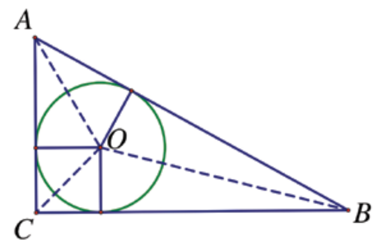
SOLUTIONS to calendar

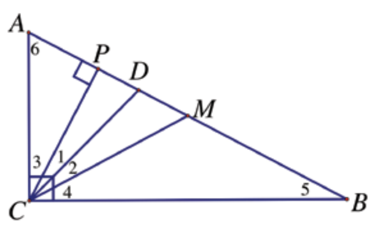
1. 6. The area of a triangle, say $\triangle ABC$, equals the sum of the areas of the three triangles formed by joining the center of the inscribed circle, O, to each of the three vertices of the triangle. That is,

$$A_{\triangle ABC} = A_{\triangle AOC} + A_{\triangle AOB} + A_{\triangle BOC}$$
$$= (r/2)(AC + AB + BC)$$

where r is the measure of the inradius. Notice that $AC + AB + BC$ is the perimeter of the triangle, so the area of $\triangle ABC$ is $(1/2)(12) = 6$. **Note:** By the same reasoning, any polygon with incircle of radius r and semiperimeter s has area rs.

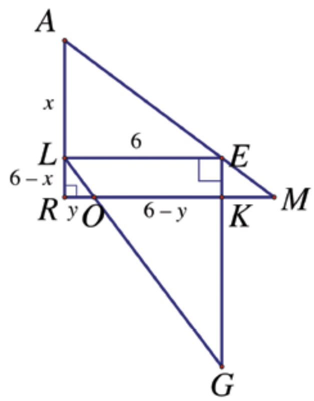


2. Because M is a midpoint, $CM = MB$. In isosceles $\triangle CMB$, the base angles $\angle 4$ and $\angle 5$ are congruent. But $\angle 3 \cong \angle 5$ because each angle is complementary to $\angle 6$. Thus, $\angle 3 \cong \angle 4$. Because $\overline{CD}$ bisects right angle ACB we have the congruence $\angle ACD \cong \angle BCD$ and by subtraction of the angle measures, $\angle 1 \cong \angle 2$. Thus $\overline{CD}$ bisects $\angle PCM$. (Note that an immediate consequence of this result is that the angle bisector of the right angle in a right triangle is a line of symmetry for the lines containing the altitude and the median drawn to the hypotenuse.)



3. $261/32 = 8.15625$. Based on the given information and AA similarity, $\triangle ALE \sim \triangle ARM$. Let $x = AL$ so $x/6 = 6/8$; then $x = 9/2$. Since $\angle RLO$ and $\angle G$ are each complementary to $\angle GLE$, we have $\angle G \cong \angle RLO$. Thus $\triangle LEG \sim \triangle ORL$. Let $y = RO$; then $(6 - x)/y = 4/3$ so $y = 9/8$ and $6 - y = 39/8$. The area of $KOLE$ is $(1/2) \cdot (3/2) \cdot (6 + 39/8) = 261/32 = 8.15625$.

***Alternative solution*:** Since alternate interior angles of parallel lines are congruent and since $\overline{LE} \parallel \overline{RM}$, we have $\angle ROL \cong \angle OLE$ and the similarity of two right triangles, $\triangle ORL \sim \triangle LEG$. The scale factor of $\triangle GKO$ to $\triangle GEL$ is $(39/8)/6 = 39/48$. The ratio of the areas of these two triangles is therefore $(39/48)^2$. Thus, the area of trapezoid $KOLE$ is $(1/2) \cdot 6 \cdot 8 \cdot (1 - (39/48)^2) = 8.15625$.



4. 84. The altitude drawn to the 14-unit side is 12, as can be found using the Pythagorean theorem. Solving

$$AD^2 = 13^2 - x^2 = 15^2 - (14 - x)^2$$

gives $x = 5$ so that $AD^2 = 144$. Thus, the area is $(1/2) \cdot 12 \cdot 14 = 84$. Notice that the 13-14-15 triangle is special because it is the result of placing a 5-12-13 triangle adjacent to a 9-12-15 triangle so that the 12-unit side is common to the two triangles.

