

cartoon corner

Name _____

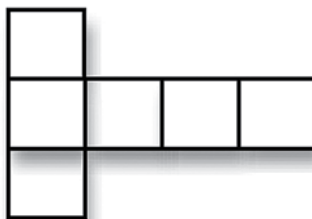
FOXTROT by Bill Amend



CUPCAKE CONVERSIONS

- Use your knowledge of units for capacity measurements to complete these "conversions":
 - 2 Pintcakes = ____ Quartcake(s)
 - 4 Quartcakes = ____ Pintcake(s)
 - 1 Galloncake = ____ Pintcakes
= ____ Cupcakes
- Suppose Paige and Nicole price their Pintcakes for \$0.50, Quartcakes for \$3.50, and Galloncakes for \$10.00. A customer buys 100 cakes for \$100.00 under this pricing system.
 - How many of each did the customer buy?
 - Are the prices of the cakes proportional to their sizes? If not, suggest a new pricing system.

- Which is larger, a Half-Galloncake or a 2Litercake? Explain.
- Paige and Nicole want to create small boxes in which to package individual Pintcakes. They plan on cutting a piece of cardboard, folding it to create a cube, and taping the sides. Below is one example of the pattern they could draw on the cardboard. Draw as many patterns as you can that could be folded to create a box for Pintcakes.



- Paige and Nicole decide to make batches of batter in 2 flavors, strawberry and lemon. Each batch is the same size and has the same amount of flavoring. Paige measures out 1 cup of strawberry batter and accidentally mixes it

into the batch of lemon batter. By the time she realizes her mistake, it is thoroughly mixed in. She tries to make up for it by moving 1 cup of the lemon batter over to the strawberry batter and mixing it in. At this point, is there more lemon in the strawberry flavoring or more strawberry in the lemon flavoring? Explain your reasoning.

CHALLENGE

- Have students provide a rationale for each of these "conversions":
 - 10 millipedes = 1 centipede
 - 1 million microphones = 1 phone
 - 1 trillion microphones = 1 megaphone
 - 453.6 graham crackers = 1 pound cake
 - 1 deck of 52 cards = 5.2 decacards

from the October 2015 issue of

mathematics
teaching in the MIDDLE SCHOOL

SOLUTIONS

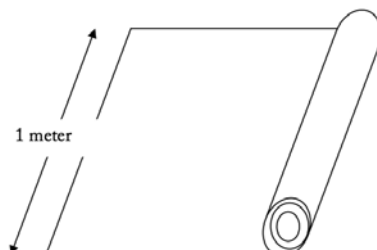
1. **a.** 2 Pintcakes = 1 Quartcake

b. 4 Quartcakes = 8 Pintcakes

c. 1 Galloncake = 8 Pintcakes = 16 Cupcakes
2. **a.** 92 Pintcakes, 4 Quartcakes, and 4 Galloncakes

b. No. Because 1 quart is twice as much as 1 pint, a Quartcake would be expected to cost twice as much as a Pintcake. Similarly, 1 gallon is 8 times as much as 1 pint, so the Galloncake would be expected to cost 8 times as much as a Pintcake. A better pricing system that reflects the size of each cake might be Pintcake: \$2, Quartcake: \$4, Galloncake: \$16.
3. A 2Litercake is larger. Because 1 liter \approx 1.06 quarts, 1 liter is more than 1 quart. Because 2 liters are more than 2 quarts, and there are 2 Half-Galloncakes in 1 Galloncake, 2 Quartcakes are larger than 1 Half-Galloncake.

4. There are 11 possible patterns, including the example pattern.



5. The two mixtures have the same amounts of the opposite flavoring. *Sample explanation:* Each mixture has 10 cups. One cup of the strawberry batter is moved to the lemon, making the lemon batter now 1 cup strawberry out of 11 cups, or $1/11$ strawberry (and $10/11$ lemon). If 1 cup of this mixture is moved back to the strawberry batter, that batter contains $10/11$ cups of lemon plus 9 and $1/11$ cups strawberry. Divide each of these quantities by 10 (the total number of cups in the mixture) to obtain the amount in each cup of the thoroughly mixed batter: $10/11$ of each cup is strawberry, and $1/11$ of each cup is lemon. Thus, each flavor of batter now contains $1/11$ of the opposite flavoring.

