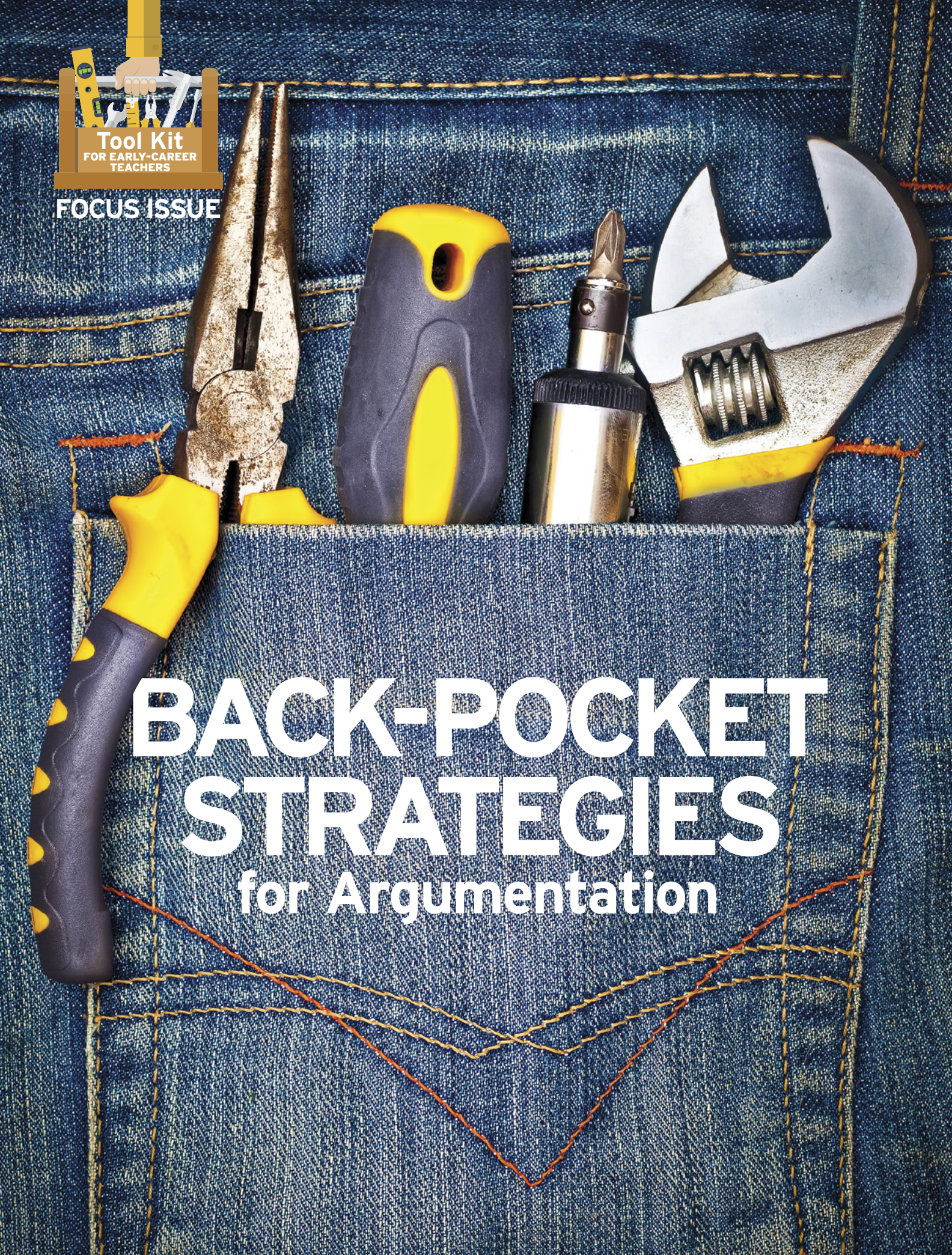
Tool Kit
FOR EARLY-CAREER TEACHERS

FOCUS ISSUE



BACK-POCKET STRATEGIES for Argumentation

New teachers can immediately begin using these classroom-tested ways to incorporate mathematical argumentation in their classrooms on a daily basis.

Melissa Graham and Kristin Lesseig

Mathematical argumentation is an essential part of the discipline of mathematics and a key indicator of mathematical proficiency. In the process of constructing arguments and critiquing the reasoning of others, students build their understanding of underlying mathematical ideas and engage in critical sense-making activity (Yackel and Hanna 2003).