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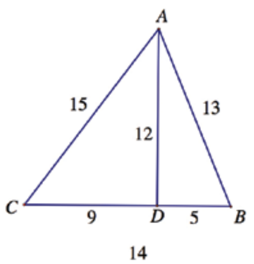
***Alternative solution*:** Apply Heron's formula

$$A=\sqrt{s(s-a)(s-b)(s-c)}$$

where s is the semiperimeter. Since $s = (a + b + c)/2 = (13 + 14 + 15)/2 = 21$, we have

$$A=\sqrt{21 \cdot 8 \cdot 7 \cdot 6}=84.$$



5. $\left(30\sqrt{61}/61\right) \approx 3.8411.$ The hypotenuse is $\sqrt{5^2+6^2} = \sqrt{61}$. Label the two parts of the hypotenuse x and $\sqrt{61}-x$. Then apply the Pythagorean theorem to the two right triangles formed by the altitude to get

$$h^2=25-x^2=36-\left(\sqrt{61}-x\right)^2.$$

Solve for x and substitute to find h, the hypotenuse.

***Alternative solution*:** The sum of the squares of the reciprocals of the legs of a right triangle equals the square of the reciprocal of the altitude drawn to the hypotenuse:

$$\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{h^2}$$

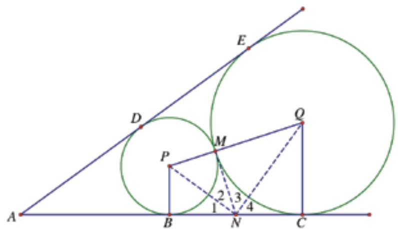
Therefore

$$\frac{1}{h^2}=\frac{1}{25}+\frac{1}{36}=\frac{61}{900},$$

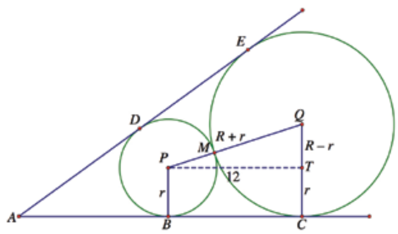
$$\text{so } h=\sqrt{\frac{900}{61}}=\frac{30\sqrt{61}}{61}.$$

***Second alternative solution*:** The area of the triangle can be calculated using $(1/2)5 \cdot 6$ or $(h/2)\sqrt{61}$. Setting these equal we get $h = 30/\sqrt{61} = 30\sqrt{61}/61$.

6. 36. Construct the common tangent to the two circles through M. Label the intersection with $\overline{BC}$ point N. Since tangent segments to a circle are congruent, $NB = NM = NC = 6$. By hypotenuse-leg congruence, we see that $\triangle PBN \cong \triangle PMN$ so $\angle 1 \cong \angle 2$. Similarly, $\angle 3 \cong \angle 4$. Since the four angles sum to 180°, we have $m\angle 2 + m\angle 3 = 90°$ and $\angle PNQ$ is a right angle. The altitude to the hypotenuse of a right triangle is the mean proportional between the two segments of the hypotenuse so $PM/6 = 6/MQ$, or $PM \cdot MQ = 36$.



***Alternative solution*:** Let $r = PB$ and $R = QC$. Drop a perpendicular from P to segment QC. and label the intersection point T. Then right triangle PTQ has sides of length 12, $R - r$, and $R + r$. Apply the Pythagorean theorem to get $12^2 + (R - r)^2 = (R + r)^2$, resulting in $Rr = 36$.

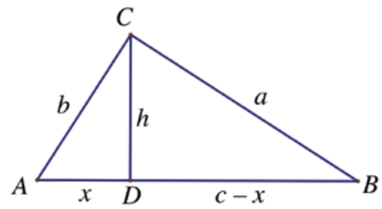


7. The proof of the statement is based upon two relationships concerning the altitude h drawn to the hypotenuse of length c, creating two segments of lengths x and $c - x$. These theorems state:
(1) Either leg is the mean proportional between the segment of the hypotenuse adjacent to the leg and the entire hypotenuse [i.e., $a^2 = c(c - x)$ and $b^2 = cx$] and
(2) The altitude to the hypotenuse is the mean proportional between the two segments of the hypotenuse [$h^2 = x(c - x)$].
Therefore

