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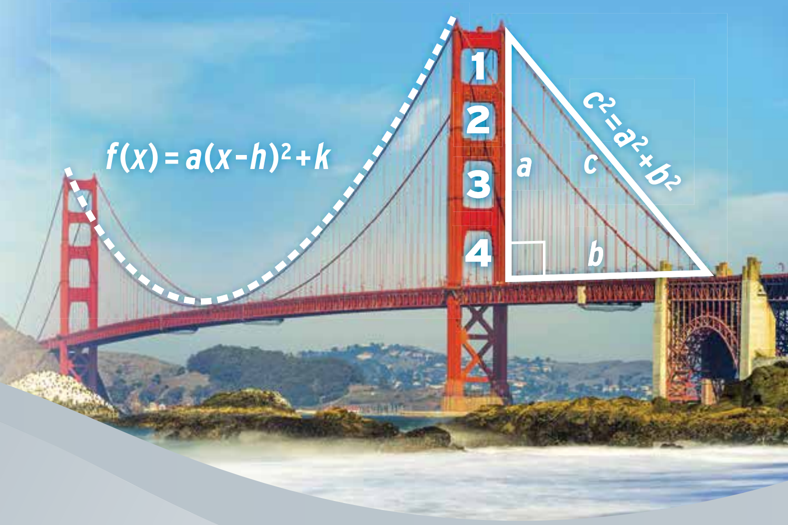

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Jo Ann Cady and Sherry L. Bair

big solutions to little problems

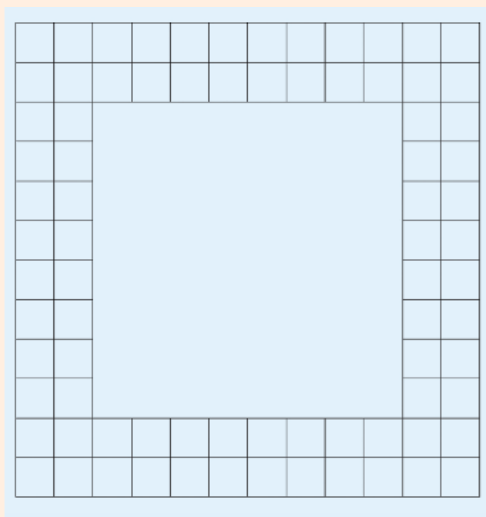
This Solve It task appeared in the December 2014/January 2015 issue:

NEIL PODOLL/THINKSTOCK

The Fire Pit

A local company installs fire pits in residential areas. The basic pit is installed in the center of an 8 ft. × 8 ft. square concrete pad. Some people want larger seating areas around the pit, so the company will add 1 ft. × 1 ft. concrete pavers around the pad. For example, to make a pad that is 12 ft. × 12 ft., builders will add 2 rows of pavers around the 8 × 8 pad, as shown below, by using a 2 foot wide border of pavers.

The owner of the company must calculate the number of pavers he needs to build a border around the original 8 × 8 pad that has any whole number for its border. Can you help him find a formula that will work?



CCSSM: Standards 6EE.2a; 6EE2b; 6EE2c; 6EE.9; 7EE.4a; and 7EE.4b

This department shares creative solutions to the problems presented in Solve It. Edited by **Jo Ann Cady**, jcady@utk.edu, and **Sherry L. Bair**, slbair@yahoo.com.

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This month, we highlight the work of seventh graders in Timothy Folger's class at Elida Middle School in Elida, Ohio. Folger provided a written description of how he introduced the problem to his students, samples of student work, and a written reflection of his experience using the Fire Pit task with his students.

