

The mathematics found in the popular adolescent book and movie gives students another way to view probability.

Sarah B. Bush and Karen S. Karp

After the end of a thought-provoking class period, one student remained in the classroom and exclaimed,

Guess what?! I figured out that when I am 18 I will have even more entries than Gale's 42. I will have 57, and that is just based on the number of family members I have living in my house right now! If my sister moves back home next week, I'll have 77 entries by the time I am 18! This reaping system is not mathematically fair!

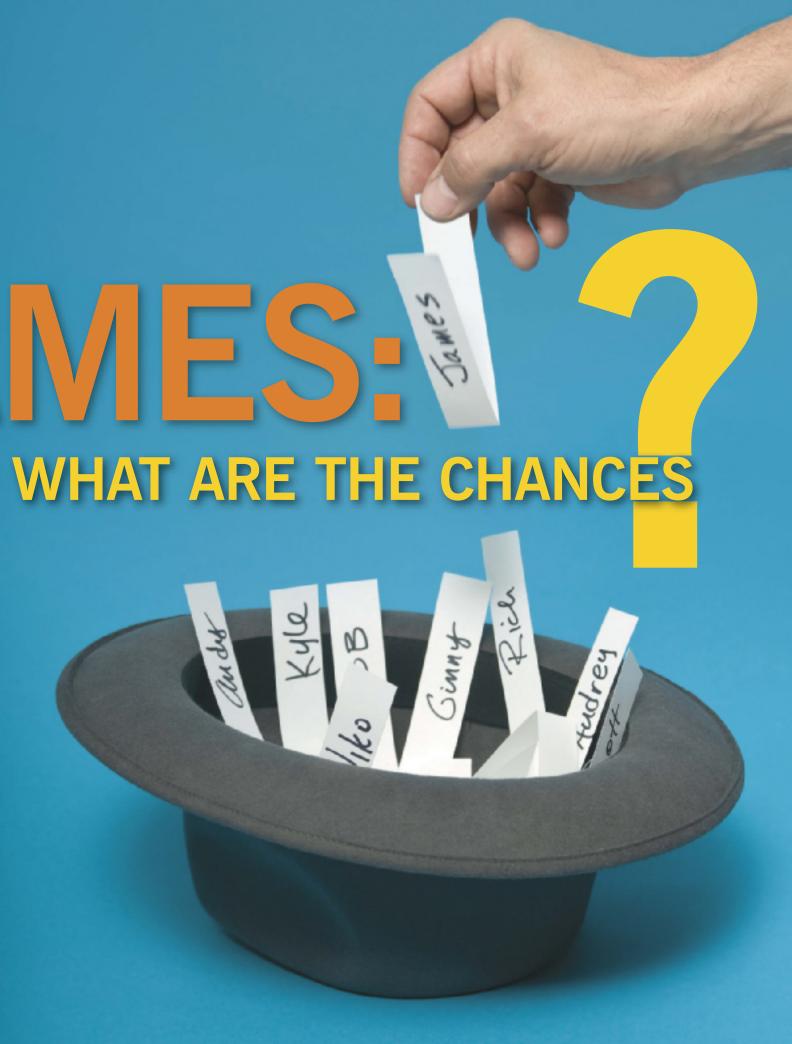
This response was a student's reaction to an activity based on the popular book and movie *The Hunger Games* (Collins 2008). The activity was designed to engage middle school students in using the mathematics

found in the book. *Catching Fire* (Collins 2009) and *Mockingjay* (Collins 2010), the other books in the series, also provided a meaningful context for probability study. We hope that this activity will guide the implementation of probability ideas in other classrooms.

The Hunger Games describes teenage heroes battling a corrupt government set in a postapocalyptic United States called Panem. This new country is divided into 12 districts, instead of 50 states, and controlled by the rulers in the capital of Denver. Leaders annually remind the districts of their authority and power through implementing hunger games. At this event, one boy and one girl, ranging from ages 12 to 18, from each of the

12 districts, are randomly selected during a ceremony to fight to the death on a live reality television show.