Despite the importance of this practice, teachers—both novice and experienced—have difficulty incorporating argumentation in the classroom. At the high school level, constructing arguments has often been equated to proof writing in geometry. Limiting argumentation to verifying formulas neglects the explanatory role that proof can play in learning mathematics at all levels (Hanna 2000). Adapting or designing activities for students to investigate why something is true and to communicate their reasoning requires intentional work on the part of teachers.

In this article, we describe strategies for promoting mathematical argumentation developed by a group of teachers as they participated in four cycles of Mathematics Studio focused on argumentation. Mathematics Studio (Teachers Development Group 2010) is a research-based professional development model that is similar to Japanese lesson study in that teachers collaboratively plan, observe/enact, and analyze a classroom lesson. The focus of Mathematics Studio is not on creating a lesson to be used by others but rather to increase teachers' understanding of mathematical argumentation and ways to develop students' abilities related to this practice (Lesseig 2016).

Teachers participating in Mathematics Studio developed a number of strategies for increasing student opportunities for mathematical argumentation. These strategies involved incorporating particular types of activities, instructional practices, and ways to promote productive student dispositions toward argumentation. Below we describe teacher-developed strategies in each of the three categories and include illustrative examples that we hope other teachers can use to increase opportunities for students to engage in mathematical argumentation.

Although increasing argumentation in one's classroom is often seen as a more advanced skill, it is also a skill that one can begin to develop in the first year of teaching. By using some of the strategies presented here, a new teacher can begin to foster a classroom in which constructing and critiquing mathematical arguments is a daily activity, which will empower students to be mathematical thinkers and doers (NCTM 2009). Daily work surrounding mathematical argumentation not only lays the groundwork for formal proofs in geometry, but also communicates to students the nature of mathematics as a discipline. Students who tinker with mathematics, who have access to activities in which they conjecture, generalize, and justify, are able to engage in mathematics in a more authentic manner, the same way mathematicians do. With a regular expectation in class that answers are not enough, that justification is important too, students will gain experience in evaluating others' arguments as well as their own, potentially leading to more sophisticated class discussion and mathematical understanding.

STUDENT ACTIVITIES THAT PROMOTE MATHEMATICAL ARGUMENTATION

Creating Posters of Justification

When students create posters that show their justification, others can then provide feedback that includes one question and one affirmation about the argumentation. The act of creating the poster forces students to put their reasoning into words. Because other students will be reading the posters when the poster creator is not present, students have to create justifications that are more thorough than they might otherwise. Mathematics Studio teachers found this strategy particularly effective when students included their names on the posters. This not only increased accountability but also caused students to put more effort into a good justification. Posters also become public records of

student justifications that can be revisited and refined. In this way, these records provide a way for students to self-assess and for teachers to track growth in students' argumentation skills over time.

Using the Challenge Cycle

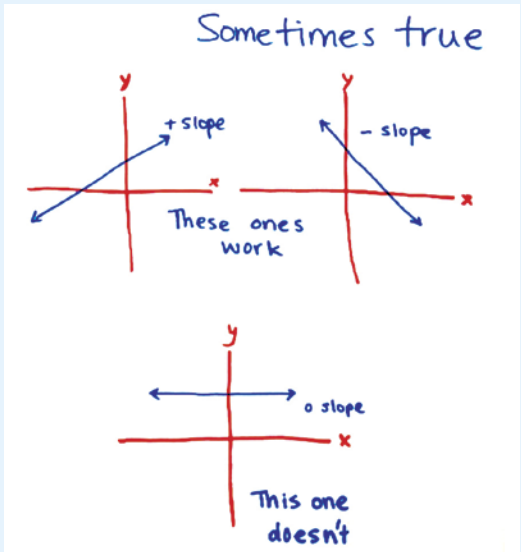
This cycle—convince yourself, convince a friend, convince a skeptic (Mason, Burton, and Stacey 1982)—can be used in different ways. One technique is first to encourage students to understand the problem well enough that they believe they have come up with a correct solution. Next they produce a justification that could be convincing to someone else in the class. The final level is a justification that is complete enough to be convincing to someone who found a different solution or might disagree with the solution provided. With this cycle, students construct arguments that grow in sophistication.