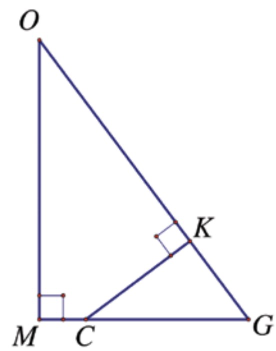
$$\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c(c-x)}+\frac{1}{cx}$$

$$=\frac{x+(c-x)}{cx(c-x)}=\frac{1}{x(c-x)}$$

$$=\frac{1}{h^2}.$$



8. 294/25 = 11.76. There are many different approaches to solve this problem. One method is to apply the theorem that the area ratio of similar triangles equals the square of the length ratio of their corresponding sides. Notice that $\triangle OMG \sim \triangle CKG$ since they are both right triangles and they share $\angle G$. The area of $\triangle OMG$ is $9 \cdot 12/2 = 54$. Thus $A_{\triangle CKG}/54 = (7/15)^2 = 49/225$ so $A_{\triangle CKG} = 294/25$.

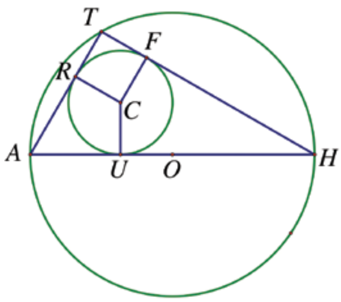


9. $(49\pi\sqrt{3}/2)$. Circle O, with radius 7, has area 49π. Since $AT = AH/2$ and $\angle T$ is a right angle, we have $m\angle A = 60°$ and $HT = 7\sqrt{3}$. Let the inscribed circle, centered at C, have radius r. Reason that $TF = r$, $RA = 7 - r$, and $FH = 7 + r$ (because $AU = AR$ and $FH = UH = AH - AU = 14 - (7 - r)$, based upon the equal lengths of tangent segments drawn from a point outside the circle). Therefore, $TH = TF + FH = r + (7 + r) = 7\sqrt{3}$ so $r = (7\sqrt{3}-7)/2$. The area of the inscribed circle is

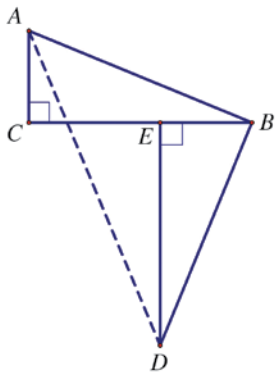
$$\pi\left(\frac{7\sqrt{3}-7}{2}\right)^2=\frac{\pi\left(98-49\sqrt{3}\right)}{2}$$

so the difference in areas is

$$49\pi-\left(\frac{98\pi-49\pi\sqrt{3}}{2}\right)=\frac{49\pi\sqrt{3}}{2}.$$



10. 26. Compute AB by applying the Pythagorean theorem: $AB^2 = 7^2 + 17^2 = 338 = BD^2$. Notice that $\angle ABC$ and $\angle EBD$ are complementary so $\angle ABD$ is a right angle and $\triangle ABD$ is a right isosceles triangle. Then $AD^2 = 2AB^2 = 676$, so $AD = 26$.



***Alternative solution*:** Superimpose a rectangular coordinate system, so that C is the origin, A has coordinates (0, 7), and D has coordinates (10, –17). By the distance formula, $AD^2 = (10 - 0)^2 + (-17 - 7)^2 = 676$, so $AD = 26$.

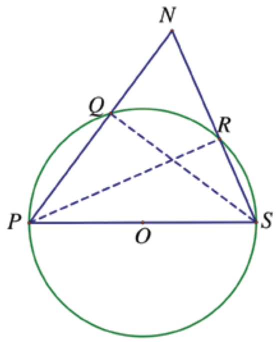
11. $56 + 4\sqrt{21}$. Find chord lengths $QS = \sqrt{25^2 - 15^2} = 20$ and $PR = \sqrt{25^2 - 10^2} = 5\sqrt{21}$. Since $\angle NQS$ and $\angle NRP$ are right angles, we have $\triangle NQS \sim \triangle NRP$. Let $x = NQ$ and $y = NR$ then

$$\frac{x}{20} = \frac{y}{5\sqrt{21}}, \text{ giving } x = \frac{4y}{\sqrt{21}}.$$