Middle-grades students readily relate to and sympathize with Katniss, Gale, and Peeta, the main characters in the book. The story begins with Katniss volunteering to take the place of her younger sister who is selected for the televised games. One student in the class said, "I would volunteer for my younger sister because although we fight often, she is still my sister and I love her. I wouldn't be able to bear her being killed on national TV." The students naturally imagined what it would be like to struggle for resources, leave their family, fight to the death, and sacrifice their life for a family member.



#### THE ACTIVITY

### Part 1: Entries in the Reaping

The lesson was completed over two fifty-minute class periods. Most students in the classes had already read The Hunger Games and were able to explain the story line to the few students who had not yet read it. We took a few minutes and encouraged students to share their personal insights about the story with the rest of the class. Students were eager to describe the main characters; discuss the inhumanity of the story; and point

### The Standards at Work

The Common Core State Standards for Mathematics (CCSSI 2010) and NCTM's Content Standards were both used as guiding frameworks. This activity addressed 4. Model with Mathematics, a Common Core Math Practice, which states that mathematically proficient students should "routinely interpret their mathematical results in the context of the situation" (p. 7).

This Hunger Games activity also addressed 1. Make Sense of Problems and Persevere in Solving Them because students must "start by explaining to themselves the meaning of a problem and looking for entry points to its solution" (CCSSM 2010, p. 6). This activity also aligns well with the Common Core Standards for seventh-grade mathematics, which state, in part, that students should "develop a probability model and use it to find probabilities of events" (CCSSI 2010, p. 51). Moreover, this activity can be related to the Curriculum Focal Points (NCTM 2006) for grades 6-8 in that students compute probabilities from simulations incorporating organized lists and tree diagrams.

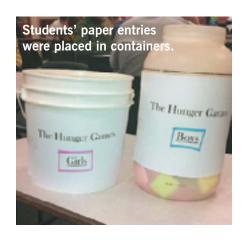
out unfair elements of the reaping system, or student lottery drawing for the televised games. Some students specifically explained how they connected the story to their own life. Students commented on what their life would be like in the Panem district:

- "When I imagine having to live in a Panem district, doing my homework doesn't seem so bad."
- "I couldn't imagine having to kill animals with bows and arrows just so my family could eat."
- "It would be awful to live in District 12."

Different discussions evolved from each class period on the basis of students' interests. This process engaged the entire class and built a high level of connection to the mathematical activity.

After discussing the plot, students were asked to imagine their class as one of the 12 Panem districts. To prepare for the lottery to select two young people from each district for the Hunger Games television show, students first had to find how many entries containing their name would be placed in the annual reaping lottery. Students' names were placed in a drawing on the basis of their age and neediness condition (resulting in the need for tesserae, or additional food resources). To find this neediness amount, each student used a calculator with a random integer function. Without telling students ahead of time what their random calculatorgenerated results meant, each student programmed their calculator to randomly select either a 0 or 1 using the calculator's random number feature.

Students were motivated by the use of technology, but the same results can be obtained by flipping a coin or rolling a standard number cube. After all students generated either a 0 or 1, the teacher explained:



If your calculator generated a 0, your family is not starving and you do not have to submit extra entries in exchange for food resources (or tesserae) each year. If your calculator generated a 1, your family is starving, and you have always had to submit the maximum number of extra entries each year since you were 12 to feed your family (all members in your household, including yourself).

Students instantly realized what their number 1 or 0 meant. The class burst into cheers and groans at the same time; the students knew that their fate had been determined for their very own hunger games.

Next, the class reviewed the process and calculated the number of times their names would be placed in the drawing for the annual reaping. See figure 1 for guidelines on how students calculated their number of entries, which matches the story. Student work examples are illustrated in figure 2.

