EPIDEMICS, Expone Mod

The phenomenon of a plague intrigues students and provides a visual model that demonstrates growth.

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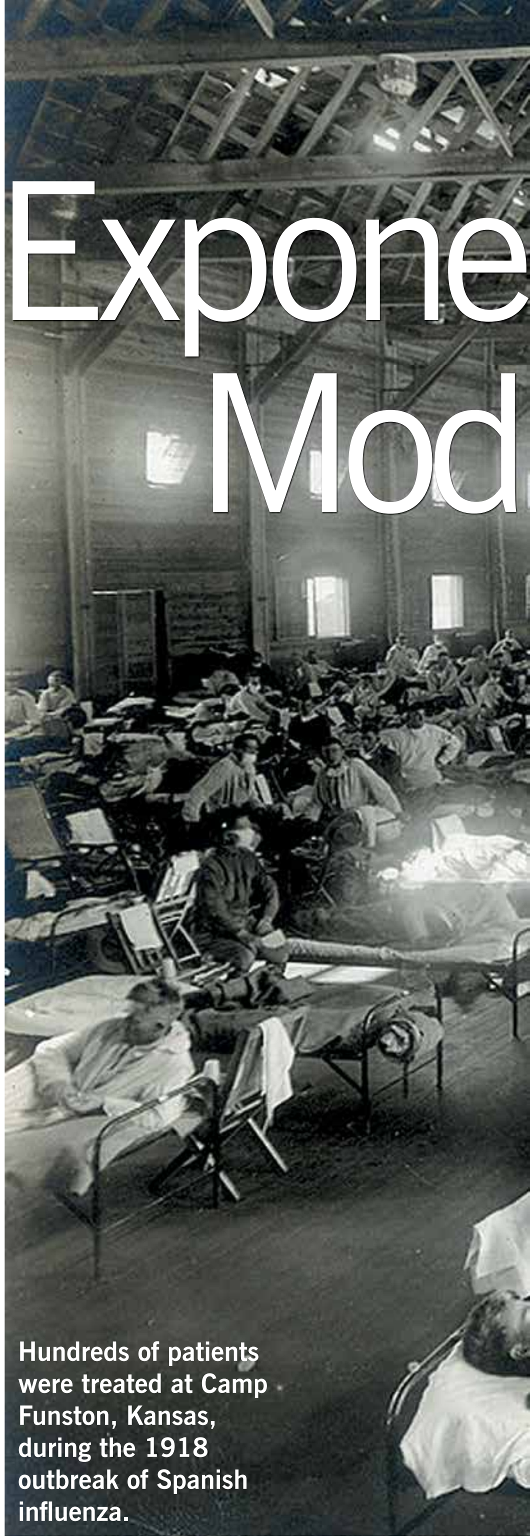

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Hundreds of patients were treated at Camp Funston, Kansas, during the 1918 outbreak of Spanish influenza.

The phenomenon of outbreaks of dangerous diseases is both intriguing to students and of mathematical significance, which is exactly why we engaged eighth graders in an introductory activity on the growth that occurs as an epidemic spreads. Various contexts can set the stage for such an exploration. Reading adolescent literature like *The Eleventh Plague* (Hirsch 2011), viewing a movie such as *Contagion* (2011), and describing the history of London's Great Plague during the 1600s are all possibilities.

In early 2015, the infectious disease Ebola was at the forefront of worldwide news. Although Ebola is not highly contagious, the mortality rate that is associated with it heightens the fear and alarm surrounding this disease (Martinez and Wilson 2014), especially in light of the fact that 70 percent of Ebola cases in 2014 were fatal (Berkowitz and Gamio 2014). We caution that focusing on Ebola as the primary real-life example to hook students may cause unnecessary anxiety and take away from the understanding of the mathematics in this activity. This activity was created for eighth-grade students as a way to simulate a rapidly spreading epidemic in today's society while providing a visual model that conceptually demonstrates to students the power of exponential growth. After conducting a classroom simulation, we asked questions about multiple representations of exponential

