Penguin Math

Daniel Green and Thomas Kearney

Many students are not motivated to learn mathematics when textbook examples contain largely nonexistent contexts or when the math is not used to solve significant problems found in real life. This article's project explores how male Emperor penguins are able to survive in Antarctica. In so doing, it addresses measurement, applies geometry, and helps foster problem-solving skills, all within a real-life context.

GETTING TO KNOW THE EMPEROR

Emperor penguins, the largest of all the penguin species, attain heights of nearly four feet and weigh up to 99 pounds. Their pale yellow breast and bright yellow ear patches distinguish them visually. As part of their breeding cycle, male Emperor penguins survive in Antarctica for four months with no food and incubate the eggs that their mates have entrusted to them.

Emperor penguins breed during the Antarctic winter. Temperatures will typically drop to –40°F, and winds can reach 90 mph. Emperors have only one mate, which they must find in a large breeding colony. Females lay one egg in May or June and then, with their nutritional reserves depleted, head back to the ocean to feed for two months while the males incubate the egg and care for the newly hatched chicks.

When the females return, the males will have gone without food for about four months and can lose up to 44 pounds of body weight. Once the females return, the males feed for a few weeks, and then parenting duties are shared.

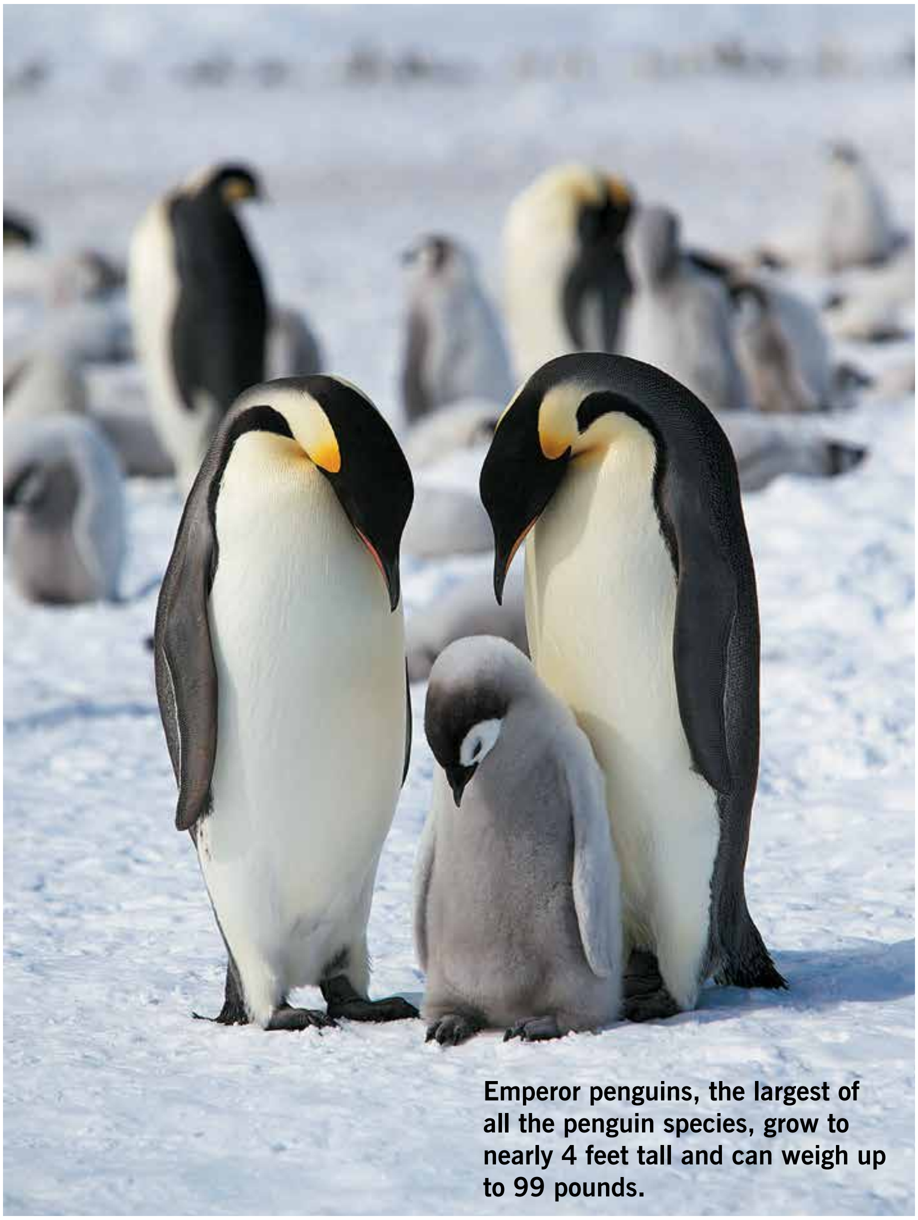
For the Emperor penguin to survive in such an extreme environment, it has developed several adaptations to the cold. Its densely packed feathers provide about 85 percent of its insulation; it also has a thick layer of up to 3 cm deep subdermal fat. The head, bill, and flippers are proportionally smaller than those of other penguins, helping maintain body heat because these areas are not as well insulated. To conserve energy, these penguins huddle in groups that can range from 10 to 5,000. Huddling not only decreases the exposed body surface area but also significantly reduces heat loss. This last aspect of huddling forms the basis for this project.