In observing a Studio lesson where students were asked to justify their work, one teacher offered another version of the challenge cycle that could be used to help students develop increasingly sophisticated justifications. This alternate version still begins with a question that students first explore privately. Once students have written down their initial thoughts, the teacher provides additional resources or sample justifications. Using the aids provided, small groups then improve on their original justifications. Last, each small group reports to the class, and through discussion, the class reflects and decides what is still needed to provide a complete justification.

Classifying a Statement and Justifying the Answer

Always, sometimes, or never true (ASN) questions naturally lend themselves to using cases, generalizing, using a diagram, and making connections across representations. Within this format, students are forced to take a stand and make a claim that they then go on to justify. Teachers found this to be a useful tool in discussing when an example or series of examples can constitute a proof, such as when justifying an answer of "sometimes" by providing an example in which the statement holds true and another example in which the statement is false.

During a Mathematics Studio session, students were asked to determine if the statement, "The graph of $f(x) = mx + b$ is a line that passes through three quadrants" is always, sometimes, or never true. After groups of students determined a claim about this statement, they constructed posters to justify their claim and then rotated to critique others' justifications (see **figs. 1a** and **1b** for sample student posters). For this particular ASN question,



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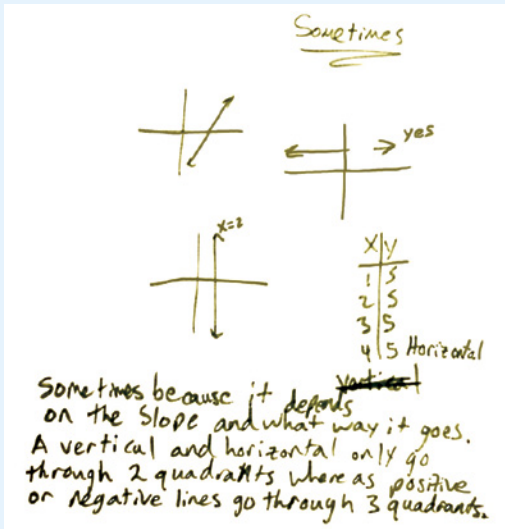

(b)

Fig. 1 Student posters justify their position regarding whether the statement "the graph of $f(x) = mx + b$ is a line that passes through three quadrants" is always, sometimes, or never true.

students were able to make connections between functions and graphs and use their understanding of functional relationships to create viable arguments. In addition to developing argumentation, posters also create an opportunity to address misconceptions, such as the misconception that vertical lines are graphs of functions of the form $f(x) = mx + b$ (see **fig. 1b**).

Ranking Work Samples

Putting different justifications for the same claim in order according to how well the result is

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justified gives students an opportunity to critique the reasoning of others and consider what constitutes an effective mathematical argument. Having student groups discuss their orderings provides students with additional opportunities to justify their thinking. This is another instance where inductive versus deductive arguments can arise if one of the arguments is a series of examples. This activity offers an opportunity to make different flaws in arguments explicit. For example, if a student believes that a series of examples constitutes a proof, this misconception regarding what constitutes a valid argument can be addressed. This could be used in conjunction with Frost and Coomes's (2014) whole-class prompt of "Find at least five ways . . ." (p. 199) to encourage students to come up with multiple arguments, which could then be the start of a discussion of what makes a good justification.

Taking Sides

Ask students with differing opinions to go to opposite sides of the room and convince others to come to their side. Physically taking sides in an argument encourages discussion and debate. It generates interaction between students and presses them to use precise mathematical language in justifying their position. This pedagogical strategy goes by many names, including "four corners," and is used in other disciplines as well. Convincing others to come to one's side involves constructing viable arguments and critiquing the reasons of others. Many other Common Core Standards for Mathematical Practice (SMP) may come into play in a task such as this as well, such as attending to precision, since a convincing argument often requires precise language.

In Mathematics Studio, a teacher used this strategy when a student asked whether a rhombus is a square. Students in her class took positions on this statement and argued their position to others, with more sophisticated arguments necessary to convince the final few students. Note that consensus does not constitute proof, though, so following an activity such as this one with a summary discussion may be in order.

