MAYA CALENDARS in the Classroom

A lesson on least common multiples helps students not only develop a perspective on an ancient culture but also draw on the cultural background of classmates.

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The Maya calendar received a lot of attention in the years leading up to December 21, 2012, because of the mythological end of "creation." Co-author Megan Rehm, a seventh-grade math teacher, had recently completed a graduate class in which she was introduced to the Maya civilization and calendar. Her prealgebra students were particularly interested in the cataclysmic Maya prophecy, so she created a lesson reinforcing the least common multiple (LCM). Since 15 percent of Rehm's students were members of immigrant families, she decided to design a culture-laden lesson. In so doing, it would give her students an opportunity to develop a compassionate understanding of their classmates from different backgrounds and would help foster an atmosphere of respect, solidarity, and collaboration.

Rehm also thought that there would be numerous occasions within the task to give students the opportunity to engage in several of the Common Core's Standards for Mathematical Practice (SMP) (CCSSI 2010), aligning to a departmental goal. The SMP are key processes and proficiencies that describe the type of


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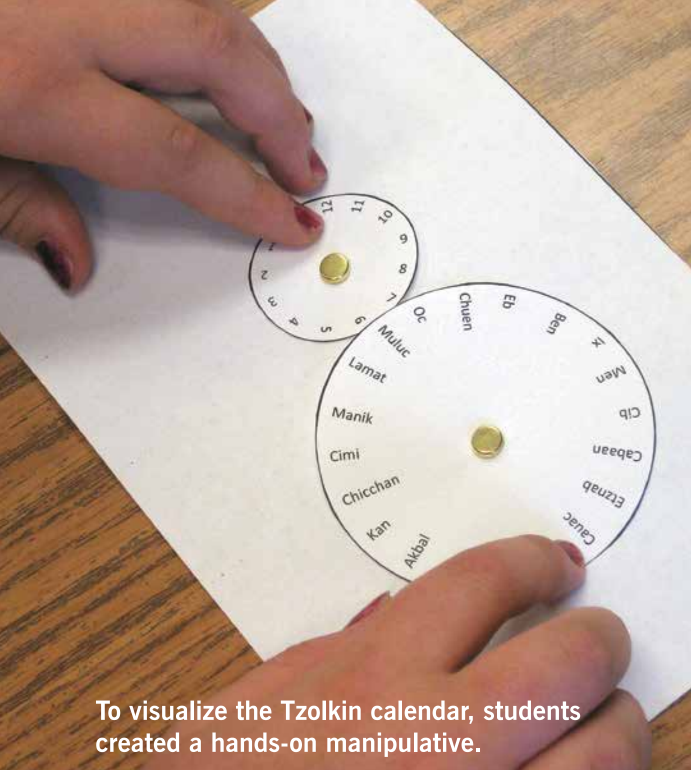
mathematical thinking and reasoning that K–grade-12 students should engage in. Rehm intended that her lesson allow students to construct their own arguments and critique others, use tools strategically, and identify and build on patterns. We describe the background of the Maya calendars and narrate the lesson that Rehm designed, specifically noting the mathematical content that students discovered and the mathematical practices they had the opportunity to develop.

HISTORY OF MAYA CALENDARS

The Maya civilization's long history is divided into three eras. The first era, the Preclassic period, began ca. 1200 BC. The second era, the Classic period, began AD 200. The last era, the Postclassic period, ended in AD 1519. This Mesoamerican civilization extended across what is now Belize, parts of Mexico, Guatemala, El Salvador, and Honduras. The Maya developed writing and number systems and excelled in several areas of science, especially astronomy. On the basis of their astronomical interpretations, they were able to develop an intricate system of calendars. The well-known date of December 21, 2012, marked the end of a grand cycle in the Long Count calendar, not the end of the world as the media advertised worldwide. It is worth noting that not a single Maya hieroglyphic text mentions disaster or the end of time (Maya Exploration Center 2015).