PREREQUISITES

We want students to estimate the body surface area (BSA) of an individual penguin and compare it with a BSA estimate of a huddled penguin to determine the heat-saving benefit of huddling. Students must be familiar with computing the surface area of basic three-dimensional solids, such as spheres, cylinders, and cones. To

Edited by **S. Asli Özgün-Koca**, aokoca@wayne.edu, Wayne State University, Detroit, Michigan, and **Marilyn Howard**, marilyn-howard@utulsa.edu, University of Tulsa, Oklahoma. Readers are encouraged to submit classroom-tested activities through **http://mtms.msubmit.net**.

Copyright © 2015 The National Council of Teachers of Mathematics, Inc. www.nctm.org. All rights reserved.
This material may not be copied or distributed electronically or in any other format without written permission from NCTM.



IAN DUFFY; HTTPS://WWW.FLICKR.COM/PHOTOS/IANDUFFY/SETS/72157622746169291

Emperor penguins, the largest of all the penguin species, grow to nearly 4 feet tall and can weigh up to 99 pounds.

complete the computations, they need to have some experience with measurement conversions. It is also helpful for students to know that energy can be measured in calories; thus, a gram of body fat provides a fixed number of calories.

DEVELOPING ESTIMATION SKILLS

The goal of the project is for students to estimate how much energy penguins can conserve by huddling in groups. Because a huddle of penguins can be enormous and almost impossible to count, we begin by showing the class some images of a packed football stadium, a petri dish with bacterial culture, and various wildlife photographs. We then ask students how they could possibly count the individual people, bacteria, and animals. We discuss strategies, such as counting rows and columns and then multiplying, counting a section and multiplying by the number of sections, and dividing into grids and counting a random selection of squares (which is exactly how biologists count some microorganisms).

We then tell them that they are researchers in Antarctica and need to count an Emperor penguin huddle because these penguins are becoming an endangered species. We have students use a classroom set of binoculars that will allow two students per group to view the huddle at a time; students can also hold their hands up in the shape of binoculars for the same effect. We tell them that our estimates must be done quickly because it is the middle of the Antarctic winter and the temperature is –40°F, so we cannot spend a lot of time outside the shelter of our vehicle. We then project an image of a large penguin huddle onto the white board for 30 seconds and ask students to count or estimate the number of penguins. (See the image at this site: http://www.arkive.org/emperor-penguin/aptenodytes-forsteri/image-G57959.html.) No one is allowed to say a word until the binoculars have been handed off to other group members. We then ask for their answers and how they estimated the number.