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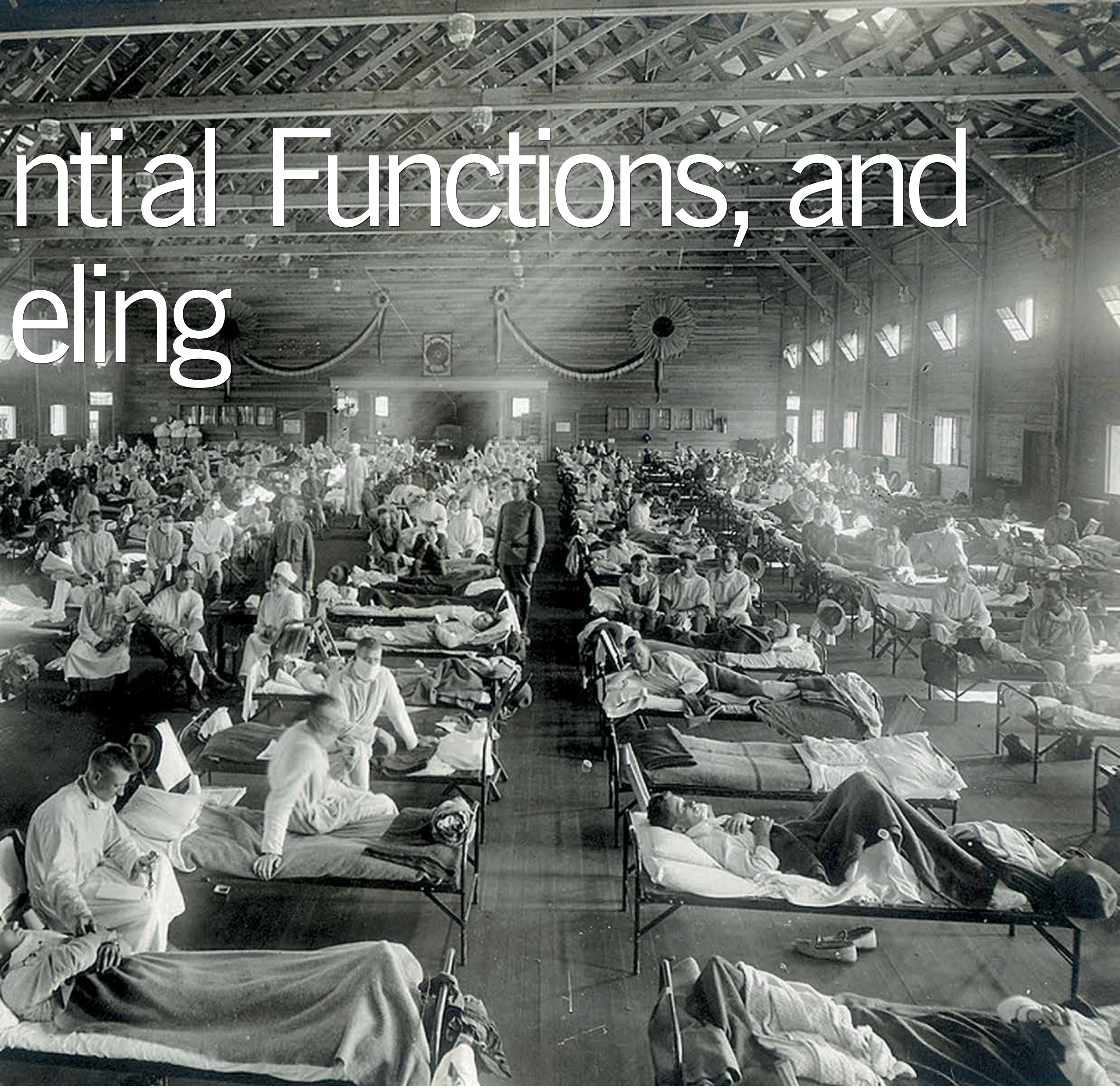

functions, comparisons in characteristics of different types of functions, applications to new scenarios, and real-life considerations regarding variables that affect the spread of epidemics. Our activity was taught in two classes of eighth-grade students.

This activity addresses several of the Common Core State Standards for Mathematics (CCSSI 2010) at both the eighth-grade and high school level in the Functions domain. Specifically, this activity closely aligns with the clusters to construct and compare linear, quadratic, and exponential models and solve problems as well as to interpret expressions for functions in terms of the situation they model. Additionally, the activity addresses modeling with mathematics, as students work with exponential models within the context of epidemics and use appropriate tools, because students considered the advantages as well as the limitations of using such technology as a graphing calculator.