Apply the Pythagorean theorem to $\triangle NQS$ so $x^2 + 20^2 = (y + 10)^2$. Replace x^2 with $16y^2/21$ and combine terms to get $5y^2 + 420y - 6300 = 0$. Equivalently, $y^2 + 84y - 1260 = 0$ so $y = 12\sqrt{21} - 42$. Substitute to find that $x = 48 - 8\sqrt{21}$. The perimeter of $\triangle PNS$ is

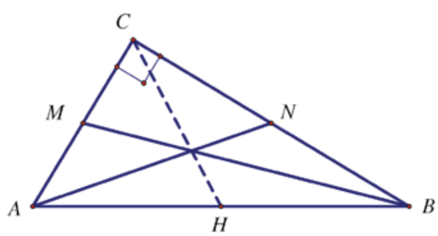
$$\begin{aligned} &x + y + 10 + 15 + 25 \\ &= \left(12\sqrt{21} - 42\right) + \left(48 - 8\sqrt{21}\right) + 50 \\ &= 56 + 4\sqrt{21}. \end{aligned}$$

***Alternative solution*:** Instead of applying the Pythagorean theorem to $\triangle NQS$, use the fact that $NQ \cdot NP = NR \cdot NS$. This process is left up to the reader.



12. 136 – 255 – 289. Find the length of the altitude to the hypotenuse of the given right triangle. One method to do this uses two ways to calculate the area of the triangle. That is, the product of the leg lengths divided by 2 must equal the hypotenuse length times its altitude divided by 2, so $8 \cdot 15 = 17a$, where a is the altitude to the hypotenuse. Thus $a = 120/17$. Scaling the triangle's sides by 17 would make the altitude 120. **Note:** Any multiple of 17 will work as well.

13. $3\sqrt{5}$. Let $AC = b$, $BC = a$, $AB = c$ and let M and N be midpoints of their respective legs. Since $\triangle BMC$ and $\triangle ANC$ are right triangles, $(b/2)^2 + a^2 = 12^2 = 144$ and $(a/2)^2 + b^2 = 9^2 = 81$. Add these equations together to get $(5b^2/4) + (5a^2/4) = 225$. Equivalently, $a^2 + b^2 = 225(4/5) = 180$. But $a^2 + b^2 = c^2$ so $c = \sqrt{180} = 6\sqrt{5}$. The median to the hypotenuse is half the hypotenuse so the median is $3\sqrt{5}$.

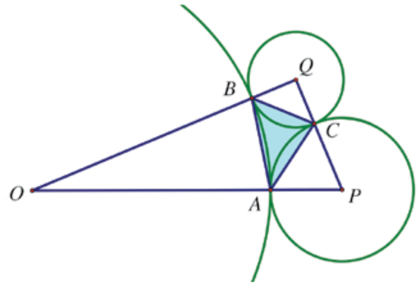


14. $60/13 \approx 4.6154$. Label the centers of the circles O, P, and Q and the points of tangency as A, B, and C. The sides of $\triangle OPQ$ have lengths $10 + 3 = 13$, $10 + 2 = 12$, and $3 + 2 = 5$ and form a 5-12-13 right triangle. The area of $\triangle OPQ$ is $5 \cdot 12/2 = 30$. To find the area of $\triangle ABC$, we can subtract the sum of the areas of the three surrounding triangles. The area of each of these can be found using the formula $A = (1/2)ab \cdot \sin C$. We have $\sin O = 5/13$, $\sin P = 12/13$ and $\sin Q = 1$. So

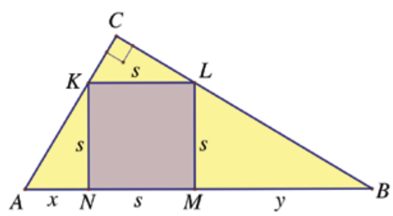
$$\begin{aligned} A_{\triangle OBA} &= (1/2) \cdot 10 \cdot 10 \cdot (5/13) = 250/13, \\ A_{\triangle PAC} &= (1/2) \cdot 3 \cdot 3 \cdot (12/13) = 54/13, \text{ and} \\ A_{\triangle QBC} &= (1/2) \cdot 2 \cdot 2 \cdot 1 = 2. \end{aligned}$$