There were 338 penguins in this modest huddle, but we received responses from 100 to more than 1,000. Although some students simply guessed, others said that they "counted the outer ring and multiplied the number of rings" or "found groups of 100, considered there to be three groups of 100" or "counted the first row and multiplied by the number of rows."

PROPORTIONAL REASONING

The next step for our Antarctic researchers was to estimate the body surface area, or BSA, of a penguin by using measurements on printed or digital images. One advantage of this approach was that it introduced students to a real-life application of proportional reasoning. They computed the scale of the image and then found the ratio, or scale factor, to convert their measured quantities to full-size measurements, at which

Fig. 1 This student explained how he found his scale-factor number.

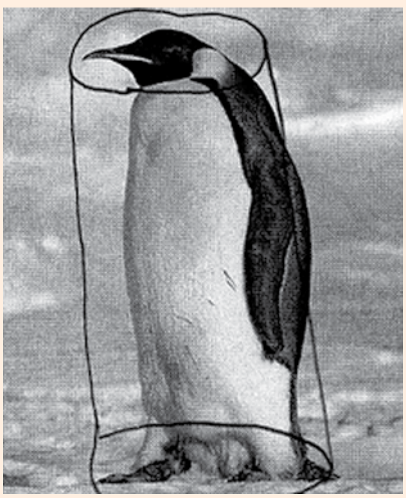
The scale factor is 120 cm.:6cm. This is also 20cm.:1cm. We got this by finding that the real height is 120cm. and the picture is 6 cm. 120 ÷ 6 = 20, so the ratio is 20cm.:1cm.

time they could use them to find the required estimates.

We asked that students try to model the body of the penguin with one three-dimensional solid that they thought would best approximate the BSA. Some students struggled to make the connection between the two-dimensional images representing three-dimensional objects. For example, some groups initially used a rectangle or a rectangular box to approximate the penguin body instead of a cylinder as we had hoped, not perceiving that the penguin body was three dimensional or round. Some students also had trouble comprehending the idea of the scale of the image compared with a life-size penguin.

To help the students make these connections, we brought in eight-inch plush stuffed Emperor penguins and gave one to each group. Although the stuffed penguins were not perfectly proportioned anatomically, they were close enough to help students see the cylindrical shape of the body. The model penguins were also valuable in helping students understand scale factors and proportion.

Fig. 2 A superimposed cylinder covered the “ovalish, 3-D” body of Sir Barkley V.



The solid that we think is a cylinder
It would be a cylinder because
the penguin has an ovalish, 3-D
type of body.

An adult Emperor penguin is about four feet tall, so we asked the students how many of the plush penguins would have to stack on top of one another to be as tall as an adult Emperor. Some students were able to do the dimensional conversion mentally and get the answer right away, but to make sure everyone understood, we asked students to participate in a hands-on demonstration. We held up a measuring tape extended to four feet and asked the groups to stack their penguins on top of one another to physically see that six stuffed penguins fit into the height of the life-size adult. This stacking illustrated a 1/6 scale, or 1:6. This activity also had the added benefit of demonstrating to the students that the radius of their image had to be multiplied by the same factor as the height to accurately describe the life-size penguin because the stack of six penguins was far too skinny to be proportional. (See **fig. 1** for one student explanation of scale.)

MEASUREMENT UNCERTAINTY

We stopped the class after the image measurements were taken, and again after they computed the penguin’s BSA, and asked them to share their results. They were surprised to see that there were many different answers for the measurements and the final BSA of the Emperor penguin. For example, in one class with six groups, the height measurement of the image ranged from 6.0–6.2 cm and the radius from 1–1.5 cm. The larger difference in the radius measurement variation resulted from the fact that two of the groups thought the approximating cylinder should not exclude any part of the beak. (See **fig. 2**.)

We asked students why the answers varied and whether or not there was just one correct answer. In mathematics, some students find variation and uncertainty foreign and puzzling. We questioned them further: “Are the images different? Are the rulers different?” They soon realized that different people can look at the same object and see something slightly different. We explained that any measured quantity will inherently have measurement error. In this case, there was not one correct answer, in part because the height and radius of the photograph could not be measured exactly with a ruler because a ruler does not have infinite accuracy or precision.