LESSON DESCRIPTION

The Eleventh Plague is a work of adolescent literature describing individuals who live in a desolate and barren United States in the aftermath of a war and an epidemic (Hirsch 2011).

Fig. 1 This activity sheet's questions framed the exploration.

Considering the Context

1. What do you already know about plagues and epidemics? What have you seen in a movie, read in a book, or learned in class related to this topic?

2. In general, comparing life in the 1600s to life today, what might increase or decrease the spread of epidemics in today's world?

Simulating the Spread

3. After the simulation is conducted, try to represent the pattern between each round and the number of infected students by creating *x*-*y* data tables. Continue each data table beyond the number of rounds completed in class for each simulation.

4. What patterns do you notice?

5. How did you choose your labels? (Hint: What variables do they represent?)

6. Create a graph for each of your data tables. Attend to precision as you label axes, consider intervals, and title your graphs.

7. What type of function is each graph? Explain mathematically how you know, specifically focusing on the *y*-values. Provide an example.

8. Suppose a simulation was conducted with an entire school of 2000 students starting with 4 infected students. How many students would be infected after the eighth round? Use drawings, models, or words to explain your reasoning.

9. Use graphing technology to represent this simulation as an equation.

10. Using your model from question 8, suppose the same simulation spread into a large city of 100 million people. How many citizens would be infected after the 22nd round? Show your work.

11. What are the advantages and disadvantages of each of the following representations when working with this scenario? Consider how the use of technology can be advantageous.

	Table	Graph	Equation
Advantages			
Disadvantages			

Culminating Questions

12. Theoretically, will the simulation pattern ever stop? Realistically? Why?

13. Predict how the graph would change if every person infected 2 new people (rather than just 1). What do you think would happen to the shape of the graph?

14. In real life, infected people might come in contact with other infected people (rather than infecting someone new). How would this change the outcome of the simulation?

15. In real life, some people might be immune. Predict how this could change the outcome of the simulation.

16. Name other real-world phenomena that could result in exponential growth or decay.

This book sparked our interest in exploring exponential functions through the context of epidemics and thus served as a catalyst for creating the activity we describe in this article. Knowing that not all students had read this book, we decided to also link to students' prior knowledge about epidemics. As noted in *Principles to Actions: Ensuring Mathematical Success for All*, it is critical to "connect new learning with prior knowledge and informal reasoning" (NCTM 2014, p. 9), which we did through the context of these outbreaks of rapidly spreading illnesses and by accessing students' prior knowledge of functions (linear and quadratic).

Setting the Context

We wanted to introduce exponential functions by using a real-life context to intrigue students. We began class with a formative assessment in which we asked students what they knew about epidemics (see **fig. 1**, question 1). Although students hesitated at first, most eventually thought of events they had learned about from history class, the news, movies, or books. As a class, we discussed students' prior knowledge and found that several had read the book *The Eleventh Plague*. We explained that the book was our inspiration for this activity, and students quickly shared the premise of the story with their peers. Even more students had seen the movie *Contagion* (2011). Next, we showed students the first minute of the trailer for *Contagion* (http://www.youtube.com/watch?v=VZGHGVIedzA). This clip provided a preview of an exponential model when it stated that "first there were two people [infected], then four, and in three months, a billion." Students began to realize that at this high rate of increase, epidemics can spread quickly, but at this point they did not notice a specific pattern. Another option for connecting students to the context was to show maps of the Great Plague epidemic of London. We concluded this initial conversation by having students answer question 2 (see **fig. 1**), which initiated a discussion about technology and modern medicine and the advantages and disadvantages they bring to society. Students determined that technology has aided research on antibiotics and developments in waste management. Students also commented, however, that such advances have not been aided by citizens who spread disease because they are too lazy to wash their hands.

Our students did not think of one of the possible responses to this question that we had hoped to hear. It involved the recognition that with advances in technology and transportation, people have become more mobile, thereby creating an environment that could foster the spread of an epidemic. After 2015, students may be more likely to give this response because the media have made frequent mention of it during the Ebola outbreak.