Thus

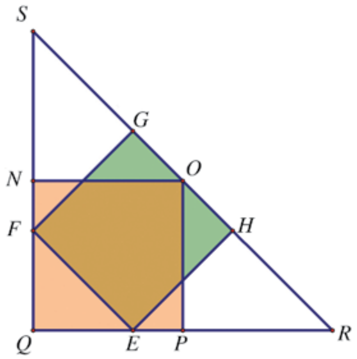
$$\begin{aligned} A_{\triangle ABC} &= 30 - (250/13 + 54/13 + 2) \\ &= 30 - 330/13 = 60/13. \end{aligned}$$



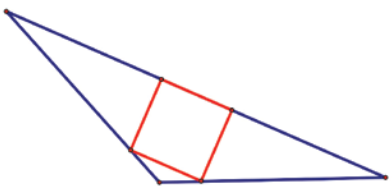
15. $s = abc/(c^2 + ab)$. The key to the solution is to write the hypotenuse length as a sum, $c = x + s + y$, and then find expressions for x and y in terms of the requested labels. In $\triangle ABC$, $\tan A = a/b$ and $\tan B = b/a$. In $\triangle AKN$, $\tan A = s/x$ and in $\triangle BLM$, $\tan B = s/y$. Since $\tan A = a/b = s/x$, we have $x = bs/a$. Similarly we find that $y = as/b$. Substitute these results into the first equation to get $c = bs/a + s + as/b$. Factor and isolate s to get $s = abc/(a^2 + ab + b^2) = abc/(c^2 + ab)$.

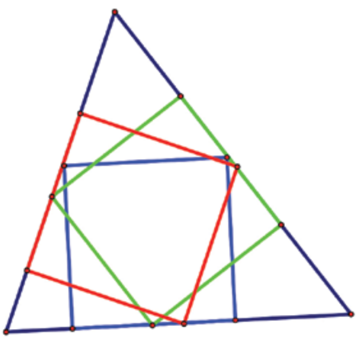


16. 8:9. Since $\triangle QRS$ is a right isosceles triangle and $EFGH$ is a square, $\triangle RHE$ is also a right isosceles triangle so $HR = HE = HG = GS = SR/3$. Without loss of generality, let $SQ = 6$; then $SR = 6\sqrt{2}$ and the area of square $EFGH = (2\sqrt{2})^2 = 8$. The area of square $NOPQ$ is one-half the area of $\triangle QRS$, that is, $(1/2)[(1/2)(6 \cdot 6)] = 9$. The ratio of the areas of the two squares is 8:9. Readers can compare this to the result of problem 15.



17. The minimum is 1 and the maximum is 3. Since all the vertices of the square must lie on the sides of the triangle, two vertices must lie on the same side. Since a triangle has three sides, there are three possible arrangements for the square when the triangle is acute but only one possible arrangement when the triangle contains an obtuse angle.





18. 0.127%. Calculate percentage error as the estimated value minus the exact value with the difference then divided by the exact value. Convert 5° into radians using $5\pi/180 \approx 0.0872664626$ to find $\sin 5° \approx 0.0871557427$. Therefore the percentage error is $(5\pi/180 - \sin(5\pi/180))/\sin(5\pi/180) \approx 0.0012703678$, which indicates that the approximation overestimates the exact value. **Note:** For small values of x, the $\sin(x)$ is very close to the value of x.

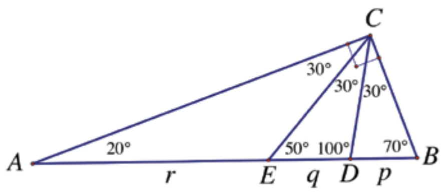
19. $AE = 6.13$, $ED = 2.13$, and $DB = 1.74$. From the fact that $\triangle ABC$ is a right triangle, we have $\sin 20° = CB/10$ so that $CB = 10 \sin 20°$ and $\cos 20° = AC/10$ so that $AC = 10 \cos 20°$. Apply the law of sines in $\triangle BCD$:

$$\frac{p}{\sin 30°} = \frac{10 \sin 20°}{\sin 80°} \text{ to find } p \approx 1.7365$$

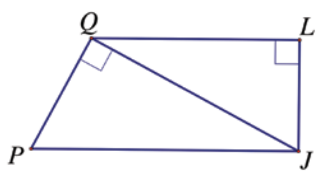
Apply the law of sines in $\triangle ACE$:

$$\frac{r}{\sin 30°} = \frac{10 \cos 20°}{\sin 130°} \text{ to find } r \approx 6.1334$$

Since $p + q + r = 10$, we have $q \approx 10 - (6.13 + 1.74) = 2.13$.