A more appropriate question is this: What is the best estimate from our experimental data? One answer would be to just state a range of values for the measurement, as we did above. A better answer would be to state that the best estimate of the answer is the average of the different measurements, and we discussed this tactic as one way to deal with the variation. For this class, the average height was $h \approx 6.03$ cm and the average radius was $r \approx 1.29$ cm. However, we should also ask what an appropriate allowed uncertainty would be for the means. Ideally, we want to express an answer as

$$\text{measurement} = \text{best estimate} \pm \text{uncertainty}.$$

We did not pursue this further with our sixth-grade classes because these students did not have the necessary statistical background. However, students in later grades could consider the standard deviation of the measurements.

After scaling up to life-size measurements, the resulting total BSA for the groups varied from 17,584 cm^2 to 28,260 cm^2, which was a significant range. (See **table 1**.) If we use the average class measurements, the scaled-up height and radius are approximately 120.07 cm and 25.83 cm, respectively, and the area estimate is

$$A \approx 2\pi(25.83)^2 + 2\pi(25.83)(120.06)$$
$$\approx 23{,}779 \text{ cm}^2.$$

This is a better estimate for the life-size BSA. Incorporating measurement uncertainty into the calculation of the BSA is a little more work. We did not go into the details with sixth graders but have included a discussion with the online solutions. However, we did emphasize that measurements involve error and that we must be aware of that error, and at least consider averaging, and not rounding, intermediate values as ways to improve our experimental results.

We saw the same proportional reasoning and measurement variation when we had the students compute the huddled BSA per penguin. We first brought our group of stuffed penguins together and formed a miniature huddle so that students could again visualize what shape the huddle would take. After realizing that the huddle, too, was fairly well approximated by a cylinder, we asked students to compute the scale factor from the photograph, measuring the radius and diameter of the huddle. After computing the surface area of the cylindrical huddle, they divided by their estimated number of penguins. On average, this class estimated a 66.5 percent decrease in the penguin's BSA (and fat burned) compared with the 49 percent and 56 percent decrease, respectively, that researchers found (Gilbert et al. 2008). This was not discouraging to us, considering how simplistic our model was. We summarize the results for the class in **table 1**.

Table 1 The class data were placed in a table for comparison.

Penguin Name	Height (cm)	Radius (cm)	BSA Alone (cm^2)	BSA Huddled (cm^2)	Decrease
Sir Barkley V	6	1.5	28,260	9,043	68.0
Peggy	6	1.25	22,765	6,894	69.7
Pablo	6	1.25	22,765	8,425	63.0
Gunter	6	1	17,584	7,206	55.8
Skinny Jimmy	6	1.25	22,765	7,206	63.3
Philbert	6.2	1.5	29,028	7,390	74.5
Mean	6.03	1.29	23,861	7,788	66.5

QUESTIONING OUR ASSUMPTIONS

To complete this project, students had to make some simplifying assumptions, as is the case in any mathematical model. Too many times, others make these assumptions for the student, and the students never question or evaluate the assumptions. Many times, the assumptions are not even acknowledged.

Along the way, we emphasized what assumptions we were making and asked students what effect they thought the assumptions would have on the outcome. For example, we approximated the surface area of a single penguin by the surface area of a cylinder. We asked students why they thought this was the best choice and whether the cylinder would overestimate or underestimate the true surface area. (See **fig. 3**.)

Students argued both ways, with most deciding that it would overestimate because there was "empty space left"; others thought it would underestimate because parts of the penguin's body were outside the cylinder. When we asked them what solid would approximate the penguin huddle, they again concluded that a cylinder would

Fig. 3 A cylinder's shape became a real-life issue of context.

A cylinder would be the best shape for the huddle. We should count the bottom as exposed because it is on the cold ice

The huddle isn't a perfect cylinder. This is because of their lumpy backs, pointy beaks, little spaces between each other and different sizes of penguins.