Simulating the Spread

After setting the context, we wanted students to visualize how an epidemic might spread at a high speed. To conduct a classroom simulation, we gave each student a numbered envelope (e.g., numbers 1–34 if there were 34 students were in the class). Students were asked to keep their number confidential so that other students would not select them on purpose to become "infected." For the simulation, we used the random integer function on a graphing calculator and selected two numbers. To begin (round 0), we called out the two numbers. If a student's number was called, he or she was "infected" and stood up. Next, for round 1, each of the two infected students randomly selected a number not yet picked, and the students with the two new numbers became infected and were asked to stand, resulting in a total of four students standing. For round 2, all four infected students randomly selected a remaining number to simulate the spread of the epidemic, resulting in a total of eight infected students standing. To stay organized, we had all possible numbers written on the board and erased them whenever they were chosen, both to narrow the selection and avoid any repeats. Having students choose numbers from the board instead of using the calculator was helpful because the calculator may display a number that was already chosen. It also helped to keep the class on track collectively.

Students identified the total number of infected students after each round. This provided a helpful visual as it allowed students to see the rate at which the epidemic was spreading. Once the number of infected students reached the maximum number possible for the given class, the students predicted the next few rounds by applying the pattern they identified.

We conducted three simulations:

> ***"When the starting number [of infected students] was higher, the number of infected students increased faster."***

The class simulated a spreading epidemic by standing up.

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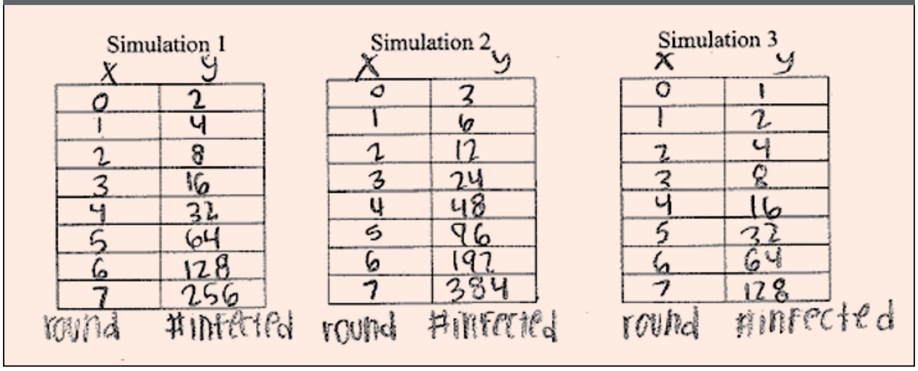
Fig. 2 A student produced these data tables from class simulations.



Simulation 1

x	y
0	2
1	4
2	8
3	16
4	32
5	64
6	128
7	256
round	#infected

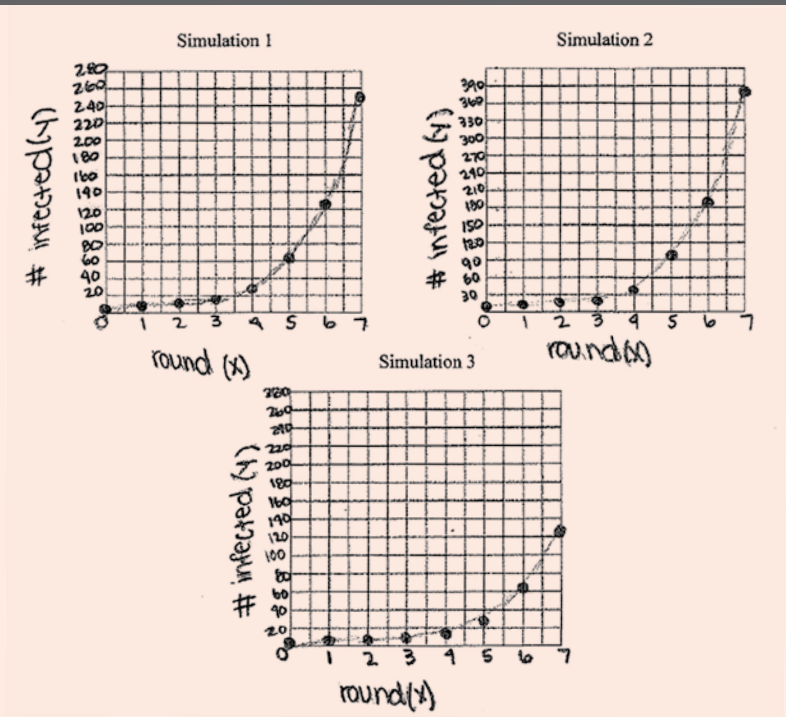
Simulation 2

x	y
0	3
1	6
2	12
3	24
4	48
5	96
6	192
7	384
round	#infected

Simulation 3

x	y
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
round	#infected

Fig. 3 The three simulations were represented by these graphs.