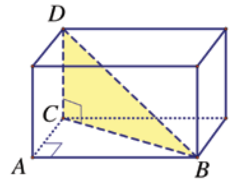


20. The trapezoid whose diagonal divides it into two right triangles is either a parallelogram or not. If the trapezoid is a parallelogram, the resulting triangles are congruent. Otherwise, let $\overline{QL}$ and $\overline{PJ}$ be parallel but $\overline{PQ}$ and $\overline{JL}$ be nonparallel. If $\angle PQJ$ and $\angle L$ are right angles, show $QJ^2 = QL \cdot PJ$. Since $\angle LQJ \cong \angle QJP$ (as they are alternate interior angles), we have $\triangle LQJ \sim \triangle QJP$. Corresponding sides of similar triangles are proportional so $LQ/QJ = QJ/JP$. Equivalently, $QJ^2 = QL \cdot PJ$, or $QJ = \sqrt{QL \cdot PJ}$, which means QJ is the geometric mean of the parallel sides of the trapezoid.



21. 11. The result is based upon an extension of the Pythagorean theorem. In a right rectangular prism with dimensions a, b, and c, the length of the space diagonal, d, is found by applying the formula $d^2 = a^2 + b^2 + c^2 = 2^2 + 6^2 + 9^2 = 4 + 36 + 81 = 121$. Therefore $d = 11$.

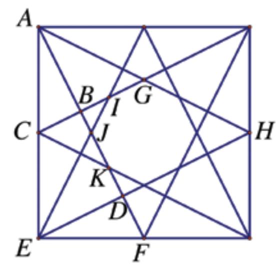
***Alternative solution*:** The space diagonal connects B to D. It is the hypotenuse of right trinagle BCD and $\overline{BC}$ is the hypotenuse of right triangle ABC. If $AC = 2$, $AB = 9$, and $CD = 6$, then $BC^2 = 2^2 + 9^2 = 85$ and $BD^2 = 85 + 6^2 = 121$ so $BD = 11$.



22. 2. In right triangle ABC where $\angle C$ is the right angle, let $BC = a$, $AC = b$, and $AB = c$. Then $\sin A = a/c$, $\sin B = b/c$, and $\sin C = 1$. Therefore $\sin^2 A + \sin^2 B + \sin^2 C = (a/c)^2 + (b/c)^2 + 1^2 = (a^2 + b^2)/c^2 + 1$. Since $a^2 + b^2 = c^2$, the total is 2.

***Alternative solution*:** Apply the fact that $\sin(90° - A) = \cos A$ to write $\sin^2 A + \sin^2(90° - A) + \sin^2 90° = \sin^2 A + \cos^2 A + 1 = 1 + 1 = 2$.

23. 56. There are eight triangles of equal size for each of the following seven triangles: $\triangle ABC$, $\triangle ADE$, $\triangle AFE$, $\triangle ABG$, $\triangle ADH$, $\triangle BIJ$, and $\triangle CBK$. (Each triangle and its reflection across a diagonal of the square can be rotated around the square's center to three other positions, for a total of eight congruent triangles.)



24. 53/45. Since the areas of the squares are 81 and 144, the side lengths of the squares are 9 and 12. But AB and BC are the lengths of the legs of right triangle ABC. Since $EC = 3$ and $\overline{EN} \parallel \overline{AB}$, we can write $3/12 = EN/9$ so $EN = 9/4$. Then

$$area(\triangle ECN) = (1/2) \cdot (9/4) \cdot 3 = 27/8.$$

It follows that

$$\begin{aligned} &area\ (AGFCND) \\ &\quad = area(GBCF) - area(ABED) \\ &\qquad - area(\triangle ECN) \\ &\quad = 144 - 81 - 27/8 = 477/8 \end{aligned}$$

and

$$\begin{aligned} area(ABEN) &= area(\triangle ABC) - area(\triangle ECN) \\ &= (1/2) \cdot 12 \cdot 9 - 27/8 = 405/8. \end{aligned}$$