To conserve energy, penguins huddle in groups that can range from 10 to 5,000.

FUSE/THINKSTOCK

be most appropriate. We then asked if we should include the bottom of the cylinder as exposed surface area and if the area of the solid would overestimate or underestimate. One class wanted to include the bottom because heat could escape through the ice; another group disagreed because the cold air and wind could not get beneath the penguins on the inside of the huddle. Most groups thought the solid would underestimate the true surface area of the huddle because the cracks between the bodies, the space between the feet, and the bumpy surface created by backs and heads were not included. We likened it to the fact that it takes more paint to cover a bumpy surface than a smooth one.

We then discussed the estimate of the fat burned by a huddled penguin. We asked what assumptions they were making, how they would affect their answer, and whether all the penguins would have the same amount of their surface area exposed. Students said that the penguins on the outside would be more exposed and were unsure how this would affect their answer. We then revealed that the penguins perform a kind of dance in which they constantly rotate their position in the huddle so that they share the burden of the cold over time. Also, we are assuming that the penguins are in the huddle continuously, for twenty-four hours a day, which is unlikely, so we were overestimating the amount of heat saved by huddling. This type of analysis is difficult for many students and sometimes requires careful questioning and class discussion, but it is absolutely beneficial to their critical-thinking skills.

REAL-LIFE MATHEMATICS

This project explained one of the ways that Emperor penguins have adapted to survive the extreme conditions of Antarctic winters. Students used a fairly small set of background skills to combine and apply geometry, measurement, conversions, and (possibly) statistics to the problem of computing penguin body surface area and heat conservation. Students became aware of the concept of measurement uncertainty and how it could affect the outcome of the problems they were solving. Finally, students learned about model assumptions and questioned their validity and effect on the outcome of their computations. They combined many areas of mathematics with science to solve a real-life problem; it was a meaningful and fun learning experience for both teacher and student.

BIBLIOGRAPHY

Australian Government Department of the Environment: Australian Antarctic Division. nd. "Breeding Cycle." http://www.antarctica.gov.au/about-antarctica/wildlife/animals/penguins/emperor-penguins/breeding-cycle

Center for Biological Diversity. nd. "Emperor Penguin." http://www.biologicaldiversity.org/species/birds/penguins/emperor_penguin.html

Gilbert, Caroline, Stephan Blanc, Yvon Le Maho, and Andre Ancel. 2008. "Energy Saving Processes in Huddling Emperor Penguins: From Experiments to Theory." *Journal of Experimental Biology*, 211 (1): 1–8. http://jeb.biologists.org/content/211/1/1.long

Kearney, Thomas. 2012. *Zoo-a-Logical Math: A Mathematical Challenge between Humans and the Rest of the Animal World.* Indianapolis, IN: Dog Ear Publishing.

National Council of Teachers of Mathematics (NCTM). 2000. *Principles and Standards for School Mathematics.* Reston, VA: NCTM.

Olivier, Fred. "Emperor Penguin" (image). http://www.arkive.org/emperor-penguin/aptenodytes-forsteri/image-G57959.html

Southeastern Louisiana University. 2007. "Measurement and Uncertainty Notes." http://www2.southeastern.edu/Academics/Faculty/rallain/plab194/error.htm

Daniel Green, dgreen@olivet.edu, currently teaches mathematics at Olivet Nazarene University, Bourbonnais, Illinois. His current interests include actuarial science and the application of mathematics to biological problems. **Thomas Kearney**, KearneyT@comcast.net, is a retired high school biology teacher and author of *Zoo-a-Logical Math*, the first of a series of books written for elementary and secondary teachers with the goal of bringing math to life by applying it to various real-life problems in the animal world.

more4U

The reproducible activity sheets and solutions to Mathematical Explorations are online at **www.nctm.org**. Online solutions are available to NCTM members only.

activity sheet

Name ______________________________

PENGUIN MATH

Emperor penguins live in the extreme cold of Antarctica. Temperatures can plummet to –40°F, and winds can reach 90 miles per hours (mph). In such extreme conditions, penguin bodies could lose heat quickly were it not for their densely packed feathers, thick subdermal fat layer, and huddling behavior.