THE LESSON

After presenting a short history of the Maya civilization to her students, Rehm began the lesson by introducing



To visualize the Tzolkin calendar, students created a hands-on manipulative.

the design of the Maya calendar, as a sort of "story . . . to capture students' imagination" (Neel 2005, p. 55). She shared that the days in the Maya Tzolkin (pronounced *sole-keen*) calendar are defined by one of 20 names (derived from names of gods) and one of 13 numbers. To convey the culture to her students, she asked them to consider the names used in their own weekly (Gregorian) calendar, both in English and Spanish, and whether the names of gods were incorporated into the days of the week. Several of her students did make some connections, as illustrated in the conversation that occurred:

Tim: "Sunday" comes from the sun, right?
Carina: My grandma told me that "domingo" in Spanish is like the word "Dios." So Sunday uses the name for God.
Esteban: And "lunes" is like "luna," the moon.
Alyssa: I wonder if "Monday" is like saying "moon-day"?

With some encouragement, students realized that "Thursday" in English is derived from the Nordic god Thor; "Saturday" from the Roman god Saturn; and that the Spanish day names "martes," "miércoles," and "viernes" reflect the Roman god names Mars, Mercury, and Venus. Students discovered through this conversation that the Tzolkin calendar is defined similarly to their own, which captured their interest. This connection was enhanced when Rehm elaborated on one name, in particular, in the Tzolkin calendar: Ahau (*uh-how*), sometimes spelled "Ahaw." This name refers to a "lord" in the Maya nobility. The English language similarly uses "lord" to refer to both a nobleman and deity.

The format of the Tzolkin calendar (see **fig. 1**) may appear foreign to students. Catepillán and Szymanski (2010, p. 2) explain how this calendar indicates a day:

> The consecutive dates are formed as follows: 1 Imix, 2 Ik, 3 Akbal, 4 Kan, . . . , 12 Eb, 13 Ben, 1 Ix, 2 Men, . . . , etc.

That is, each day rotates to a new name (i.e., the right dial rotates counterclockwise) as well as to a new number (i.e., the left dial rotates clockwise). When considering the familiar Gregorian calendar (see **fig. 2**) of approximately 30-day months, this daily rotation of names appears strange unless viewed as paralleling the Gregorian day names, Sunday through Saturday, combined with the numerical position in a month. Thus, today might be "Saturday the 30th" and tomorrow will be "Sunday the 1st" One would then move forward through both a cycle of names and a cycle of numbers.

Students see the familiar Gregorian calendar (see **fig. 2**) as a pair of day-name and day-number cycles, followed by the Tzolkin calendar in a similar format (see **fig. 1**). The Tzolkin calendar visual aid led to some discussion among the students as they sought to understand the different calendar system. The following discussion occurred among one group of students:

Andrew: So the names in the Maya calendar are like the days in a week, and the numbers are like the days in a month?
Celia: Only 13 days in a month? That's short.
Kristen: How many names are there?
Celia: Twenty. So there are 20 days in a week?

Rehm emphasized that the Tzolkin calendar does not actually count weeks or months. However, she wanted to make the connection to the Gregorian calendar and help students understand the Tzolkin cycle. She thought that her students might get stuck on the idea of seven-day weeks later on, so it was important to emphasize that the Maya did not use a seven-day week.

THE TZOLKIN CALENDAR

After introducing this calendar, Rehm posed the main question of the lesson: How many days does it take for the Tzolkin calendar to complete one full cycle? She anticipated that students would eventually recognize that the LCM answers the question. She asked her students to make a prediction and justify their answer, which meant that they had to construct a viable argument to support their prediction. Some of the student responses were 365 days (same as our calendar); 20 days (the number of names in the Tzolkin calendar); 260 (13 multiplied by 20); and 33 (13 added to 20). The responses that her students shared all

Fig. 1 The Tzolkin calendar comprised 13 days and 20 names.