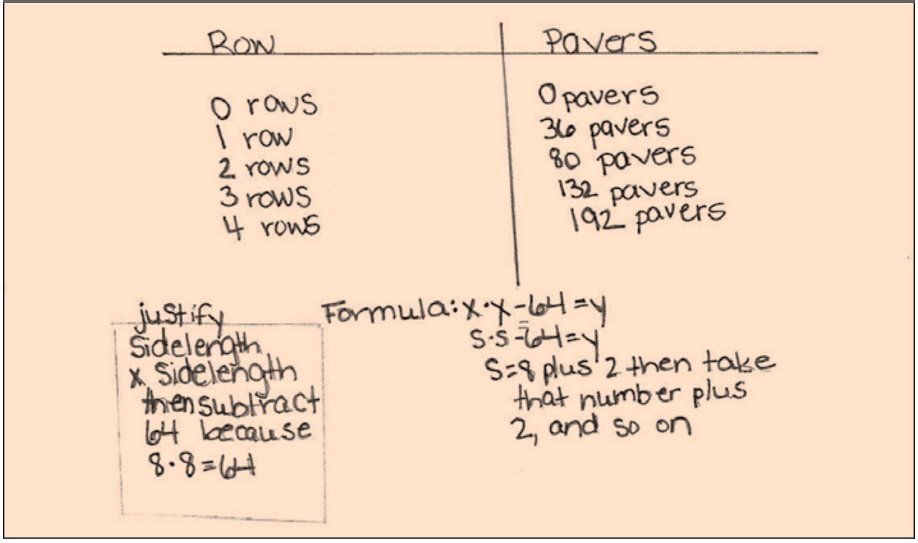
Folger saw this problem as an opportunity to have students model with mathematics (CCSSM's Standard for Mathematical Practice 4; CCSSI 2010) and, therefore, devoted several days to the problem. On the first day, he provided only the first paragraph of the problem and the illustration, hoping that students would focus on making sense of the scenario and developing their own questions about the context of the problem. This strategy and the task's high level of cognitive demand (Stein and Smith 1998) produced a variety of student responses. Folger said the questions that students developed regarding the scenario ranged from lower to higher levels of cognitive demand.

Folger reported that it was the second day of the activity when students were presented with the task of developing a formula to determine the total number of pavers needed. The students had not been formally introduced to expressions and equations, so the Fire Pit problem was their first foray into this domain. The problem also presented an opportunity for students to learn new ideas through problem solving. His students did have prior experiences working with situations involving independent and dependent variables when they used ratio tables to develop the constant rate of change in proportional relationships.

Although this task did not represent a proportional relationship, their prior experience did offer an immediate entry point into the exploration of the problem. "By the end of this day, the majority of students had constructed a table, comparing the rows of pavers and the total number of pavers needed [see **figs. 1** and **2**]. Some students accomplished this by drawing pictures [see **fig. 3**]. Other students discovered patterns in the increase of pavers required for an additional row." A common mistake that Folger noticed occurred when students recorded the increase in the number of pavers from one row to another as the total number of pavers rather than the total number of pavers in all for *x* rows. The use of recursive reasoning is expected of students at this level, and the mistake that he noticed was typical. Folger explained that his students were able to see the difference between these quantities and progressed toward a generalization of the pattern and an explicit formula describing the relationship between rows of pavers and total number of pavers.

Folger also noted that many students showed an immediate interest

Fig. 1 This example of student work contained an explicit formula for the total number of pavers as $x \bullet x - 64 = y$.

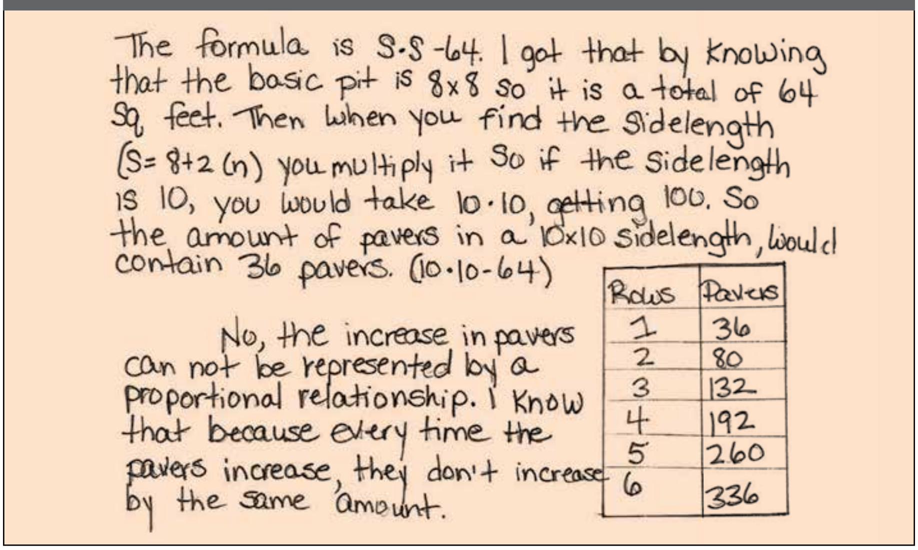


Row	Pavers
0 rows	0 pavers
1 row	36 pavers
2 rows	80 pavers
3 rows	132 pavers
4 rows	192 pavers

justify
Sidelength x Sidelength then subtract 64 because 8·8=64

Formula: x·x-64=y
S·S-64=y
S=8 plus 2 then take that number plus 2, and so on

Fig. 2 This student constructed a table, comparing the rows of pavers with the total number of pavers needed.