INSTRUCTIONAL PRACTICES THAT PROMOTE MATHEMATICAL ARGUMENTATIONS

Opt for Fewer Problems

Being explicit about a focus on justification leads to better justification from students. For students to have time to create such justifications, they have to spend less time working problems. Studio teachers employed this technique when planning for a card sort with representations of linear functions. The original version of the task had ten graphs for which students were asked to find matching cards. Teachers modified the task by limiting the number of graphs and presenting a specific structure within which groups would work collaboratively. Each group of four students was given three graphs and four bundles of cards that contained (a) equations in standard form, (b) equations in slope-intercept form, (c) tables, or (d) verbal descriptions. Student were given the following directions:

1. As a group, choose one of the three graphs to explore. Place that graph in the center and give each person one bundle of cards.
2. Within each bundle, one of the ten cards fits the graph. Each person needs to find the card that matches the graph and then convince your group why you know it matches.
3. Each group must then make a poster justifying why all the cards match the graph.

Teachers found that the original task was too long for a focus on justification. Limiting the number of graphs used allowed students to spend more time discussing their reasoning and communicating their mathematical arguments verbally and in writing.

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Use Mathematically Less-Challenging Tasks

When first developing argumentation, choosing a task that is mathematically accessible for all students in the class allows students to focus on their justification and allows everyone to participate in the discussion of how to justify solutions. Teachers found that when the problem was too difficult, many students were unable to participate in a discussion of how to justify their answer because they were still making sense of the problem. This is connected to the challenge cycle (convince yourself, convince a friend, convince a skeptic). If the mathematics used to develop argumentation is too difficult for students to convince themselves, students will not make progress in developing maturity in their argumentation.

Use Sentence Starters

Give students an entry point into a justification by using such statements as "I know . . . because. . . .". Students working in groups can each come up with an "I know . . . because. . . ." statement. Students may find that many justifications make sense in a particular situation. As students consider these statements together, big ideas may emerge that are common to each of these justifications.

In Mathematics Studio, this strategy was suggested to students who were having difficulties writing justifications on their posters during the

card-sort activity described in the instructional practice above. Student justifications that resulted from this prompt included, "I know this table matches the graph because I checked the points and the line on the graph goes through them" and "I know this table matches the graph because I calculated the slope from the table and it matches the slope of this line." Many justifications make sense in this case, and students who initially struggled to come up with the language to describe their reasoning found success once they received this prompt.

Give an Expectation of Length for Justification

If students know the expectation is a paragraph of 3–5 sentences (or whatever is fitting for the task at hand), they may provide a more complete justification than without a clear expectation. Offer a framework, such as CER (claims, evidence, reasoning), for justifications. Giving students a structure of quadrants with which they separate their description of the investigation, what claims they were making, evidence supporting those claims, and their justification has benefits for students. This structure helps students enter the task. The structure also prompts discussions surrounding what quadrants need to be done before others. The teacher leading the task may add specifications, such as, "Claims and reasoning quadrants must involve words."

Mathematics Studio teachers used this approach during a task to introduce systems of equations in an eighth-grade algebra class. Student pairs were given small containers of change. Students could not open the containers but were told what types of coins were inside along with additional information, such as the total dollar amount in the box. For example, students were told that one box had only nickels and dimes, 10 coins for a total value of $0.85. Another box contained some pennies and one more nickel than dimes, 26 coins with a total value of $0.82. The challenge for students was to determine how many of each coin were inside and support their answer using the investigation-claim-evidence-and reasoning structure that teachers provided. Student work in **figures 2a** and **2b** illustrates how students used this template to record their strategies and corresponding justification for two of the problems posed.

