It's a Girl! Random Numbers, Simulations, and the Law of Large Numbers

Chris Goodwin and Enrique Ortiz

Edited by **Rebecca Damas**, rdamas@trevor.org, who teaches at the Trevor Day School, New York, New York. Readers are encouraged to submit manuscripts through **http://mtms.msubmit.net**.

Modeling using mathematics and making inferences about mathematical situations are becoming more and more prevalent in most fields of study. When we want to generalize about a population or make predictions of what could occur, we cannot use descriptive statistics. Instead, we turn to inference. Simulation and sampling are essential in building a foundation for statistical inference.

This activity addresses the NCTM Standards about students using proportionality and having a basic understanding of probability to make and test conjectures about the results of experiments and simulations. This activity also addresses the Common Core State Standards for School Mathematics (CCSSM) directive about statistics. It states that students understand that statistics can be used to gain information about a population by examining a sample of the population. It also states that generalizations about a population from a sample are valid only if the sample is representative of that population. Students are to understand, as well, that random sampling tends to produce representative samples and support valid inferences.

This activity addresses how to estimate a theoretical probability from sample statistics when compiling an entire class's data. Students will have to figure out how to sample and how to use different tools to simulate an outcome. As the number of trials increases, the law of large numbers says that their experimental probability will approach the theoretical value.

THE SCENARIO

Suppose that a couple is to have three children, with each child's gender independent of the next and with an equal chance of obtaining a child with either gender. What is the probability that the couple will have exactly 1 girl and 2 boys in any order?

SIMULATION

Ask students how to model this situation. Give students simulation tools (coins, dice, a number spinner, a random-digit table, or a graphing calculator with a random integer function) to help in problem solving. Have students brainstorm ideas on how to model the

situation using each of these tools and then have them settle on a method to use for themselves after all suggestions have been discussed in class.

Once the students have selected a simulation method, have them run the simulation for different numbers of trials in which n = 1, 5, 10, 15, and 20. Have them tally their results and then record their values as percentages to share with the class. Monitor their work to make sure that they are sampling properly.

RESULTS

Three classes of ninth-grade and tenth-grade students were taken through the "It's a Girl!" activity. By generating sets of random integers, we decided to let 0 = boy and 1 = girl to simulate a couple having three children.

For example, {1, 0, 1} is GBG, which is a failure as there is not exactly 1 girl, whereas the following are successes: {1, 0, 0}, {0, 1, 0}, and {0, 0, 1}. We discussed these possibilities as a class to ensure that we got appropriate data. In my first class, I tried having the students decipher the pattern by themselves. In the second and third classes, I implicitly showed them these successes and failures, and we discussed it as a class. Some common misconceptions were that students thought that each "1" represented a success, regardless of the other digits. Close monitoring and assistance were necessary, as well as help from other students to clarify how to record the data.

The next day, the experimental results were compared with the theoretical probability. Given the following possible results, where B = boy and G = girl, there are 8 equally likely combinations of which 3 work for our successful solution. Hence, our probability is 3/8, or .375, or 37.5 percent. See the sample space below with successful outcomes in bold:

Fig. 1 This solution failed to incorporate the number of combinations of two boys and one girl.

$\frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}$ $\frac{1}{8}$ because you are having 3 children and you have to multiply 50% by 3 to get your answer

Fig. 2 Students believed that their results could become more accurate with increased numbers of trials.

Trials	1	5	10	15	20
Percent Success (Rounded to whole percent)	0%	80%	50%	35%	20%

I think it's a little unlikely but it's always possible as is shown by the overwhelmingly 80% in a group of five, but as the numbers got larger, the chance of one girl in a family went down.

Fig. 3 A majority of students decided to average all their results together.

$\frac{18 \text{ sucess}}{50 \text{ trials}}$ = 36% of having one girl

{BBB, **BBG**, **BGB**, **GBB**,
BGG, GBG, GGB, GGG}

When asked the last question in the activity, about what the students think the probability truly is, many different responses and techniques were noted. Some students averaged their percentages across the number of trials (without giving any weight to the trials themselves), and others stated that the value was approximately 30–50 percent, given their data. They justified the last result by saying that with more trials, the percentage would and should be more accurate. These techniques and thought processes were discussed in relation to the law of large numbers. Some sample answers are given in **figures 1–4**, which highlight misconceptions.