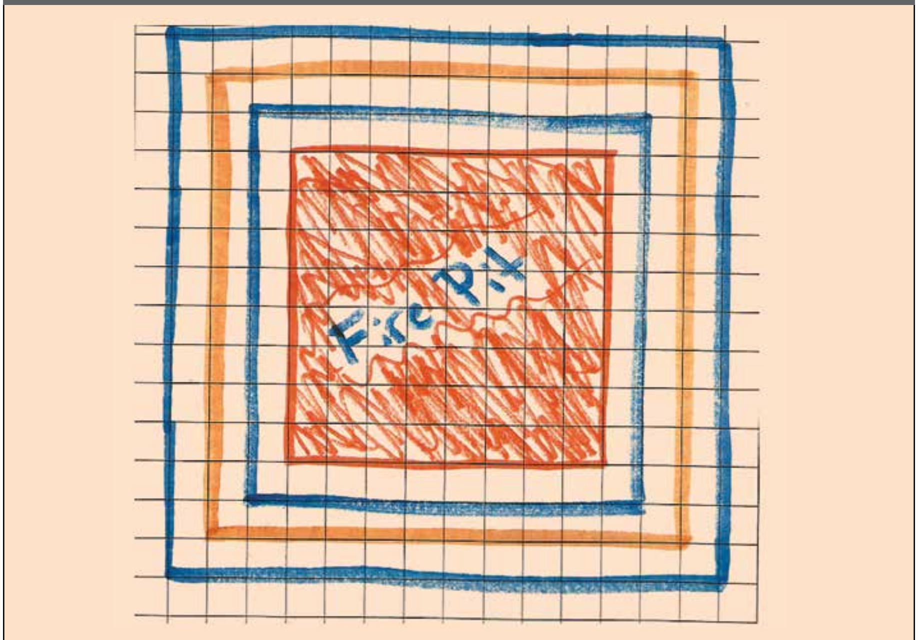


The formula is S·S-64. I got that by knowing that the basic pit is 8x8 so it is a total of 64 Sq. feet. Then when you find the Sidelength (S= 8+2 (n) you multiply it So if the Sidelength is 10, you would take 10·10, getting 100. So the amount of pavers in a 10x10 Sidelength, would contain 36 pavers. (10·10-64)

No, the increase in pavers can not be represented by a proportional relationship. I know that because every time the pavers increase, they don't increase by the same amount.

Rows	Pavers
1	36
2	80
3	132
4	192
5	260
6	336

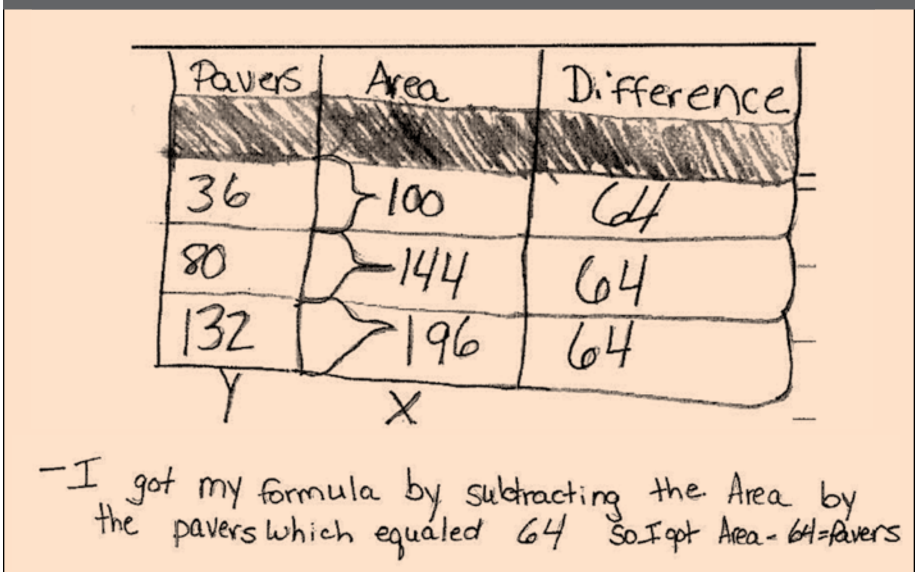
Fig. 3 A student drew a fire pit to provide a visual before generalizing.



