

A Geometric Path to the Concept of Function

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The approach described in this article is supported by a number of Sketchpad activities, including worksheets, activity notes, and sketches. Several of the sketches can be downloaded from the article's summary page. All the worksheets, activity notes, and sketches are available from the Dynamic Number Project website at <http://www.kcptech.com/dynamicnumber/welcome.html>. (Click the "curriculum" link to find the Geometric Functions activities.)

These sketches give a flavor of some of the activities, but keep in mind that many of the activities are not based on prepared sketches but rather are designed so that students can create, manipulate, and investigate their own geometric functions.

ACTIVITY 1: IDENTIFY FUNCTIONS

Students use a prepared sketch to experiment with several examples

and nonexamples of functions and formulate their own definition of function.

ACTIVITY 2: IDENTIFY FUNCTION FAMILIES

Students play several games in which they determine which one of four functions is different from the other three and in the process identify similarities and differences in the relative rate and direction in which variables change, and in the presence and locations of fixed points (locations where the independent and dependent points come together).

ACTIVITY 3: THE TRANSLATION FAMILY

Students create, manipulate, and investigate several different translations, with special attention to the behavior of these functions (the relative rate and direction of the variables and the presence and locations of fixed

points). After investigating their own constructions, students undertake prepared challenges in which their job is to create a new function to match an existing translation. (The full activity requires the worksheet and activity notes; only the prepared challenges can be downloaded.)

ACTIVITY 4: THE REFLECTION FAMILY

Students create, manipulate, and investigate several different reflections, with special attention to the behavior of these functions (the relative rate and direction of the variables and the presence and locations of fixed points). After investigating their own constructions, students undertake prepared challenges in which their job is to create a new function to match an existing reflection. (The full activity requires the worksheet and activity notes; only the prepared challenges can be downloaded.)

ACTIVITY 5: THE ROTATION FAMILY

Students create, manipulate, and investigate several different rotations, with special attention to the behavior of these functions (the relative rate and direction of the variables and the presence and locations of fixed points). After investigating their own constructions, students undertake prepared challenges in which their job is to create a new function to match an existing rotation. (The full activity requires the worksheet and activity notes; only the prepared challenges can be downloaded.)

ACTIVITY 6: THE DILATION FAMILY

Students create, manipulate, and investigate several different dilations, with special attention to the behavior of these functions (the relative rate and direction of the variables and the presence and locations of fixed points). After investigating their own constructions, students undertake prepared challenges in which their job is to create a new function to match an existing dilation. (The full activity requires the worksheet and activity notes; only the prepared challenges can be downloaded.)

ACTIVITY 7: FUNCTION FAMILY DANCES

In this two-part activity, students dance several different functions from the four families they know (translation, reflection, rotation, and dila-

tion). In the first part, groups of four students perform a dance for the class, with one student as the independent variable, another as the dependent variable, and the remaining two as the assistants who determine the particular member of the function family to be danced. In the second part, pairs of students use their mouse (or finger) to dance the role of the dependent variable as the independent variable dances along a restricted domain. (The full activity requires the worksheet and activity notes; only the sketch for the second part can be downloaded.)

ACTIVITY 8: FUNCTION DETECTIVE

Students review the characteristics of a city's four crime families (the Translation, Reflection, Rotation, and Dilation families) and use the evidence on each page of the sketch both to identify the family to which the criminal belongs and to determine the specific member of that family.

The eight activities described above are designed to introduce fundamental function concepts (emphasizing function behavior and families of functions) in a visual, dynamic way that is accessible to middle school students.

Additional activities address ways of combining functions (particularly composition of functions and inverses of functions) and the idea of a function as a mapping that can, in a single step, take a given set of input values

and construct the entire set of corresponding output values (as shown in **figs. 2, 3, and 4** in the article). Both are areas of study that are commonly regarded as advanced high school topics; however, the geometric approach makes them surprisingly accessible to middle school students.

All these activities are drafts, in various stages of field-testing and refinement. We will continue to polish these activities and support this geometric approach to functions with additional activities during the course of the Dynamic Number project. Check the Dynamic Number Project website for newer versions of these activities and for additional Geometric Functions activities as they become available.

All of the prepared sketches for these activities require Sketchpad 5. You can explore these sketches using the free preview version of Sketchpad 5 available at <http://www.keypress.com/x24795.xml>.

To help us improve the Geometric Functions activities, please send your suggestions and comments to Scott Steketee at stek@kcptech.com.

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