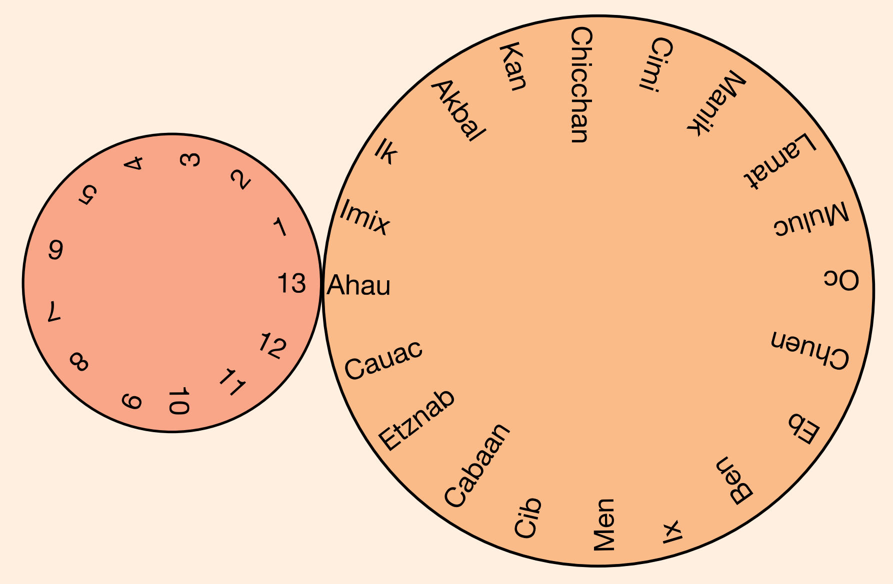
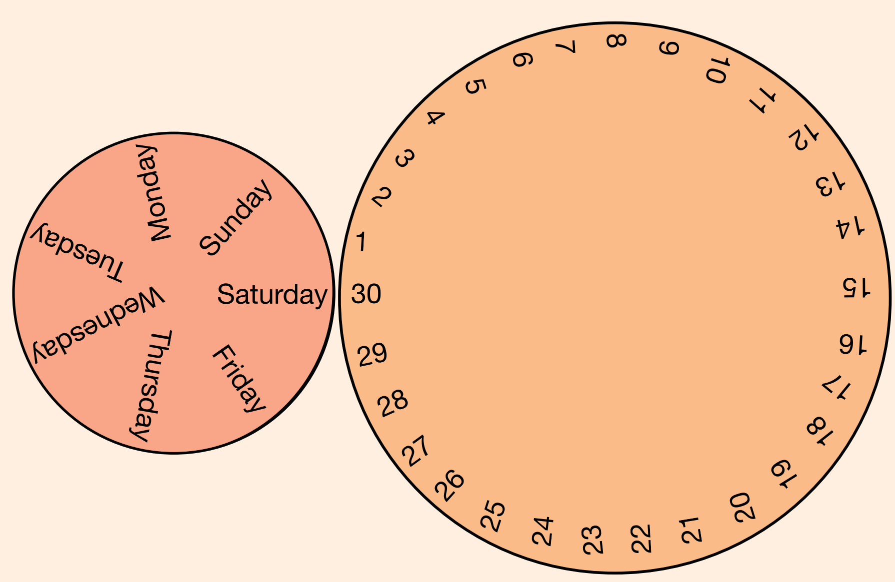


Fig. 2 The Gregorian calendar comprises seven names and approximately thirty days.



gave reasonable explanations for why they chose the estimate for the length of the calendar cycle.

Rehm then asked the students to analyze the calendar in search of an exact response to the posed task. To help her students visualize the Tzolkin calendar, she asked her students to create a hands-on manipulative while working in pairs. They used brad fasteners to create their own Tzolkin calendar "dials," which could be rotated by hand. See the **sidebar** (on page 112) for a template. Numerous students used the manipulative they created by rotating the dials toward each other to track the Tzolkin calendar cycle, an example of students using tools strategically.

As shown in **figure 1**, the dials are positioned at 13 Ahau, and the days start with 1 Imix. The cycle ends when the dials return to 13 Ahau. Several students preferred to observe an animated rotation in an on-screen applet that Rehm created. She encouraged students to record the dates that included either 13 or Ahau as the calendar cycle progressed. Each date that included 13 represented a full rotation of the left dial of the calendar manipulative, and each Ahau date represented a full rotation of the right dial. By tracking the completion of each rotation independently, students could see that those completions first coincided after the number of days that was the smallest multiple of both 13 and 20 (i.e., the LCM). A completed list of those dates appears in **figure 3**. These lists not only helped

Fig. 3 Students recorded the dates that included either 13 or Ahau.