Biologists have found that the amount of heat loss of the penguin's body depends on the amount of exposed body surface area (BSA). By huddling in tightly packed groups, Emperor penguins decrease their exposed BSA and decrease heat loss significantly. This is a necessity during the months of the winter that the males are deprived of food as they incubate eggs that the females have laid. Your job as a researcher is to form a model to estimate how much energy Emperor penguins conserve by huddling.

1. The exact BSA of an Emperor penguin can be difficult to measure. Although the penguin's trunk is smooth and regular, some of the BSA comes from the feet, flippers, and head. However, the BSA can be estimated by using one geometric solid to approximate the penguin's body.

a. Look at the image of an Emperor penguin. Which solid do you think would approximate its body the best?

b. Do you think the surface area of the solid will overestimate or underestimate the penguin's BSA? Explain.

2. The average height of an adult Emperor penguin is about 120 cm.

a. Find the scale factor of the penguin (see below) to the nearest whole number. (*Hint*: Scale factor is the ratio of actual size to model size, so a scale factor of 6 means a 1 cm length on your model is 6 cm in actual size. This is called a 1/6 scale.)

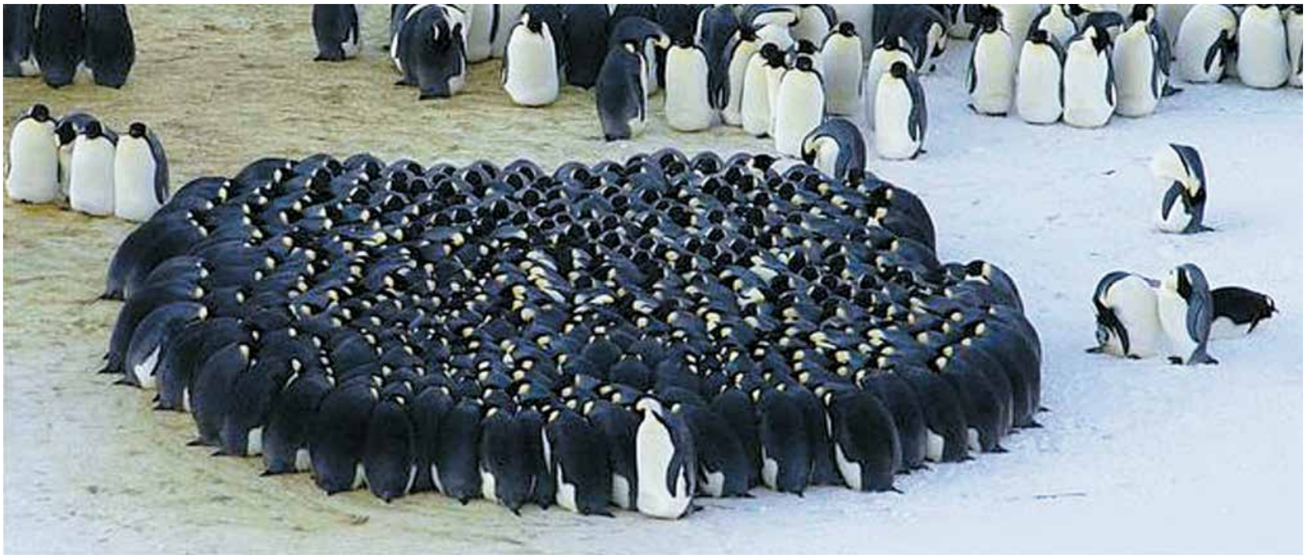
IAN DUFFY; HTTPS://WWW.FLICKR.COM/PHOTOS/IANDUFFY/SETS/72157622746169291

b. Use your scale factor and the solid you chose in question 1 to estimate the BSA of a life-size average Emperor penguin in cm^2. (Hint: Scale all lengths before finding area.)

