 include the following:

- What are you considering to be the total number of electoral votes, 435 or 538? Why did you choose this total?
- Did you accurately calculate the number of electoral votes for your state and for California? Why or why not?
- Did you have to round up or down in your calculations? Which way did you choose and why? Is this fair? Why or why not?
- Does this voting process favor bigger states, smaller states, or neither?
- Are the total votes of California directly proportional to the total votes of your state? Why or why not?

These questions are explored in the discussion guide for teachers, which can be downloaded from **www.nctm.org/mt**.

WRAP UP

After this mathematical discussion, allow students to reflect on the entire process—why the United States uses an Electoral College, what the Electoral College is and how it works, and the "fairness" of

The political and historical undertones make this activity an ideal addition to a larger, cross-disciplinary study of elections and voting systems.

voting for the president using the Electoral College (DQ11). Students' responses will vary, but this is an excellent opportunity to show students how mathematical ideas, like proportional reasoning, help determine important national decisions. "Effective teachers understand how contexts, culture, conditions, and language can be used to create mathematical tasks that draw on students' prior knowledge and experiences . . . leading to increasing engagement and motivation in mathematics" (NCTM 2014, p. 17).

For an additional challenge, as homework, or as the next class's activity, students can use PBS NewsHour's electoral calculator map (PBS News Hour 2012) from the 2012 presidential election to calculate just how important the number of electoral votes for each state are in determining the next U.S. president. Students can also tackle the cross-multiplication algorithm, justifying why it works (see Boston, Smith, and Hillen 2003). Another possible extension of this activity would be to investigate the difference in the results between the direct popular vote and the Electoral College vote; there has been a difference between them only four times in U.S. history: 1824, 1876, 1888, and 2000. This extension would connect to the different voting methods that introduced the activity.

REFERENCES

Boston, Melissa, Margaret Smith, and Amy Hillen. 2003. "Building on Students' Intuitive Strategies to Make Sense of Cross Multiplication." *Mathematics Teaching in the Middle School* 9 (3): 150–55.

National Council of Teachers of Mathematics (NCTM). 2000. *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

———. 2014. *Principles to Actions: Ensuring Mathematics Success for All*. Reston, VA: NCTM.

PBS NewsHour. 2012. "Vote 2012: Electoral Calculator." http://www.pbs.org/newshour/spc/vote2012/map/calc.html

Steen, Lynn Arthur. 1999. "Numeracy: The New Literacy for a Data-Drenched Society." *Educational Leadership* 57 (2): 8–13.

U.S. Census Bureau. 2016. "United States QuickFacts." http://www.census.gov/quickfacts/map/PST045215/00

U.S. National Archives and Records Administration. 2016. "What Is the Electoral College?" http://www.archives.gov/federal-register/electoral-college/about.html

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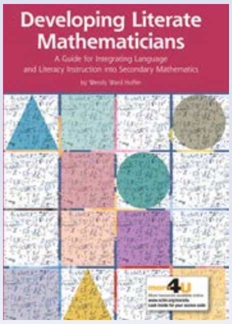

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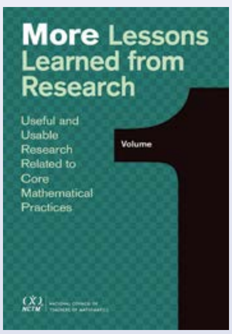

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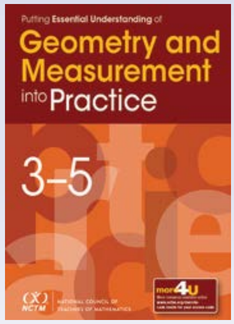

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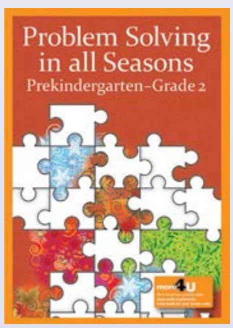

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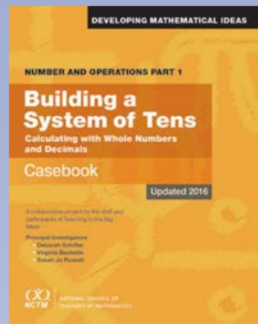

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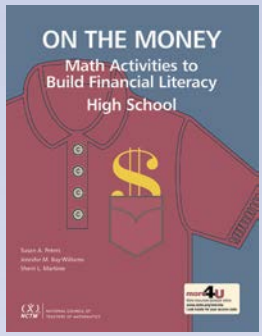

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