Dates That Include 13	Dates That Include Ahau
13 Ben	7 Ahau
13 Cimi	1 Ahau
13 Cauac	8 Ahau
13 Eb	2 Ahau
13 Chicchan	9 Ahau
13 Etznab	3 Ahau
13 Chuen	10 Ahau
13 Kan	4 Ahau
13 Cabaan	11 Ahau
13 Oc	5 Ahau
13 Akbal	12 Ahau
3 Cib	6 Ahau
13 Muluc	13 Ahau
13 Ik	
13 Men	
13 Lamat	
13 Ix	
13 Imix	
13 Manik	
13 Ahau	

students answer the lesson's main question—"How many days are in the Tzolkin cycle?"—but also allowed students to look for patterns in a sequence, which ultimately helped them to better understand LCMs.

Rehm monitored the progress of each pair of students while they completed these lists. Many were confused about which dates they should record, so Rehm animated the on-screen Tzolkin calendar applet and demonstrated how to identify several entries. Some students struggled to physically coordinate the rotation of both dials together, resulting in lists of dates being off track. If Rehm noticed students completing the first couple of entries incorrectly, she asked them to begin again. However, if others were further along, she indicated what the most recent entry should have been and encouraged them to begin their rotations again from that date. She periodically asked students how many days they counted to remind them of the main question. Once a pair recorded four or five entries, she temporarily shifted the student focus by encouraging them to look for patterns.

Students quickly saw a pattern in the dates that included Ahau (the right column of **fig. 3**), which allowed them to complete their list without rotating the dials. Students articulated the pattern of Ahau in one of three methods. The first two methods describe sequences. In method 1, students articulated the pattern as a pair of interwoven sequences, each adding 1 (e.g., 7 Ahau increases to 8 Ahau, and 1 Ahau increases to 2 Ahau). In method 2, students verbalized the pattern as a single sequence, alternating between subtracting 6 and adding 7.

The third method surprised Rehm. Students noticed that the numbers attached to Ahau formed a pattern in modulo 13, which is akin to arithmetic on a 13-hour clock. When counting past the number 13 in modulo 13, instead of labeling the dates 14, 15, 16, and so on, the numbering starts over with 1, 2, and 3, respectively. In the case of the Tzolkin dial, when adding 7 to the date 7 Ahau, rather than writing the new date as 14 Ahau, it is written as 1 Ahau because in modulo 13, the number 14 is 1 (mod 13). It is mathematically written as

$$14 \equiv 1 \pmod{13}.$$

Students articulating this third method described the patterns they found by recording +7 beside each entry in their list to show that the date consistently increased by 7 (see **fig. 4**). They then showed a conversion to modulo 13. Beside the date 2 Ahau, they recorded the number 15 to show that in the cycle of 13 numbers, 15 is equivalent to 2, or

$$15 \equiv 2 \pmod{13}.$$

Attempting to describe this conversion,

Fig. 4 Modular arithmetic was used to find the date.