As student work was checked to see whether they were correctly calculating their own number of entries, a common misconception emerged. Several students wanted to add the total number of entries for their age and add the number of multiplicative entries for tesserae in the same way. For example, a 14-year-old student incorrectly stated that six entries would result for his or her age because



$$1 \text{ (age 12)} + 2 \text{ (age 13)} + 3 \text{ (age 14)} = 6.$$

However, age is not calculated multiplicatively, and this student should have only 3 entries for age—1 for each year. This issue was resolved when student volunteers created the chart found in **figure 1**, which highlighted how students should calculate the entries based on age.

After all students had calculated their final number of entries based on age and received tesserae, they repeatedly wrote their names on pieces of paper and placed the determined number of entries with their name into either the boy reaping or the girl reaping lotteries.

### Part 2: What Are the Chances?

After all entries were placed into the lotteries by gender, students were asked to identify what information they needed to calculate their probability of being selected for the hunger games. Students quickly decided that the class needed to know the total number of entries in each collection, which we tabulated together as a class.

After the total number of entries for boys and girls were found, the teacher posed a series of questions in an activity sheet to engage students in thinking about the fairness (or unfairness), probabilities, and possible outcomes

**Fig. 1** This formula allowed students to calculate the number of entries into the annual reaping for the hunger games.

PART 1: AGE

Your baseline entries for each year are determined by your age.

| Age | Number of<br>Entries |  |
|-----|----------------------|--|
| 12  | 1                    |  |
| 13  | 2                    |  |
| 14  | 3                    |  |
| 15  | 4                    |  |
| 16  | 5                    |  |
| 17  | 6                    |  |
| 18  | 7                    |  |

### **PART 2: TESSERAE**

In addition to the baseline number of entries (for your age), you must add 1 extra entry for every family member (including yourself) that received tesserae. These extra entries are cumulative.

For example, if you are 14 years old, your baseline number of entries would be 3 (1 for each year—12, 13, and 14). Added to this number would be your tesserae. For example, if you have 5 members in your family, the entries for tesserae at age 14 would be  $5 \times 3 = 15$ .

Number of entries = 3 (current age baseline)

- + 5 (extra for tesserae age 12)
- + 5 (extra for tesserae age 13)
- + 5 (extra for tesserae age 14) = 18.

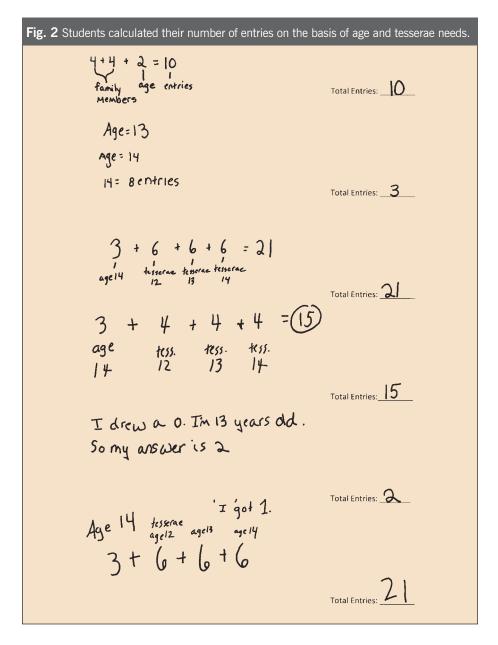
### **IMPLICATIONS OF TESSERAE**

A 14-year-old student with 5 family members receiving tesserae (if he or she received a calculator-generated 1) would have 18 entries. A 14-year-old student with 5 family members *not* receiving tesserae (if he or she received a calculator-generated 0) would have 3 entries.

of the hunger games. See the **activity sheet** at the end of this article:

- Items 1–2 align with Part 1: Entries in the Reaping.
- Items 3–4 align with the individual student's chances of being selected for the hunger games.
- Items 5–12 are follow-up questions that relate the mathematics to the story and help students imagine themselves as characters in the story.