3. A calorie is a unit to measure energy. Researchers estimated that an Emperor penguin loses heat energy at a rate of 0.002868 calories/cm^2 of exposed BSA per hour in Antarctica. How much heat, in calories, will your average Emperor penguin lose over its whole body in 1 day?

4. If 1 gram of fat provides 8 calories, how much fat (in grams) will an Emperor penguin burn in 1 day to maintain its body temperature?

5. Suppose a group of "average" Emperor penguins huddle as pictured below.



© FRED OLIVIER; NATUREPL.COM

a. What three-dimensional shape approximates the exposed surface area of the group?

b. Will you include the bottom of the shape? Explain.

c. What assumptions will simplify the problem, and how do they affect your answer?

6. Estimate or count the number of penguins in the huddle. Explain your strategy.

7. Estimate the average exposed BSA per penguin by dividing the total surface area of your three-dimensional shape from problem 5 by the number of penguins in the huddle. Use 120 cm as the estimated height of a penguin in finding your scale measurements.

8. Compare the BSA of an individual penguin (from question 2b) with the average exposed BSA for a penguin in a huddle (from question 7).

a. How much did huddling decrease the BSA per penguin?

b. By what percentage did huddling decrease the BSA per penguin?

9. Assume heat loss at a rate of 0.002868 calories/cm^2 per hour

a. How much heat in calories will an average huddled Emperor penguin lose over its entire body in 1 day?

b. How many grams of fat will it burn in a day?

c. What assumptions are you making, and how do they affect your answer?

10. What is the percentage reduction in heat loss and fat burned per penguin by huddling?

Apply for Grants and Awards

***The NCTM Mathematics Education Trust** channels the generosity of contributors through the creation and funding of grants, awards, honors, and other projects that support the improvement of mathematics teaching and learning.*

Did you know? As a member of NCTM, you have access to grants and awards to enhance your mathematics teaching and learning. The Mathematics Education Trust (MET) provides funding opportunities to focus on classroom action research, projects that engage students in learning mathematics, professional development, and graduate study to improve teaching skills and classroom practice.

Begin your search at www.nctm.org/met, where you will find current grants and awards grouped by grades pre-K–5, 6–8, 9–12, and more. Click on any title to see a description of the award or grant, comments from a previous awardee, and eligibility and proposal requirements. The following are examples of MET awards:

- **Future Leaders Initial NCTM Annual Meeting Attendance Awards:** Grants of up to $1,200 plus meeting registration provide for travel, subsistence expenses, and substitute teacher costs of members who are classroom teachers and have never attended an NCTM annual meeting.
- **School In-Service Training Grants:** Elementary, middle, or high schools receive up to $4,000 for support of in-service mathematics programs.
- **Mathematics Coursework Scholarships:** Scholarships of up to $2,000 are awarded to classroom teachers working to pursue courses to improve their mathematics content knowledge.
- **Pre-K–6 Classroom Research Grants:** Awards of up to $6,000 support collaborative classroom-based action research in precollege mathematics education involving college or university mathematics educators.
- **Engaging Students in Learning Mathematics Grants:** Awards of up to $3,000 are given to grades 6–8 classroom teachers to incorporate creative use of materials to actively engage students in tasks and experiences designed to deepen and connect their mathematics content knowledge.
- **Connecting Mathematics to Other Subject Area Grants:** Awards of up to $4,000 are awarded to grades 9–12 classroom teachers to develop classroom materials or lessons connecting mathematics to other disciplines or careers.

A proposal to the Mathematics Education Trust is typically no longer than five pages. Two deadlines occur per year: the first week of May and the first week of November. The MET Board of Trustees reads proposals and notifies awardees by letter in July and February.

The MET Board of Trustees strives to distribute all awards in each funding cycle. Some funds go unused because applications are not received for all grants each year. Take advantage of this opportunity to obtain funding for you or your school. Visit the website on a regular basis to check for updates.

The MET also accepts donations and is always looking to establish new grants and awards. MET is an asset of NCTM and can be an asset for you.

NCTM® | MATHEMATICS EDUCATION TRUST

Visit www.nctm.org/met