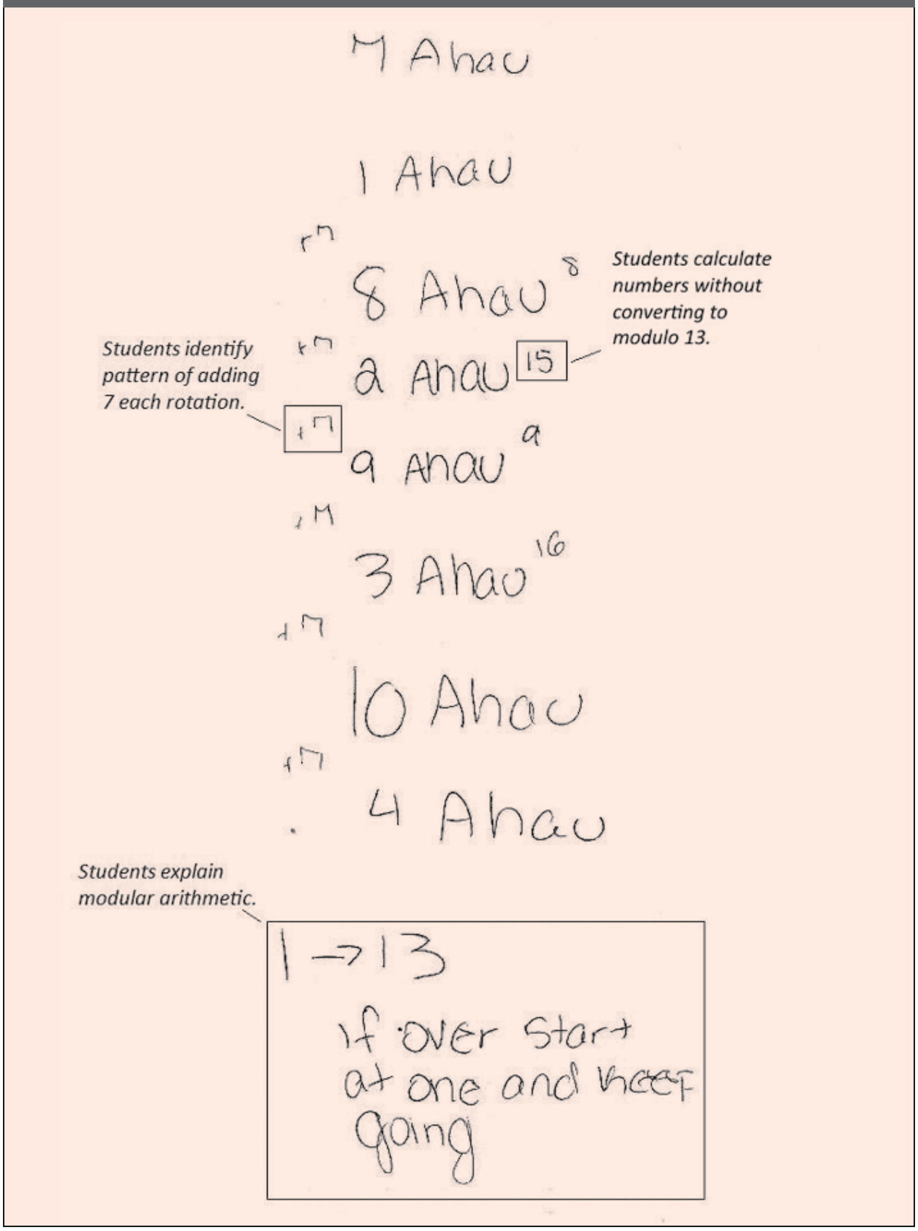


Fig. 5 Students analyzed the 13 list to notice that consecutive names were 7 spaces apart.

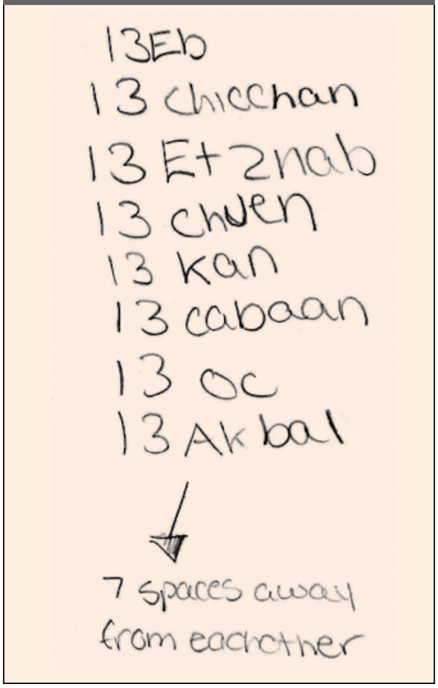
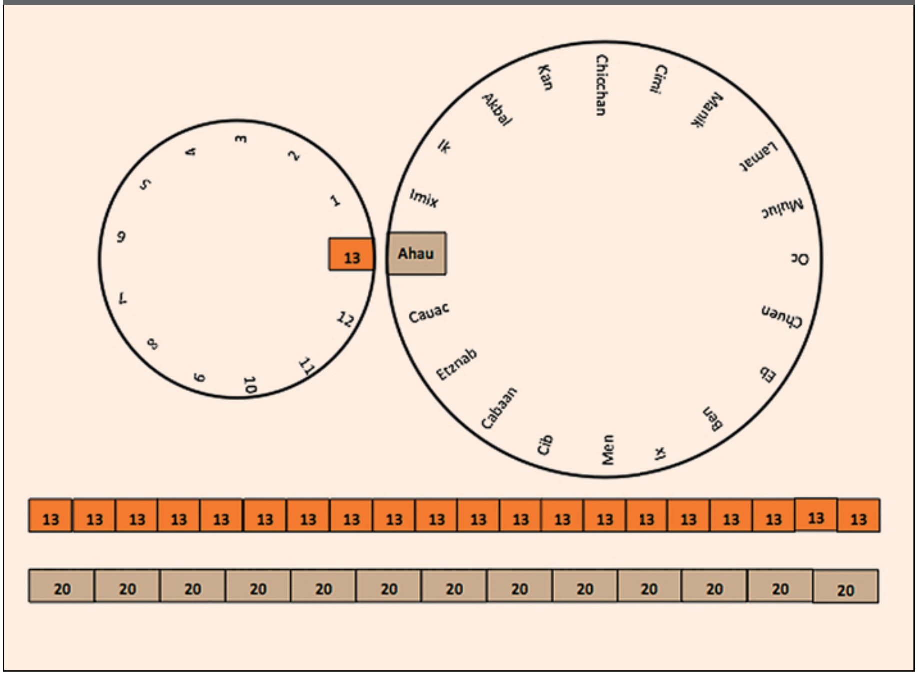