Students immediately took an interest in the activity sheet and were placed in groups of three. Any student who had not read *The Hunger Games* was grouped with at least one other student who had. They were eager to compare their own chances of being drawn in the reaping with those of their classmates. Rachel stated, "Not only is the number of entries you have not fair, it's also not fair that 18-year-olds have to go against 12-year-olds." We overheard a conversation between



a boy and girl who both had three entries in the reaping. Although they had the same number of entries numerically, their entries were not the same proportionally.

The boy's chances of getting drawn were much higher than the girl's chances because of the difference in the total number of boy and girl entries in that class. Students also discussed the probability differences between different class periods on the basis of class sizes, boy-girl ratios, and the number of entries from each student. Students told us:

- "My probability of being chosen in another class period [district] would be different from my class period. This is because the population of the class periods is different, so the chance for me changes."
- "The chance for being selected would be different in another class period because of different variables such as boy-girl ratio, number of students, students drawn for tesserae, and with different numbers of family members to support."

### FINDINGS FROM THE ACTIVITY

After groups completed the activity sheet, the class reunited and discussed which questions were challenging and which results were interesting or surprising. Item 5, "Write an algebraic equation representing a person's total number of entries for a given year," was difficult for students. Generally, they were able to calculate their total number of entries arithmetically but struggled to represent the operations symbolically (adding for age and multiplying for tesserae) in the form of an equation.

Some students struggled to form an equation until we helped them define the possible variables. To help them think about the variables that determined their total number of entries, we asked, "What factors cause you to have entries?" "What operation occurs with that variable?" "What if your calculator generated a 0. Would your equation still work?" See figure 3 for various student-created equations.

Students found a question involving permutations (item 9) and another on grouping (item 12) especially challenging. Most students thought that the solution to item 9 was extremely large, so we took this opportunity to discuss how switching the order of just 2 people (out of the 24) counts as another possibility. Some students incorrectly wanted to find the number of combinations, rather than permutations, of this event.

To help students realize the difference between a combination and a permutation, we wrote several examples on the board of nearly, but not exactly, identical orders for the 24 contestants. Making comparisons seemed to help students conceptualize how there could be so many more different possibilities in a permutation. For item 12, once students visualized and illustrated what the question was asking, many students

stated that it reminded them of the well-known Handshake problem. They then displayed creative ways to find the solution. See the organized list and diagram in **figure 4**.

Rounding percentages was another common student misunderstanding. When calculating probability on their calculators, students wanted to round the decimal solution instead of changing the decimal to a percentage first and then rounding. For example, if a student's chances of being drawn were 10 out of 248, he or she would find  $10 \div 248 = 0.0403225806...$  on the calculator. Next, he or she would note that the directions asked students to round to the nearest hundredth percent. However, instead of changing the decimal to a percentage first (4.03225806 . . . percent) and then rounding their solution (4.03 percent), students incorrectly rounded their decimal to the nearest

**Fig. 3** These student-invented equations answered question 5, which asked for an expression representing the number of entries a person would submit in a year.

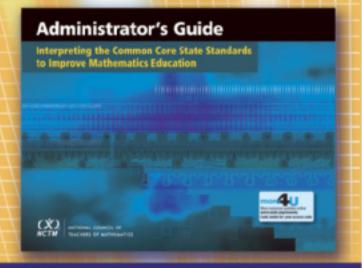
# **NEW from NCTM**—A Common-Sense Approach to the Common Core State Standards for Mathematics

Administrator's Guide: Interpreting the Common Core State Standards to Improve Mathematics Education by Matthew R. Larson

This book includes answers to frequently asked questions and detailed lists of resources to support school mathematics programs.

Visit **www.nctm.org/catalog** for a table of contents and sample pages.