the one described above (with two students initially infected) and two others (one with three and one with one student initially infected). When considering the three simulations and the equation of an exponential function, $y = a \cdot b^x$ (which students did not yet recognize), varying the number of students infected at the start of the simulation changed the *a* value (2, then 3, then 1), but the value of the base, *b,* remained the same (it was 2 because the pattern was doubling each time). During the simulation, we asked students to consider (but wait to document) patterns they noticed when running the simulation. Some students spoke of their initial thoughts, saying that "two infected people picked two more, which is two squared"; "it is doubling each time"; and "when the starting number [of infected students] was higher, the number of infected students increased faster."

Exploring the Mathematics

After the simulation, students worked in groups of four to complete questions 3–6 (see **fig. 1**). Examples of tables and graphs created by students are shown in **figures 2** and **3**. At one point, we called a huddle (asking one student from each group to conference with the teacher) to remind students of the importance of making a graph understandable. This involved precision in the use of a coordinate grid, keeping a consistent scale, labeling axes, and providing a meaningful title. We overheard students having conversations such as the following when they determined how to create their graphs:

Student 1: "How do we get to 256 on the graph [for the *y*-axis]? Count by 15s or 30s?"
Student 2: "But this causes many points to be clustered at the bottom of the graph."
Student 1: "Yes, but there is no other way to graph it because that is the shape of the graph."

This conversation as well as others enabled us to note the areas that should be discussed at the close of the lesson.

Next, we engaged the class in a discussion on the shape of the graphs. Until this activity, students had not previously worked with exponential functions. We asked students what their graphs for question 6 looked like. They responded with "it looks like half of a parabola"; it looks "like a parabola . . . but with a parabola there is a negative side, but this graph doesn't have that"; it is "not linear because it is increasing as a curve"; "the function doesn't grow at a constant rate"; and "it starts out looking like a horizontal line, then becomes more of a vertical line." We transitioned this conversation into an introduction to exponential functions.

When we asked students what they noticed about the patterns of our exponential functions, they quickly determined that it was a doubling pattern. Students considered the doubling effect as the reason why disease spreads at such a fast rate, as described in the *Contagion* (2011) trailer ("two, four, . . . a billion").

Students did well applying the doubling pattern to question 8, which was similar to the simulations that we conducted. Students used different methods (i.e., tables, equations, and so on) to show their thinking. A sampling of student solution strategies, including one in which a student deduced the equation $y = 4 \bullet 2^x$, is in **figure 4**.

We used the TI-SmartView™ displayed on the interactive whiteboard to guide students through the sequence of keystrokes. The students then entered their question-8 data and found the exponential regression equation to model the situation (see **fig. 5**). Students applied their previous knowledge of finding the linear regression equation of a scatter plot to identify similar keystrokes.

Once students saw the exponential regression equation, we discussed what each part of the parent function $y = a \bullet b^x$ represented (because this was an introduction to exponential functions). We asked, "How is this equation different from the linear and quadratic parent function?" and "What part of the equation causes this function to have a doubling pattern?" Additionally, we guided the class through several examples modeling how substituting x-values from their tables into the equation resulted in the y-values. Students noticed that instead of x being the base as they saw with linear and quadratic functions, x was now an exponent. We also found that this scenario presented an ideal opportunity to review important function vocabulary such as input-output and independent variable-dependent variable.