Fig. 6 An animation shows when the cycle completes.



students referred to the numbers 1 through 13 and wrote, "If over [13], start at 1 and keep going."

Because some pairs of students completed their Ahau lists before others, Rehm encouraged them to look for a pattern in the 13 list (the left column of **fig. 3**). This pattern was harder to discern, but when students compared the 13 list with the Tzolkin dials, they noticed that consecutive names in the list were consistently 7 spaces apart in the dial (see **fig. 5**).

Using the completed Ahau list, Rehm redirected students' attention to the main question of determining the number of days in the Tzolkin cycle: Ahau is one of 20 day names, and so each occurrence of the name Ahau represented a rotation through 20 days. Thus, the occurrence of Ahau 13 times represented a rotation through 13 times 20 days, or 260 days. Likewise, although less likely to come up naturally in student discussion, the dates that included 13 eventually cycled through all 20 day names, representing a rotation through 20 times 13 days.

The final calculation of 260 days occurs by multiplying 13 by 20, as some students had predicted earlier. However, by physically using the manipulative and looking for structural patterns in the Tzolkin calendar cycle, many students developed an intuitive calculation of the LCM. For example, one student wrote the ratio 20:13 on her paper and explained, using her hands, that as the days increased by chunks of 20 and 13, they needed to meet at the same number. Rehm showed students another animation of the Tzolkin calendar (see **fig. 6**) to help solidify this intuitive understanding of LCMs. The animation built rows with blocks of length 13 and 20, adding a block each time the respective side of the calendar completed a rotation.

Rehm posed several questions for students to address as they observed the animation. First, she asked, "Why do the LCM and the product of 13 and 20 yield the same result?" Her students were quick to respond that 13 and 20 "aren't alike," suggesting that they are relatively prime.

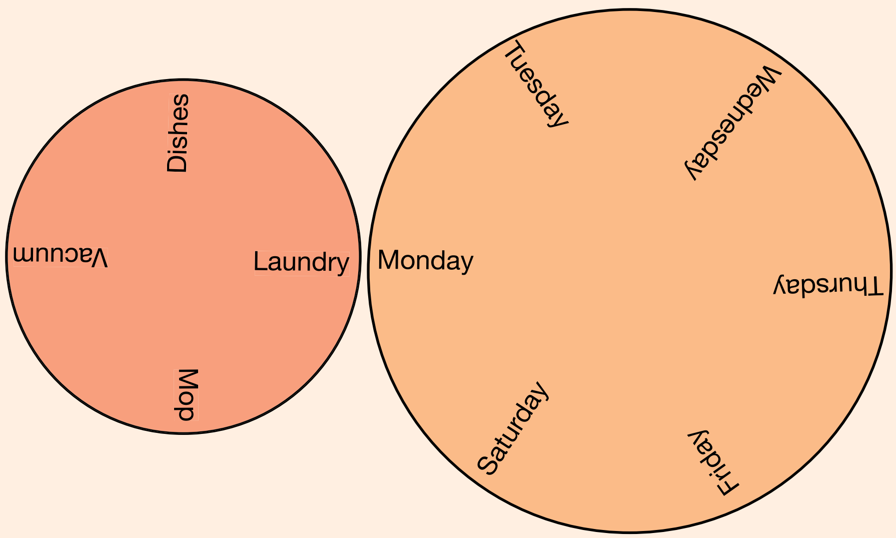
Next, Rehm asked, "Will the LCM and product of two numbers always yield the same result?" This question tested students' understanding of relative primes and led to some debate, allowing them to construct viable arguments and critique one another's reasoning, as illustrated in this dialogue:

Corinne: Yes, the product and LCM are always the same.
Sarah: No, they're not. It only worked last time because 13 and 20 aren't alike.
Corinne: So, some numbers are alike?
Sarah: Yeah, like 5 and 10. The product is 50, but the LCM is 10.