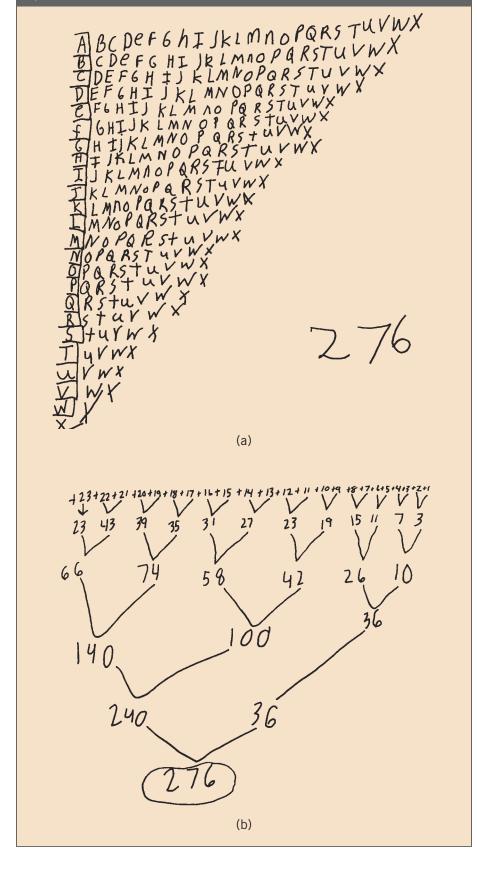
Stock # 14288 List Price \$21.95 NCTM Member Price \$17.56





For more information or to place an order, please call (800) 235-7566 or visit www.nctm.org/catalog.

Fig. 4 Question 12 asked students to count the number of "confrontations" in systematic ways.



hundredth (.04) and wrote 4 percent as their answer. This problem provided an ideal opportunity to help students by providing additional examples and showing connections to work done earlier in the school year. We circulated around the room, identifying students who had this misunderstanding and conducting quick assessments. From their responses, we were quickly able to ensure that they grasped the concept.

### **CONCLUDING REMARKS**

This activity provided a meaningful way to connect probability to a work of adolescent literature that related to, was interesting to, and motivated middle-grades students. The students who completed this two-day activity were engaged, eager to find solutions, and personally invested in the story line and main characters. They contributed valuable input and took ownership of the activity. They also took pride in the fact that they remembered more facts about the story than the teacher.

But more important, this lesson gave students the opportunity to investigate the notion of chance events and develop, use, and evaluate probability models as emphasized in both the Common Core State Standards for Mathematics and the Curriculum Focal Points. Students' perspectives on the fairness of the hunger games were both interesting and surprising. We were most surprised by how students viewed fairness in two different ways—as the fairness of one entry in the reaping as a random selection or as the fairness for the contestants competing in the games. We read and heard a variety of student responses:

- "This game would not be fair because everyone had different skills and different strategies for competing."
- "Considering that it's a random

- selection, yes, it is fair as far as choice, but it is unfair because not all contestants are equally matched."
- "Yes, because it is random, but it's also sort of not fair because some people have 18 entries while others have 2."

### **GENERAL REMARKS**

Students used reasoning and sense making during this activity to address mathematical situations from an engaging context in meaningful ways. Throughout the activity, students expressed their disbelief at some of the small calculated probabilities and some of the large total possible outcomes, as found on their activity sheet. These discoveries provided a nice transition to extended conversations about very small and very large numbers and the probability of winning a game of chance, such as the lottery.

In some classes, we talked about some games being based solely on chance, such as the lottery; some based on multiple variables, such as the skill needed in horse racing; or on both, such as found in card games. It was not surprising that the issue of fairness dominated students' initial discussions about the story. As their understanding of the underlying probability concepts became more sophisticated, their ability to separate various nuances in fairness increased.

Students quickly applied their new understanding to consequences of finding a different total number of entries for each class period. This understanding and application grew naturally as students grappled with these concepts in the context of the story and discussed the implications of the antagonists with respect to their personal situation.

Their ability to blend their personal and story contexts and share the meaning of each in terms of fairness as defined by probability contributed to a very powerful learning experience. Students were happy that the hunger games were only fiction, as reflected in such comments as "I would compete in the hunger games if for no other reason than to escape my fate in a district" and "It is tragic that 23 people are going to die during each annual hunger games."

