solutions to palette

(Alternative approaches to those suggested here are encouraged.)

(*Continued from pp. 82–83*)

ANSWERS

1. 210 committees

2. No

3. 212 times

4. 11/9/99

5. $17.60

6. 7.5 minutes

7. Approximately 44 mph

8. $71.75

9. 2520

10. 42

11. 1/36

12. 14 cm^2

13. The two ratios are equal; both are 2:1.

14. 1/8, or 0.125, or 12 1/2%

15. Answers will vary; 24%

16. $7,500

SOLUTIONS

1. Let's name the 5 boys A, B, C, D, and E. Boy A can be with 4 other boys on the committee (AB, AC, AD, and AE). Boy B can be with 3 other boys (not counting boy A because that pairing has already been accounted for). Boy C can be with 2 other boys, which leaves only 1 remaining pairing for boy D that has not already been accounted for (DE). Thus, there are 10 pairings of boys for the Sustainability Committee. For the girls, there are 6 + 5 + 4 + 3 + 2 + 1 = 21 pairings on the Sustainability Committee, which makes 21 × 10, or 210, possible committees.

2. According to the table, the plan that the Cantadd family has picked costs $90 for the data plus $50 for the two phones, or $140. If the family had picked a 10 GB plan instead, the cost would be $100 per month plus $30 for the two phones, or $130. That plan would give them more data for less money.

Data Use per Month (GB)	Data Cost per Month ($)	Bill with 2 Smartphone Charges ($)
0.3	20	70
1	45	95
2	55	105
4	70	120
6	80	130
8	90	140
10	100	130
15	130	160
20	150	180
30	225	255
40	300	330
50	375	405

3. The key is to identify when the product of the month and day would be 100 or greater because then the product would not match a two-digit

year. This would never happen in January, February, or March (2/29/58 would not work because 58 would not be a leap year). The number of days satisfying the condition from every other month can be found by dividing 100 by the month and identifying the first whole number less then the quotient. For example, April is the fourth month and 100/4 = 25. The first whole number less than 25 is 24.

Month	Dates That Satisfy Condition	Number of Days
January	All days in the month	31
February	All days in the month	28
March	All days except 2/29	31
April	4/1–4/24	24
May	5/1–5/19	19
June	6/1–6/16	16
July	7/1–7/14	14
August	8/1–8/12	12
September	9/1–9/11	11
October	10/1–10/9	9
November	11/1–11/9	9
December	12/1–12/8	8
	Total days	212

4. The dates that would be in 1999 would be the numerical month and date that would multiply to 99; 9/11 and 11/9 are the only dates that meet that condition. The date 11/9/99 occurs closest to 12/31/99.

5. The license plate and the gum cost \$1.76(4), or \$7.04; \$7.04 is 2/3 of the half that he had remaining, or 2/6 of his starting amount. He had \$7.04(3), or \$21.12, to start, and he spent all but the 1/3 of 1/2, or 1/6, of the money that he lost. Thus, he spent \$21.12 × 5/6, or \$17.60.

6. Because the moving sidewalk is doubling his pace, he covers the portion of the path with the moving sidewalk in half the time it would normally take to walk the path. The 6 minutes that it takes him to travel to the airport is the same as walking 4/5 of the route normally, 3/5 of the route without the moving sidewalk, and 1/2 of 2/5 of the route with the moving sidewalk. His return time is then 6 min. ÷ 4/5, or 7.5 min.

7. If we use s to represent Rachel's original speed in miles per hour (mph), $68/s$ is how many minutes it would take her to arrive at her destination, and $68/(s + 6)$ is how many minutes it will take her at 6 mph faster. Using this information, the equation

$$\frac{68}{s} - \frac{68}{s+6} = \frac{11}{60}$$

can be generated to represent the 11 minutes her faster speed will get her to her destination. To solve for s, students can plug in numbers until the result is close to 11/60th of an hour, or 11 minutes. An alternative is to find a common denominator for the left-hand side of the equation, $s(s + 6)$, and then the quadratic formula can be used to find the original speed as well.

8. 12 issues of the magazine with the \$10 shipping and handling fee would cost \$1.79(12), or \$21.48. Thus, the cost of the magazine only would be \$21.48 – \$10, or \$11.48. The price tag of \$11.48 represents a one-year subscription with 84% off the cover price. Thus, the cost of the magazine for the year at the cover price would be \$11.48 ÷ 16%, or \$71.75.

9. The desired integer must be divisible by 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. We can construct the number by considering the prime factorization of these 10 divisors:

$$2 = 2$$
$$3 = 3$$
$$4 = 2^2$$
$$5 = 5$$
$$6 = 2 \cdot 3$$
$$7 = 7$$
$$8 = 2^3$$
$$9 = 3^2$$
$$10 = 2 \times 5$$

The smallest number that meets the requirements of divisibility by 1–10 will have a prime factorization that includes 2^3 (to meet the requirement of divisibility by 8), 3^2 (to meet the requirement of divisibility by 9), 5 (to meet the requirement of divisibility by 5), and 7 (to meet the requirement of divisibility by 7). A positive integer possessing these factors will be divisible 1, 2, 3, 4, 6, and 10, so no other factors are required. The desired positive integer is $2^3 \times 3^2 \times 5 \times 7 = 2520$.

