Name _____

# PENGUIN MATH

Emperor penguins live in the extreme cold of Antarctica. Temperatures can plummet to –40°F, and winds can reach 90 miles per hours (mph). In such extreme conditions, penguin bodies could lose heat quickly were it not for their densely packed feathers, thick subdermal fat layer, and huddling behavior.

Biologists have found that the amount of heat loss of the penguin's body depends on the amount of exposed body surface area (BSA). By huddling in tightly packed groups, Emperor penguins decrease their exposed BSA and decrease heat loss significantly. This is a necessity during the months of the winter that the males are deprived of food as they incubate eggs that the females have laid. Your job as a researcher is to form a model to estimate how much energy Emperor penguins conserve by huddling.

**1.** The exact BSA of an Emperor penguin can be difficult to measure. Although the penguin's trunk is smooth and regular, some of the BSA comes from the feet, flippers, and head. However, the BSA can be estimated by using one geometric solid to approximate the penguin's body.

**a.** Look at the image of an Emperor penguin. Which solid do you think would approximate its body the best?

**b.** Do you think the surface area of the solid will overestimate or underestimate the penguin's BSA? Explain.

**2.** The average height of an adult Emperor penguin is about 120 cm.

**a.** Find the scale factor of the penguin (see below) to the nearest whole number. (*Hint*: Scale factor is the ratio of actual size to model size, so a scale factor of 6 means a 1 cm length on your model is 6 cm in actual size. This is called a 1/6 scale.)

IAN DUFFY; HTTPS://WWW.FLICKR.COM/PHOTOS/IANDUFFY/SETS/72157622746169291

**b.** Use your scale factor and the solid you chose in question 1 to estimate the BSA of a life-size average Emperor penguin in $cm^2$. (Hint: Scale all lengths before finding area.)

**3.** A calorie is a unit to measure energy. Researchers estimated that an Emperor penguin loses heat energy at a rate of 0.002868 calories/$cm^2$ of exposed BSA per hour in Antarctica. How much heat, in calories, will your average Emperor penguin lose over its whole body in 1 day?

**4.** If 1 gram of fat provides 8 calories, how much fat (in grams) will an Emperor penguin burn in 1 day to maintain its body temperature?

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**5.** Suppose a group of "average" Emperor penguins huddle as pictured below.



**a.** What three-dimensional shape approximates the exposed surface area of the group?

**b.** Will you include the bottom of the shape? Explain.

**c.** What assumptions will simplify the problem, and how do they affect your answer?

**6.** Estimate or count the number of penguins in the huddle. Explain your strategy.

**7.** Estimate the average exposed BSA per penguin by dividing the total surface area of your three-dimensional shape from problem 5 by the number of penguins in the huddle. Use 120 cm as the estimated height of a penguin in finding your scale measurements.

**8.** Compare the BSA of an individual penguin (from question 2b) with the average exposed BSA for a penguin in a huddle (from question 7).

**a.** How much did huddling decrease the BSA per penguin?

**b.** By what percentage did huddling decrease the BSA per penguin?

**9.** Assume heat loss at a rate of 0.002868 calories/$cm^2$ per hour

**a.** How much heat in calories will an average huddled Emperor penguin lose over its entire body in 1 day?

**b.** How many grams of fat will it burn in a day?

**c.** What assumptions are you making, and how do they affect your answer?

**10.** What is the percentage reduction in heat loss and fat burned per penguin by huddling?

# Penguin Math

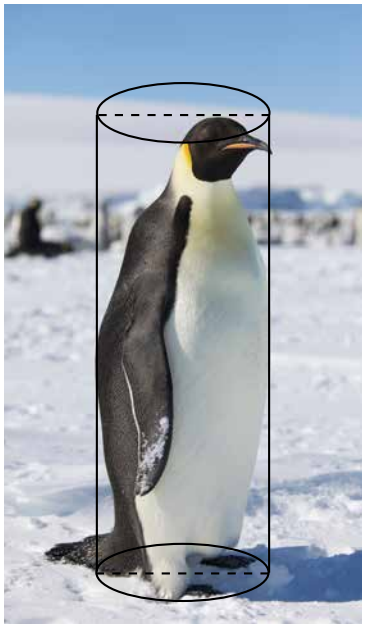
Daniel Green and Thomas Kearney

## SOLUTIONS

Instructional note: Viewing the video clip "Emperor Penguins Huddle for Warmth" (https://www.youtube.com/watch?v=OL7O5O7U4Gs) can be helpful to start the lesson and provide background information about why and how Emperor penguins group themselves together.

**1. a.** A cylinder approximates a penguin's body (see below).

**b.** Its use will probably overestimate the BSA because the head, legs, and feet are small in relation to the trunk. There is leftover volume around the head. However, it does not include the surface area on the underside of the flippers.



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**2. a.** Answers will vary because of students' potential measurement error. Ask each group to measure the height and radius, discuss the results, and then use the class average for the height and radius.

*h:* 6.0 cm; scale factor:
120 cm/6.0 cm
= 20 (the scale is 1:20)

**b.** Image height: 6.0 cm
*h:* Life-size height =
image height × scale factor
= 6.0 × 20 = 120 cm
Image radius: 1.15 cm
$d$ = 2.5 cm, so $r$ = 2.3 cm ÷ 2
= 1.15 cm
*r:* Life-size radius
= image radius × scale factor
= 1.15 × 20 = 23.0 cm

For a cylinder: $A = 2\pi r^2 + 2\pi rh$
Life-size BSA: $A = 2\pi(23.0)^2 + 2\pi(23.0)(120) \approx 20{,}665.4$ cm$^2$ or 21,000 cm$^2$, if we use significant figures.