Students used the equation they found with the technology ($y = 4 \bullet 2^x$) to complete question 10. Within their groups, students then completed question 11 by considering the advantages and disadvantages of three ways to represent an exponential function. This allowed students to apply their previous knowledge about multiple representations specifically to exponential functions. Students described common advantages and disadvantages of using tables, graphs, and equations in **table 1**. As students considered how the disadvantages could be overcome, an interesting whole-class conversation occurred regarding how technology can be advantageous in accomplishing certain tasks in a more efficient and precise way. This gave students an opportunity to think about the strategic and appropriate use of a technological tool.

Fig. 4 Students answered, "Suppose a simulation was conducted with an entire school of 2000 students, starting with 4 students infected. How many students would be infected after the eighth round? Use drawings, models, or words to explain your reasoning."

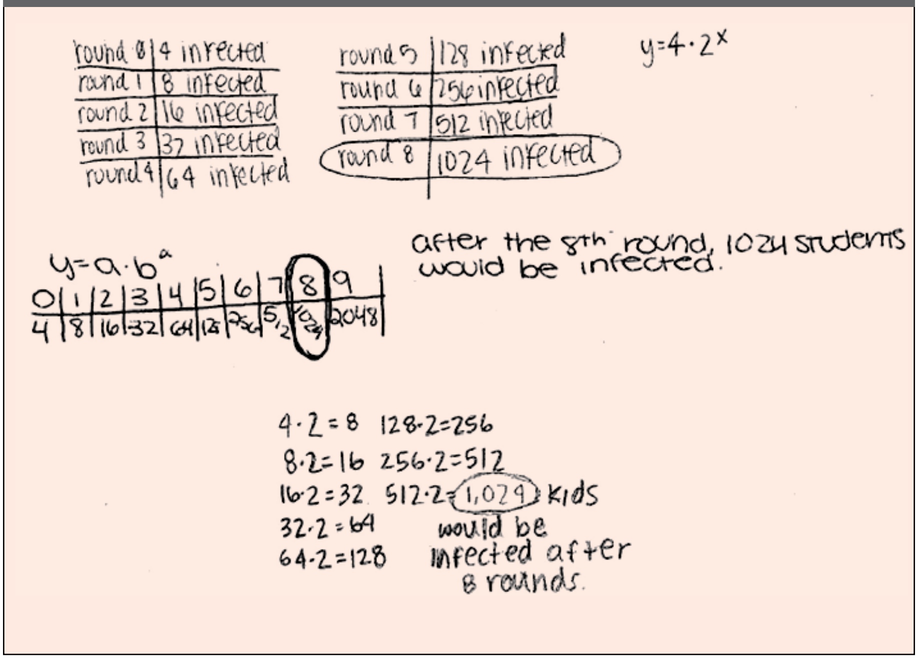
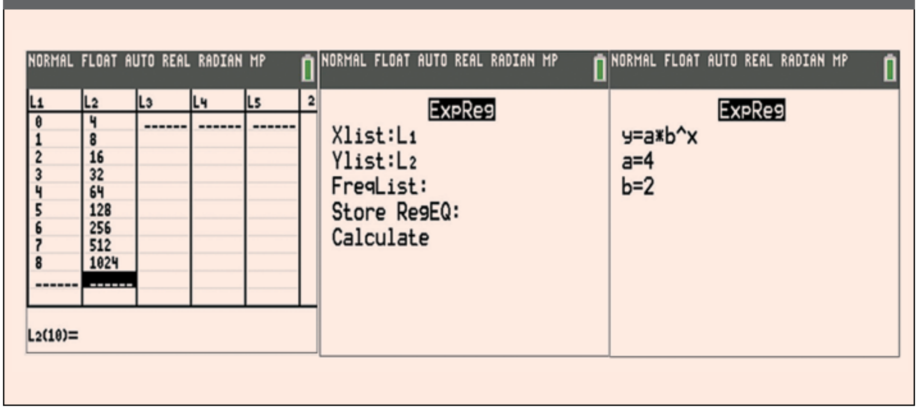


Fig. 5 The TI-SmartView illustrated data from the activity.



CONNECTING TO REAL LIFE

For the final portion of our activity, we found it critical to focus on the

Table 1 Students listed both advantages and disadvantages when using a table, graph, and equation.