in applying the concept of perimeter to calculate the total number of pavers. Students were capable of describing the number of pavers on one side as 8 plus 2 times the number of rows (r) and the perimeter as $4(8 + 2r)$, but struggled to determine the total number of pavers needed using the idea of perimeter. Finding the total number of pavers using the idea of perimeter is beyond the scope of the middle school curriculum, and some students had begun deriving a formula by looking at the area of the fire pit. Therefore, Folger steered the class discussion toward the concept of area and how it was measured in square units.

As a result, each sample of student work that Folger provided shows a formula derived using an area argument.

Fig. 4 This student work did not connect the number of rows of pavers with the area of the entire square.



The student work in **figure 1** included the formula for the total number of pavers as $x \cdot x - 64$, where x represents the side length of the entire square. Although this formula answered the original question, it did not describe the explicit relationship between rows of pavers and total pavers, as shown in the table. The work described how the side length, s, can be determined recursively, given the number of rows ("s = 8 plus 2, then take that number plus 2 and so on").

The student work in **figure 2** contained an explicit formula similar to the other group's work, relating side length with total number of pavers, but it provided an explicit formula for determining the length of a side (s) of the entire square given the number of rows (r) (e.g., $s = 8 + 2r$). This student also described the relationship as being nonproportional because it did not grow at a constant rate.

The student work in **figure 4** contained a table relating the number of tiles with the area of the entire square. Although the formula found by the student, "Area – 64 = Pavers," correctly described the table shown, the work did not show an attempt to connect how the number of rows of pavers can be used to determine the area of the entire square. All student work samples shown demonstrated how students made sense of new situations and how they solved complex problems by connecting new information with past experiences on proportions and measurement.

In his reflection on the task, Folger stated,

> It should be noted that none of my students were successful at deriving a formula by analyzing the numerical progression of the pavers, but this was expected because students in seventh grade are primarily responsible for identifying and creating linear relationships [e.g., CCSS.Math.

Content.7.RP.A.2.C: "Represent proportional relationships by equations"]. It is interesting that students who applied a conceptual understanding of geometric concepts could also explain what the numbers and variables of their formula represented. The Fire Pit problem was a great task that will lead us further into the domains of Expressions and Equations, and Geometry.

THE EDITORS' REFLECTION

We provided a challenging task that could be approached in a variety of ways. It also presented opportunities for whole-class discussions about some key mathematical concepts at a variety of levels, as evidenced by Folger and his seventh-grade students. His reflection was indicative of teachers knowing their students best and adapting the task based on their knowledge of their students' mathematical understanding to help students focus on important ideas and build on prior knowledge. Modifications, scaffolding, and clarification can be done without lowering the cognitive demand of the task.

The length of the class period can also be a constraint that needs to be addressed; high-level tasks require several shorter class periods (45–60 minutes) or one longer class period (90 minutes). The Common Core State Standards for Mathematics are challenging for students and teachers alike, but these types of problems and experiences can help develop the atmosphere needed in the classroom that will help teachers feel successful in implementing the mathematical practices in their classroom and help students achieve the goals of the Common Core's content standards.

We appreciate the time and effort that Folger took to submit a response and his students' willingness to share their work with *MTMS* readers. Examples of actual middle school students engaging in high-level tasks provide the focus for the Solve It department.

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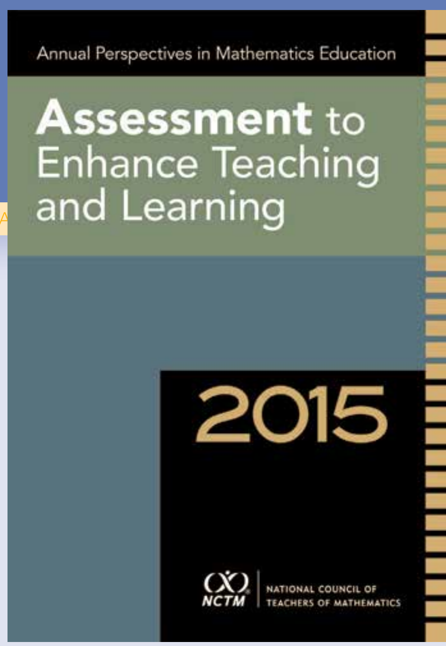

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