10. There are a variety of ways to do this; one is by guessing and systematically modifying your guesses. Another way is to use the fact that the product of the two numbers' greatest common factor and the two numbers' least common multiple is equal to the product of the two numbers. That is,

$$\text{GCF}(a, b) \times \text{LCM}(a, b) = ab.$$

This theorem can be applied here according to:

$$\text{GCF}(a, b) \times \text{LCM}(a, b) = ab$$
$$6 \times 462 = 66b$$

Solving for b yields 42.

11. Each die has 6 possible outcomes, so there are $6 \times 6 \times 6 = 216$ different outcomes for the three-die roll. Of these 216 outcomes, only 6 correspond to winning in the game: (1, 1, 1); (2, 2, 2); (3, 3, 3); (4, 4, 4); (5, 5, 5); and (6, 6, 6). Because every outcome is equally likely, the probability of winning a free cup of coffee is $6/216 = 1/36 \approx .0278$.

12. A square of area 9 cm^2 must have a side length of

$$\sqrt{9} = 3 \text{ cm}.$$

Similarly, a square of area 25 cm^2 must have a side length of

$$\sqrt{25} = 5 \text{ cm}.$$

Using this information, we can assign lengths to the unshaded right triangle in the figure. The horizontal leg of the triangle must be $5 + 3 = 8$ cm, and the vertical leg of the triangle must be 5 cm. The area of the unshaded triangle must be $1/2 \times 8 \times 5 = 20 \text{ cm}^2$. Therefore, the area of the shaded region must equal the total area of the squares minus the area of the unshaded right triangle: $(25 + 9) - 20 = 14 \text{ cm}^2$.

13. Let the radius of the inner circle equal 1 unit. Then, the area of the inner circle is given by

$$\pi \cdot (1)^2 = \pi \text{ units}^2.$$

The side length of the inner square is twice the radius of the inner circle, so the area of the inner square is given by $(2 \cdot 1)^2 = 4$ units^2. The diameter of the outer circle is equal to the diagonal of the inner square. The diagonal of the inner square can be found with the Pythagorean theorem as

$$\sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2} \text{ units.}$$

The radius of the outer circle is half the length of this diagonal and is equal to $\sqrt{2}$ units. The area of the outer circle is then

$$\pi \cdot (\sqrt{2})^2 = 2\pi \text{ units.}$$

The side length of the outer square is equal to the diagonal of the inner square, so the area of the outer square is given by

$$(\sqrt{8})^2 = 8 \text{ units}^2.$$

Having found all the areas, we can now construct the ratios in question. The ratio of the outer square's area to the inner square's area is 8:4, or 2:1. The ratio of the outer circle's area to the inner circle's area is $2\pi : \pi$, or 2:1. Therefore, the two ratios are equal. Alternate solution: The outer square and outer circle are the image of the inner square and inner circle under a 45 degree rotation and a dilation by some scale factor d centered at the center of the concentric circles. Because rotation preserves area, and dilation by a scale factor d scales the area by a factor of d^2, the ratio of the outer circle and outer square's areas to the inner circle and inner square's area, respectively, are both equal to d^2, so the ratios are equal. (The previous solution provides $d = \sqrt{2}$.)

14. Pepperoni comprises

$$12\ 1/2\ \% = 12.5/100 = 1/8.$$

Sausage is on 12/32 = 3/8 pizza. Green peppers are on 0.375 = 375/1000 = 3/8 pizza. Therefore, Canadian bacon and pineapple must be 1 – 1/8 – 3/8 – 3/8 = 1/8.

15. To explain to the customer his or her error, look at a simple example. Suppose that an item costs \$10.00. Then, 60% off on the item means that the customer must pay for the remaining 40% of the item's cost, so the item will cost 0.4(\$10.00) = \$4.00. Applying the 40% off coupon means taking 40% off of the item's sale price, so the customer must pay 60% of the sale price, or 0.6(\$4.00) = \$2.40. Generally, if an item costs C dollars, it will cost

$$0.6(0.4(C)) = 0.24C, \text{ or } 24\%,$$

of its original price after the 60% off sale and the 40% off coupon have been applied.

16. If the mean annual income for 10 people before the raises is \$40,000, then the 10 earned a combined income of 10 × \$40,000 = \$400,000. If we let a represent the pay raise of each of the 4 who received it, then the new combined income for the 10 people is \$400,000 + 4a. The new mean income is

$$(\$400{,}000 + 4a)/10 = \$43{,}000.$$

Solving for a yields \$7,500.