This debate led naturally into Rehm's final question, "Which method will accurately determine the number of days in a calendar cycle?" At this point in the class discussion, many students now understood the connection of an LCM to a calendar cycle. However,

some students were still uncertain whether the product or the LCM of 13 and 20 had determined the 260 days in the Tzolkin calendar.

Fig. 7 A chore calendar can provide another context for reinforcing LCM.



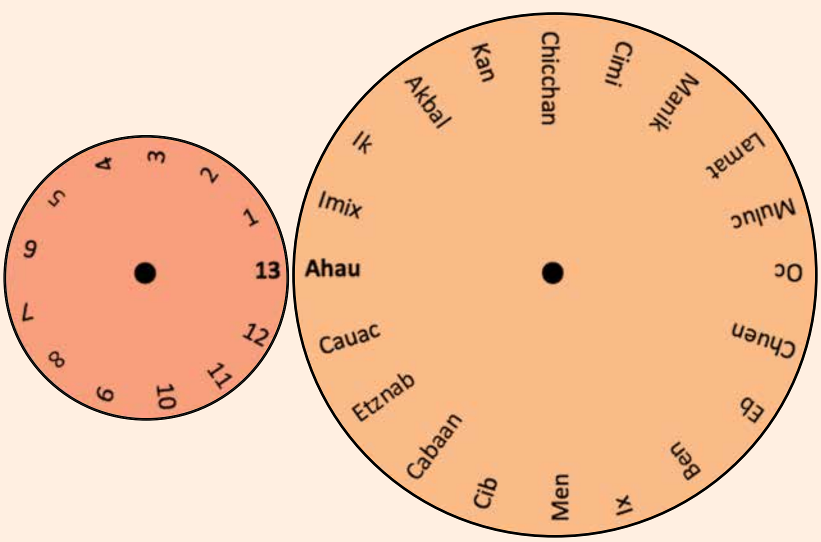
REINFORCING THE LCM CONCEPT

To help solidify students' understanding about using an LCM versus a product, Rehm posed a smaller-scale problem. She shared the chore calendar (see **fig. 7**) and asked, "In how many days will you have to do laundry on Monday again?" This calendar included four chores repeated over a 6-day cycle (the calendar week excluding Sunday).

Rehm asked all students to make a prediction, one by one aloud, providing the opportunity for each individual to state an initial argument to be either justified or rejected. At the beginning, several students shared that they multiplied 4 by 6 just as they had 13 by 20, and so they incorrectly predicted 24. However, after a few students correctly predicted 12 as the LCM, others followed suit. When the chore calendar was animated, students counted the number of days that passed before Laundry and Monday coincided, again proving to themselves that the answer was 12, not 24. After the animation, Rehm asked students who had predicted 12 to explain their correct arguments, and they described the calculation of the LCM of 4 and 6.

Manipulative Tzolkin Calendar

This template can be used to create a hands-on manipulative. Use brad fasteners to attach the dials to poster board.



REFLECTING ON THE MAYA CALENDAR LESSON

Rehm realized that several students who did not typically engage in her mathematics class had embraced the material's cultural basis and had participated enthusiastically.

One student excitedly shared her knowledge of the Spanish language. Another student compared the Maya mythology with that of his parents' native country in Southeast Asia. The inclusion of such culturally relevant material in the math classroom seemed to reach students in a unique way. This lesson was designed to enrich student understanding of LCMs and help students develop several of the SMP.

Furthermore, the cultural focus of this lesson helped students develop a perspective of an ancient society by drawing on "students' culture, conditions, and language to support and enhance mathematics learning" (NCTM 2014, p. 63).

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Any thoughts on this article? Send an email to **mtms@nctm.org**.—*Ed.*



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