	Table	Graph	Equation
Advantages	Organize ordered pairs See exact values to determine pattern Easy to convert to a graph See the change clearly	See the slope or pattern See the rate at which the data increase (or decrease) Provides a visual	Once you have it, you can quickly calculate your answer You have the exact rate of increase (or decrease) Fastest method for working with larger numbers
Disadvantages	Time-consuming Inefficient for higher numbers Cannot see the shape of the graph	Different scales (mainly on the y-axis) Can be confusing or misleading Time-consuming to create by hand Cannot always tell the exact ordered pairs when looking at a graph made by hand	Does not visually show the growth Harder to convert to a graph by hand (than a table) Difficult to find the equation (without technology)

spread of epidemics in real life, beyond the results of our class simulation. Students worked in their groups on questions 12–16 to brainstorm how other real-life variables might cause the spread of an epidemic to occur differently from our classroom simulation. Students were given various circumstances that might cause changes in their data and were asked to discuss how such changes would affect the overall rate of infection. This scenario required them to think outside the box.

As we allowed students to deliberate over their ideas as a class, we truly saw how our activity had not only introduced them to exponential functions but also piqued students' interest in the phenomenon of epidemics. The culminating questions at the end allowed for differentiation of instruction in that all students exercised their ability to think critically within a real-life context. They, therefore, responded with varying levels of sophistication. For example, on question 13, some students realized that the pattern would triple rather than double; other students simply noted that the graph with these conditions would be steeper and the number of infected people would increase at a faster rate. Students thought of many different real-world phenomena that might result in exponential growth (question 16) such as the purchase rate of a popular product, development of land over time, and spread of diseases in animals. Although the exponential model is useful for this grade level, a model of logistic growth is an extension that could be explored at higher grades. An exponential model assumes an infinite number of people may become infected. In reality, the population limits how many people can actually be infected. This idea was the basis for question 12.

As closure, we asked each class two questions: "What did you learn about exponential functions?" and "Why is an epidemic represented by an exponential function?" For the answer to the latter, students explained that "because not every round infects the same amount of people," "because it increases at a doubling rate," and "it increases rapidly." This final discussion helped us recognize that students were not only more interested in the spread of epidemics but also able to apply the exponential function as an explanation for the math behind this phenomenon. This introductory lesson positioned us for future exponential explorations. One suggested exploration is the TI Math Nspired algebra 1 activity called Spreading Doom (TI 2011), which focuses on the exponential spread of a computer virus. In this activity, students derive an exponential equation that models the spread of the virus.

MODELING MEANINGFUL MATHEMATICS

This activity provided a meaningful avenue for introducing exponential functions in a way that connected students to their prior knowledge of both functions (the mathematics) and epidemics (the context). Students had the opportunity to be part of a classroom simulation to experience how an epidemic spreads rapidly. This activity also provided an avenue for eighth graders to work within the eighth-grade and high school CCSSM Functions domain with interdisciplinary connections to well-known history, current events, books, and movies. More important,

Suggestions for the Classroom

After completing this activity with students, we took time to reflect and created the following list of classroom suggestions:

1. Find a way to designate students who are newly infected for that round from the students who were previously infected. One possible approach is to have newly infected students hold up a "newly infected" card for the remainder of that round. This clarification will help students better visualize the growth of the function as the simulation moves through each round (e.g., 2 infected, 4, 8, and so on).
2. Once students complete question 6, use a graphing calculator or spreadsheet to display all three graphs on the same coordinate plane, perhaps using a different color for each simulation. In that way, comparisons among the three graphs can be easily made and will add to the conversation on effective use of technology.
3. Consider posing the following as a final culminating question: If you start with 2 people infected (as the movie trailer suggests), how long until 1 billion people are infected? Is it actually three months, as the movie trailer suggests? Is that a reasonable time frame?

students were genuinely engaged in the standards for mathematical practice as they modeled simulations using multiple representations (tables, graphs, and equations), explicitly discussed the advantages and appropriateness of using technology, and attended to precision throughout the activity. Students were challenged to think critically within the context of the mathematics and connected their knowledge of epidemics and society to the mathematics, which resulted in an engaging learning experience.

CCSSM Practices in Action

- Construct and compare linear, quadratic, and exponential models and solve problems (F.LE.2)
- Interpret expressions for functions in terms of the situation they model (F.LE.5)

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Any thoughts on this article? Send an email to **mtms@nctm.org**.—*Ed.*




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