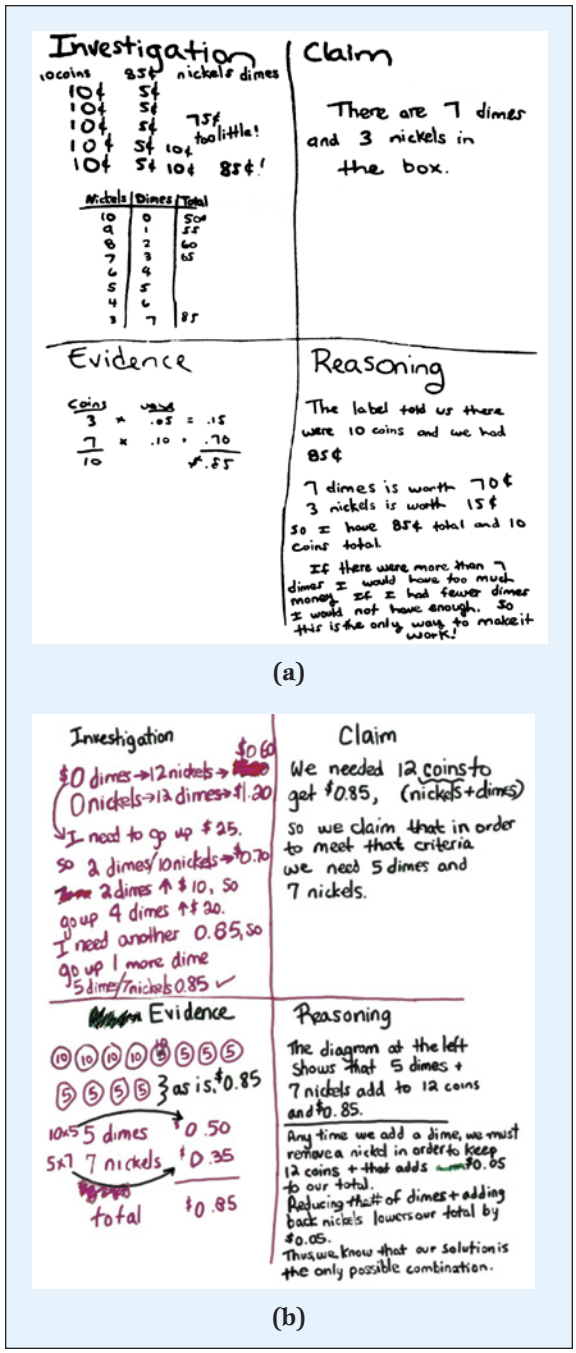

Fig. 2 Teachers in the Math Studio completed these posters, anticipating how students might use the claims, evidence, and reasoning structure with a coin problem.

Be Aware of Subtle Change Effects

Subtle changes or modifications in a problem can lead to changes in how students might justify their answers. Consider the graphs in **figures 3a** and **3b**. In a lesson studied by the teachers, students were given a system of equations and a graph similar to **figure 3a**. They were asked to determine how many solutions the system had and to justify their answer. Initially, the teachers were surprised that no group of students used slope in their justification. Students were able to point to the intersection point of the lines on the graph, and used that to justify their answer. Teachers then wondered if a system whose graph looked more like **figure 3b** would elicit answers that discussed slope, since the intersection point was not visible on the graph. Being aware of how a task might limit student thinking and/or might encourage particular forms of argumentation can help teachers be more intentional in planning. These final strategies arose as Mathematics Studio teachers realized that they

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needed to attend to more "affective" dimensions of argumentation to support productive dispositions toward argumentation.

FOSTERING STUDENT DISPOSITIONS THAT PROMOTE MATHEMATICAL ARGUMENTATIONS

Train Students to Be Skeptics

Teachers thought it would be helpful to discuss being a skeptic as part of a lesson. Questions explored would include the following:

- What is a skeptic?
- What kinds of questions do skeptics ask?
- Why do we need skeptics?
- What does a guiding skeptic look like?

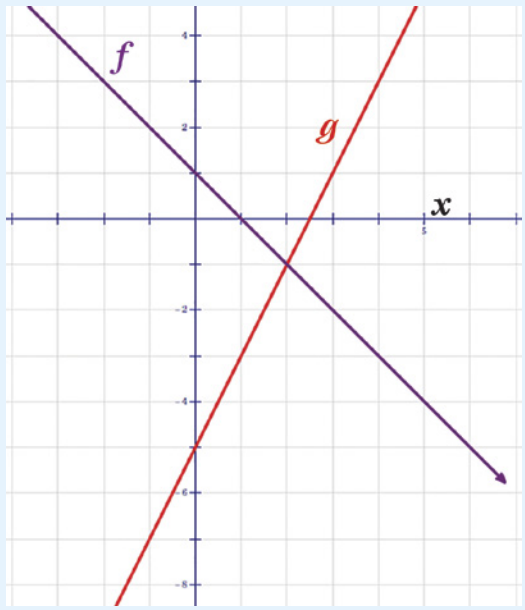
Teachers also found it important to note that skeptics *can* be convinced. Assigning one student the role of skeptic when students are working in groups can be helpful in encouraging students to justify their reasoning.