Ask different groups to share their answers with the class, and ask students to explain any variation in answers (because of measurement error). We recommend using the class's average height and radius to not only get a more accurate estimate but also keep answers consistent across groups.

**3.** Use the unrounded BSA because it is part of an intermediate calculation to get calories per day.
Heat loss per hour: 20,665.4 cm$^2$ × 0.002868 calories/cm$^2$
≈ 59.27 calories per hour
Heat loss per day: 59.27 calories/hr. × 24 hours = 1,422.4 calories/day ≈ 1,400 calories/day

**4.** Fat burned per day: 1,422.4 calories/day × 1 g/8 calories = 177.8 ≈ 180 g/day

**5. a.** A cylinder approximates the exposed surface area of the group (see the image on the next page).

**b.** We recommend including the bottom because some heat will be lost to the ice below the penguins' feet.

**c.** We are assuming that the penguins are all the same size. We are ignoring the space between their feet and the irregularities in the surface of the huddled group created by the space between the bumpy heads on top and bodies on the sides. This would make our answer smaller than the actual value.

**6.** Approximately 330 penguins are in the large huddle. Consider asking some groups to count and others to estimate, then share the results.

**7.** We need a new scale factor because the photograph is a different size. Imagine that an individual penguin in the huddle is about 1.3 cm, and the huddle diameter is about 12.5 cm.

$$\text{Scale factor} = \frac{\text{real-life height}}{\text{photo height}}$$
$$= \frac{120 \text{ cm}}{1.3 \text{ cm}} = 92.308$$

Image huddle height: 1.3 cm
*h:* real-life height = photo height × scale factor = 1.3 cm × 92.308
= 120 cm
Image huddle radius: 6.25 cm
*r:* real-life radius = photo radius × scale factor = 6.25 cm × 92.308
= 576.925 cm
Real-life area of huddle: $2\pi r^2 + 2\pi rh = 2(3.1416)(576.925)^2 + 2(3.1416)(576.925)(120)$
≈ 2,526,302.0 cm$^2$

The BSA per penguin:
2,526,302.0 cm$^2$/330
= 7,655.46 cm$^2$ ≈ 7,700 cm$^2$

**8. a.** Amount of decrease:
20,665.4 cm$^2$ – 7,655.46 cm$^2$
= 13,009.94 cm$^2$

**b.** Percentage decrease:
13,009.94 cm$^2$/20,665.4 cm$^2$
≈ 63%

**9. a.** Heat loss in 1 hr.: 7,655.46 cm$^2$ × 0.002868 calories/cm$^2$
≈ 22 calories/hr.

**b.** 21.956 calories/hr.
× 24 hr./day
≈ 530 calories/day
526.9 calories/8
≈ 66 g/day

**c.** We are assuming that the penguins stay huddled 24 hours per day and that all penguins have the same exposure. Underestimate the calories burned. To see the penguins giving each a turn to stay warm, show the churning penguin huddle at https://www.youtube.com/watch?v=dJQmWYZzg5M.

**10.** Decrease in heat lost:
1,422.4 calories/day – 526.9 calories/day = 895.5 calories/day
Percentage decrease:
895.5 calories/ 1,422.4 ≈ 63%
Decrease in fat burned:
177.8 g – 65.86 g = 111.94 g
Percentage fat decrease:
111.94 g/177.8 g ≈ 63%
(This is comparable to researchers' results of 49%–56%.)

## DISCUSSION OF MEASUREMENT UNCERTAINTY

To incorporate measurement uncertainty into the calculation of BSA, we can use the upper-lower bound method, which looks at the smallest and largest possible values of BSA given by measurements in the ranges of uncertainty for the variable inputs. In this example, the smallest scaled-up values of *r* and *h* are

$$r = 1.0 \text{ cm} \times 20 = 20.0 \text{ cm}$$

and

$$h = 6.0 \text{ cm} \times 20 = 120 \text{ cm},$$

giving a BSA of

$$A = 2\pi(20.0)^2 + 2\pi(20.0)(120)$$
$$\approx 17{,}592.9 \text{ cm}^2,$$

and the largest values are

$$r = 1.5 \text{ cm} \times 20 = 30.0 \text{ cm}$$

and

$$h = 6.2 \text{ cm} \times 20.0 = 124 \text{ cm},$$

giving a BSA of

$$A = 2\pi(30)^2 + 2p(30)(124)$$
$$\approx 29{,}028.3 \text{ cm2. cm}^2.$$

To get our best estimate of BSA, we average

(17,592.9 + 29,028.3)/2
= 23,310.6 cm

with an uncertainty that is half the difference of the max and min:

(29,028.3 – 17,592.9)/2
= 5,717.7 cm.

Since our error is greater than 1,000, it does not make sense to state the BSA values with more accuracy than the uncertainty. Thus, we would round to the nearest thousand and state the BSA as

$$23{,}000 \text{ cm}^2 \pm 6{,}000 \text{ cm}^2.$$

Again, we did not go into this much detail with sixth graders. However, we did emphasize that measurements involve error and that we must be aware of that error, and at least consider averaging, and not rounding, intermediate values as ways to improve our experimental results.