Some students correctly concluded that the result implied taking

$$1/2 \times 1/2 \times 1/2$$

because each probability is 50 percent but failed to incorporate the number of combinations of two boys and one girl (see **fig. 1**). Others concluded that their results could become more accurate with increased numbers of trials and that their 80 percent success rate for 5 trials is possible but not likely (see **fig. 2**). A majority of students decided to average all of their results from the trials together (see **fig. 3**). This was a good approach, but in another response (see **fig. 4**), students did not account for the weights of the numbers of trials. A few students used combinations and multiplied

Fig. 4 Some students did not account for the weights of the numbers of trials.

Trials	1	5	10	15	20
Percent Success (Rounded to whole percent)	0%	20%	20%	47%	15%

20.4% , I added all the percentages together then divided by the Number of percentages.

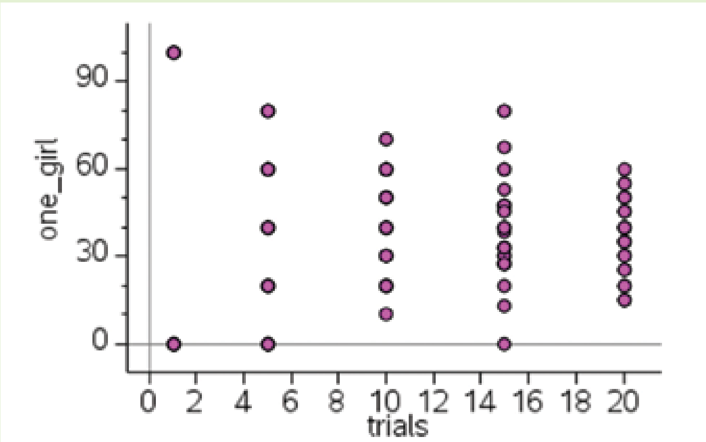

Fig. 5 This graph illustrates the results of all students' simulations.

$$1/2 \times 1/2 \times 1/2 \times 3,$$

since there was a 50 percent chance of getting either 1 boy or 1 girl and three combinations of 2 boys and 1 girl. Their result was 37.5 percent.

The students were shown that the values should converge to the theoretical value. The results of all students' trials are shown in **figure 5**. We discussed ways to improve the simulation by increasing the trials and repeating the results, by running it more than once per student or by having more students participate in the activity. When I assign this activity again, I will spend more time going over combinations and sample space; I will also tweak the activity sheet. I will have an increased number of trials, such as n = 25 and n = 30, to get better results.

LESSON EXTENSIONS

Extensions of this activity can lead to many discussions about sampling, statistics, proportions, the law of large numbers, binomial distribution and even inference. It can also be adapted to more advanced courses. For example, the binomial formula may be introduced to get the result

$$\binom{3}{1}(0.5)(0.5)^2 = 0.375$$

using the coefficient, "3 choose 1." Statistical inference can also be used for proportions to test if the results of the simulation are statistically significant in their difference from .375 as the hypothesized proportion.

Chris Goodwin, cgoodwin@lhps.org, teaches at Lake Highland Preparatory School in Orlando, Florida. He is interested in problem solving, probability, statistics, and the use of technology in teaching. **Enrique Ortiz**, enrique.ortiz@ucf.edu, teaches at the University of Central Florida. He is interested in brain-based teaching strategies, teachers' development of teaching goals, and use of manipulative materials to teach mathematics.

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The **activity sheet** and solutions for Mathematical Explorations are online at **www.nctm.org**. Online solutions are available to NCTM members only.

Student Mathematician's Name ______________________

IT'S A GIRL!

A couple has 3 children. If the gender of each child is independent, with a probability of 50 percent for each gender, what is the probability that they will have exactly 1 girl and 2 boys *in any order*?

1. What are some ways to simulate this scenario?

2. Use your method to simulate a couple having 3 children. Record the percentage of times you get exactly 1 girl. A trial is considered having 3 children. A success is having exactly 1 girl.

Trials	1	5	10	15	20	25	30
Percent success (rounded to whole percent)							

3. How many possible outcomes will there be?

4. List all the possible combinations and note which ones are successes.

5. What do you think the probability is for a couple having 3 children to have 1 girl? Explain your reasoning.