Increase Student Agency

Teachers were troubled when students accepted others' work without verification or went through the motions of problem solving without really understanding why. Rather than just going along with what they are being told, students need to have the confidence to actively pursue justifications that make sense to them and allow the mathematics to be the authority. As described in *Developing Essential Understanding of Proof and Proving* (Ellis et al. 2012), "A proof is not an argument based on authority, perception, popular consensus, intuition, probability, or examples" (p. 36). Teachers can help develop students' agency by responding to student questions or requests for solutions with further questions or by directing students toward mathematical resources.

Begin with Argumentation on Day One

Engaging students in mathematical argumentation and noticing subtleties in student thinking surrounding proof can be challenging even for veteran teachers. Such skills require a teacher not only to have a deep knowledge of proof but also to be familiar with what students are likely to find difficult and which mistakes students will typically make. Facilitating student discourse surrounding proof requires a teacher to think quickly on his or her feet, making sense of students' arguments and responses and deciding in the moment which to pursue. Because of these challenges, we suggest that new teachers begin with strategies that are less demanding in terms of teacher facilitation. Have students create posters of their justifications



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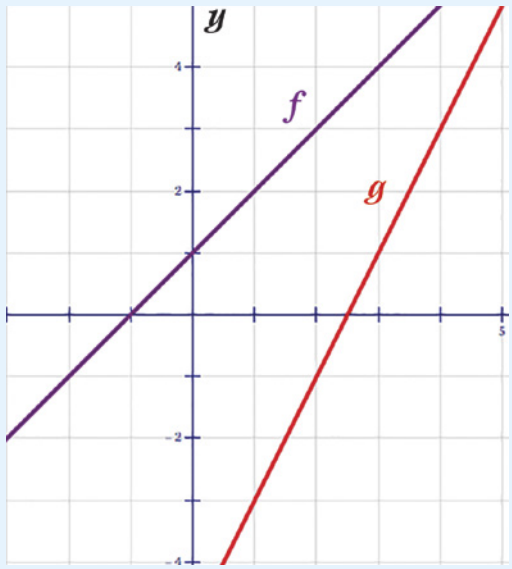

(b)

Fig. 3 The two graphs shown may elicit different justifications from students regarding the number of solutions to the linear system.

as a start, with sentence starters on hand in case students struggle to begin their justifications. ASN questions can be used in conjunction with posters as can providing an expectation for length of justification. Asking students to order samples of work according to how well the result is justified can be used as an intermediate activity. Ordering samples may require more teacher facilitation, but it can be less demanding than other activities because the arguments are already present. As a teacher becomes comfortable anticipating student thinking

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surrounding argumentation, she or he can begin to incorporate the more demanding activities of using the challenge cycle and having students convince one another to come to their side of the room using argumentation.

WRAPPING UP

Many teachers, novice as well as expert, find teaching mathematical argumentation challenging; however, taking up this challenge is essential. Setting up a culture where argumentation is part of the daily activity in a classroom gives students the opportunity to engage with mathematics the way mathematicians do, sets them up for success in future coursework, deepens their understanding of the content they are learning, and creates the expectation that mathematics should make sense (NCTM 2009; Schoenfeld 1994).

Given the necessity of teaching mathematical argumentation as well as the struggle many teachers have implementing it, having several go-to strategies to include and enhance argumentation in the classroom is a must. Many of the approaches discussed in this article are not novel or unique to the Mathematics Studio group, but they are strategies that participating teachers found useful. They are strategies that can be planned for or may come up organically. With structures and expectations surrounding argumentation that promote it, student expertise in constructing viable arguments will increase. With no structure or intentional planning in place, however, students have little chance of developing this essential skill. We urge teachers to look at existing tasks with an eye toward argumentation and to be flexible so that students' genuine questions or confusions can be capitalized on with a strategy, such as choosing sides and constructing arguments. We hope that what we discussed here will provide tools and purposeful structures to enhance argumentation for all teachers, but especially for novice teachers who are looking for a few "back-pocket" approaches to argumentation that can be drawn on in the moment.

Let's Chat about Back-Pocket Strategies

On Wednesday, November 28, at 9:00 p.m. ET,

we will discuss "Back-Pocket Strategies for Argumentation," by Melissa Graham and Kristin Lesseig (pp. 172–178).

Join the discussion at #MTchat.

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