Thus

$$\frac{area(AGFCND)}{area(ABEN)} = \frac{477/8}{405/8} = \frac{53}{45}.$$

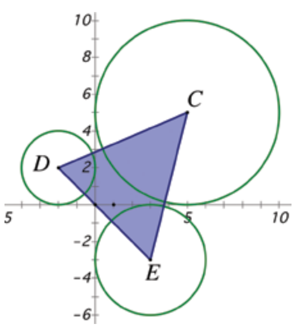
25. 25. Use completing the squares to find the centers of the circles. The three equations become

$$\begin{aligned} (x+2)^2 + (y-2)^2 &= 4, \\ (x-5)^2 + (y-5)^2 &= 25, \\ \text{and } (x-3)^2 + (y+3)^2 &= 9 \end{aligned}$$

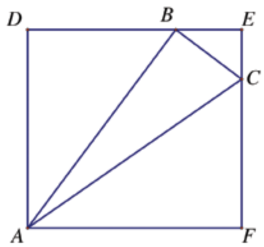
so the centers are (–2, 2), (5, 5), and (3, –3). Use one of several techniques to find the area of the triangle. One standard approach is to draw a rectangle using vertical and horizontal lines that passes through all the vertices of the triangle. Then find the area of the rectangle and subtract the areas of the three surrounding right triangles. The result is

$$\begin{aligned} 7 \cdot 8 - (5 \cdot 5/2 + 3 \cdot 7/2 + 2 \cdot 8/2) \\ = 56 - 31 = 25. \end{aligned}$$

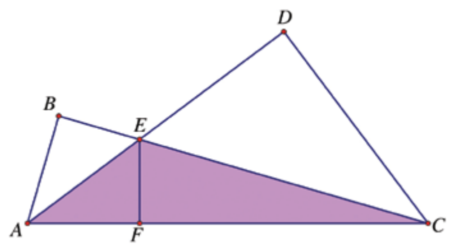
(You might want to check out the "shoelace" theorem to find the area of a polygon given the coordinates of the vertices.)



26. $5\sqrt{10}$. Angle B in $\triangle ABC$ is a right angle so $\angle DBA$ and $\angle EBC$ are complementary. Angle D is a right angle so $\angle DBA$ and $\angle DAB$ are complementary. Therefore $\angle EBC \cong \angle DAB$ and $\triangle EBC \sim \triangle DAB$. Let $x = BE$ and $y = CE$ then $y/x = (13 - x)/12$. Furthermore, by the Pythagorean theorem, $x^2 + y^2 = 25$. Solving this system of equations yields the fourth-degree polynomial equation $x^4 - 26x^3 + 313x^2 - 3600 = 0$. This can be solved by factoring as $(x - 4) \cdot (x + 3) \cdot (x^2 - 25x + 300)$ or by graphing the quartic function to find that the only possible solution is $x = 4$. Another solution that does not require factoring involves graphing the original two equations: $x^2 + y^2 = 25$ and $y = (13x - x^2)/12$. The only point of intersection of the circle and the parabola yielding a positive value for x is (4, 3). With $x = 4$ and $y = 3$, we find $BD = 13 - 4 = 9$ so $AB = 15$. Finally, apply the Pythagorean theorem to $\triangle ABC$ to get $AC = \sqrt{15^2 + 5^2} = \sqrt{250} = 5\sqrt{10}$.



27. $525/8 = 65.625$. Let $h = EF$ be the altitude of $\triangle AEC$ from E, and let $k = AF$. Since $\triangle AEF \sim \triangle ACD$, we have $k/h = 20/15$ so $k = 4h/3$. Since $\triangle CEF \sim \triangle CAB$, we have $h/(25 - k) = 7/24$. Substitute for k and solve $24h = 175 - 7(4h/3)$ to find $h = 525/100 = 5.25$. The area of $\triangle AEC = (1/2)(25)(5.25) = 65.625$.

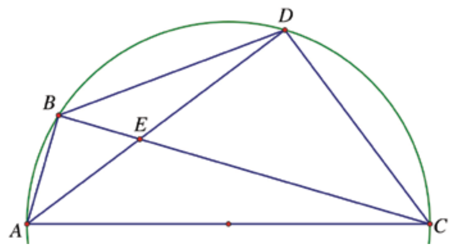


28. 192. Since $\angle ABC$ and $\angle ADC$ are both right angles, they can be inscribed in a circle. Thus $ABDC$ is a cyclic quadrilateral with $\overline{AC}$ as the diameter of the circle. The area of a cyclic quadrilateral may be calculated using Brahmagupta's formula:

$$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

where a, b, c, and d are the sides and s is the semiperimeter. To find BD we apply Ptolemy's theorem that states: The sum of the products of opposite sides in a cyclic quadrilateral equals the product of its diagonals. Therefore $AB \cdot CD + AC \cdot BD = AD \cdot BC$ or $7(15) + 25(BD) = 20 \cdot 24$, resulting in $BD = 15$. The semiperimeter is $s = (7 + 25 + 15 + 15)/2 = 62/2 = 31$. It follows that

$$\begin{aligned} A &= \sqrt{(31-7)(31-25)(31-15)(31-15)} \\ &= \sqrt{24 \cdot 6 \cdot 16 \cdot 16} \\ &= 192. \end{aligned}$$



29. In the solution to problem 28 we found that $BD = 15$. Since congruent chords intercept congruent arcs, $\overset{\frown}{BD} \cong \overset{\frown}{CD}$. Since $\angle BAD$ and $\angle DAC$ are inscribed angles intercepting congruent arcs, they are congruent angles, so $\overrightarrow{AD}$ bisects $\angle BAC$.

Alternative solution: In the solution to problem 27 we let $h = EF$ and $k = AF$. We found that $h = 5.25$ so $k = 4h/3 = 7$. Thus $\triangle BAE \cong \triangle FAE$ by hypotenuse-leg congruence. Hence, $m\angle BAE = m\angle FAE$ and $\overrightarrow{AD}$ bisects $\angle BAC$.

Second alternative solution: In $\triangle AEC$, draw the altitude from E. In the solution to problem 27 we saw that $EF = 5.25$. If we can show that BE is also 5.25, then E is equidistant from the sides of $\angle BAC$ so $\overline{AD}$ is the bisector of the angle. Let $x = BE$ and $y = AE$. Since $\triangle ABE \sim \triangle CDE$ by AA similarity, we can set up the following two proportions:

$$7/x = 15/(20 - y) \text{ and } 7/y = 15/(24 - x)$$

Solve the system of equations $7x + 15y = 168$ and $15x + 7y = 140$ to find $x = 5.25$. Since $BE = EF$, we see that $\overrightarrow{AD}$ bisects $\angle BAC$.

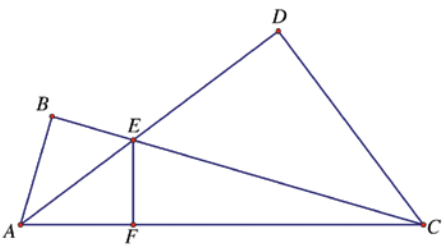
Third alternative solution: Since $\triangle AEF \sim \triangle ACD$, the sides of $\triangle AEF$ are in the ratio 3:4:5. Let $AF = 4p$ and $EF = 3p$. Similarly, $\triangle CEF \sim \triangle CAB$ so we let $CF = 24q$ and $EF = 7q$. Therefore, $3p = 7q$ or $q = 3p/7$. Furthermore, $4p + 24q = 25$ so

$4p + 24(3p/7) = 25$, or $p = 1.75$. Thus, $EF = 3(1.75) = 5.25$. Complete the proof as above.

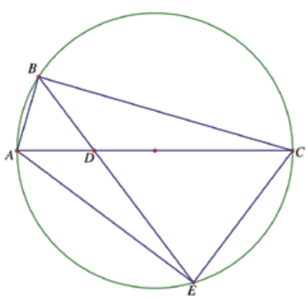
Fourth alternative solution: Apply the half-angle formula or double-angle formula to show that $m\angle DAC = m\angle BAC/2$. For example:

$$\begin{aligned}\cos\left(\frac{BAC}{2}\right) &= \sqrt{\frac{1+\cos(BAC)}{2}} \\ &= \sqrt{\frac{1+7/25}{2}} = \sqrt{\frac{32}{50}} = \frac{4}{5} \\ &= \cos(DAC)\end{aligned}$$

Because both angles are acute and their cosines are equal, the angle measures must be equal.



30. $117/5 = 23.4$. Quadrilateral $ABCE$ is a cyclic quadrilateral since $\angle ABC$ and $\angle AEC$ are each right angles. Applying Ptolemy's theorem we get $AB \cdot CE + BC \cdot AE = AC \cdot BE$ or $7(15) + 20(24) = 25 \cdot BE$. Thus $BE = 117/5 = 23.4$.



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