The use of this piece of adolescent literature in the mathematics class-room helped students become more mathematically proficient in many ways. They developed a conceptual understanding of probability concepts, learned flexibility while using various procedures, solved mathematics problems strategically, engaged in logical thought and reflection, and believed that they could do math.



Note: The Hunger Games, an adolescent novel, includes elements of violence, so it may not be suitable for all middle-grades students.

### **REFERENCES**

Collins, Suzanne. 2008. The Hunger Games. New York: Scholastic Press. -. 2009. Catching Fire. New York: Scholastic Press.

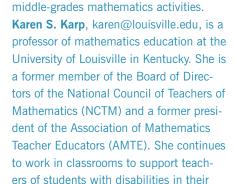
-. 2010. Mockingjay. New York: Scholastic Press.

Common Core State Standards Initiative (CCSSI). 2010. Common Core State Standards for Mathematics. Common Core State Standards (College- and Career-Readiness Standards and K-12 Standards in English Language Arts and Math). Washington, D.C.: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www .corestandards.org.

National Council of Teachers of Mathematics (NCTM). 2006. Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence. Reston, VA: NCTM.



Sarah B. Bush. sbush@bellarmine.edu, is an assistant professor of mathematics education at Bellarmine University in Louisville, Kentucky. She is a former middle-grades mathematics teacher who is interested in relevant and engaging



mathematics instruction.

## activity sheet



| Name |  |
|------|--|
|      |  |

### **HUNGER GAMES QUESTIONS**

- 1. On the basis of your current age and whether your calculator randomly generated a 0 or 1, calculate how many entries you would have in the reaping lottery this year. (If you received a 1, do not forget to add cumulative entries of tessarae from past years.) Show all work.
- 2. Place your entries in the boy drawing or girl drawing (the total from question 1) using small pieces of paper from your teacher.

| Check when comple | ete |
|-------------------|-----|
|-------------------|-----|

- **3.** Given the grand total number of entries in your class (district) and for your gender, what is the probability that your name would be selected? Express your answer as a percentage rounded to the nearest hundredth. Show all work.
- **4.** Suppose you were a student in another class period. Would your chances (or probability) of being selected for the Hunger Games be the same? Why, or why not?
- **5.** Write an algebraic equation representing a person's total number of entries for a given year. Define all possible variables, and write your equation below.
- **6.** If 11,500 total entries were in your district, and 42 of that number were your entries (similar to Gale in the book), what is the probability that your name would be drawn for the Hunger Games? Assume that the 11,500 entries are the same gender. Express your answer as a percentage rounded to the nearest hundredth. Show all work.
- 7. How many entries would you have if you were 18 years old, had 9 family members (including yourself), and received tesserae for each of them (and yourself) every year since you were 12? Show all work.
- **8.** Suppose you were Katniss and the name of your same-sex younger sibling was drawn for the Hunger Games. Would you volunteer in his or her place? Why, or why not? Be serious.
- **9.** During the Hunger Games in the book, 24 contestants fight to the death until only 1 is standing. This last person standing is declared the winner. How many orders are possible in which the contestants could have been eliminated? Assume that only 1 contestant is eliminated at a time. Show all work.
- 10. Similar to horse racing, citizens at the capital placed bets on who would win the Hunger Games. How many orders are possible for the first, second, and third person eliminated? Assume that only 1 person can be eliminated at a time. Show all work.
- 11. Suppose you were in a mathematics class of 24 students, and each student randomly draws the name of a contestant from the Hunger Games. If your contestant wins the Hunger Games, you win a prize.
  - **a.** Is this a fair game? Why, or why not?
  - **b.** If it were a fair game, what is the probability of your contestant winning the Hunger Games? Assume that there is only one winner. Express your answer as a percentage rounded to the nearest hundredth. Show all work.
- **12.** Suppose that each contestant at the Hunger Games confronts all the other contestants but none die in the process. How many total confrontations would there be for each contestant to confront all the other contestants exactly one time? Use drawings or lists to help organize your thoughts. Show all work.