Challenge

6. Sample answers are given.
 - a.** In the metric system, the prefix “milli-” means one thousandth and “centi-” means one hundredth. Because one hundredth is 10 times as large as one thousandth, 10 millipedes = 1 centipede.
 - b.** “Micro-” is the prefix for one millionth. Because one millionth times one million is equal to 1, 1 million microphones = 1 phone.
 - c.** One trillion microphones = (1 million) \cdot (1 million) microphones, or 1 million phones. “Mega-” is the prefix for 1 million, so 1 megaphone = 1 million phones. Thus, 1 trillion microphones = 1 megaphone.
 - d.** One pound is approximately the same as 453.6 grams.
 - e.** “Deca-” is the prefix for 10. There are 5.2 groups of 10 in 52, so 1 deck of 52 cards = 5.2 decacards. (Note that 5.2 decacards should not be confused with 5.2 decks of cards, which is 5.2 times as much as 5.2 decacards.)

FIELD-TEST COMMENTS

“Cupcake Conversions” provided many challenges for my sixth-grade prealgebra students. About half the students had no recollection of the unit conversions. The other half entertained us with the tale and with completed illustrations of the land of G [gallon], where there are four queens [4 quarts] and where each

queen has a prince [pint] and a princess [another pint]. The prince and the princess each have two cats [cups] that only have 8 lives!

Students tackled the cakes’ calculations in the second problem with guess and check. I suggested to several groups that using a table to organize their work might help, and eventually students arrived at a solu-

tion. They quickly realized that they needed more Pintcakes. Paige and Nicole’s strange pricing was recalculated easily.

Paige and Nicole gave us another discussion topic with problem 5, creating strawberry-lemon and lemon-strawberry batter. Many students felt that there was more strawberry in the lemon. The best student explanation

for the equal flavorings used 16 cups of each flavor. He drew an arrow, showing 1 cup going into the lemon batter. At that point, there were 15 cups of strawberry and 17 cups of lemon-strawberry batter, which meant that $\frac{1}{17}$ was strawberry. When 1 cup was removed and returned to the strawberry, the ratio remained the same. Although the student was very concerned about the variety of the consistency of the batter, he reluctantly agreed it was the same ratio.

Judy Kraus
Hyde Park Middle School
Las Vegas, Nevada

I used this cartoon with my sixth graders. The students shared the same science teacher and had begun work on a measurement unit in science, so I provided this as an interdisciplinary activity in math. The students were allowed to work in pairs or individually. They liked this cartoon, maybe because cupcakes are a popular treat.

Students were able to easily complete problems 1 and 3. It took some time for them to realize that there were multiple ways to arrive at the answer for problem 2. Once they realized that, there were some fantastic conversations about how the money could be spent.

Most of my students did not think the pricing system was proportional. In their literal thinking, the Quartcakes should have cost twice as much as the Pintcakes, and the Galloncakes should have been priced four times as much as the Quartcakes. Most of them realized that if you wanted a “deal,” then you could have seven Pintcakes for the same amount of money as one Quartcake. In our class, I focused a lot on helping my students learn to be better consumers, so this was a great question for discussing that topic.

Because we were in Europe, my students were familiar with liters. So

for problem 3, most of them knew that a 2Litercake would be more than a Half-Galloncake.

Most students easily completed problem 4. Although nets were to be touched on later in the year in sixth grade, the topic had been introduced in fifth grade, so many students knew exactly what to do.

The students had a difficult time with problem 5. Although the topic of ratio and proportion was to be studied later in the school year, students did try to solve the problem by drawing pictures. One student even asked if she could use cups and food coloring to try it out. Once she started, she realized that the colors would mix.

It was interesting to me to hear how they attempted to solve these problems. It will be fun to use this question after we study proportions.

The Challenge problems blew most of the students away. However, some easily solved them because of their experiences in science.

Carol Fears
Landstuhl Elementary/Middle School
Armed Forces Post Office, Area Europe

I had very small classes and decided to test this cartoon with each of my sixth-, seventh-, and eighth-grade math classes. My sixth-grade math class spent the most time on these problems because they most directly related to a recently studied topic: ratio and proportions. We had not used capacity measurement, so students found it helpful to review that information before setting out to solve the problems.

At each grade level, the students were engaged and interested by the premise of the problems. Once they refreshed their understanding of the measurement conversions, they jumped around in the problems, finding problem 2 the most engaging overall. Students tended to make tables to try to sort out how many of

each cupcake would be purchased. They found it challenging and exciting to meet both the criteria of the number of cakes and the total dollar amount. Although most problems were clear, problem 5 was harder for students to decipher. Overall, students in all three classes enjoyed the challenge and look forward to future math-related cartoons and the accompanying problems!

Kate Burstein
Chicago Jewish Day School
Chicago, Illinois

OTHER IDEAS

Extend this Cartoon Corner task with these ideas:

- Have students research and calculate the actual cost of the ingredients for each size cupcake and then determine if the prices will allow the baker to make a profit. (Many thanks to field-test reviewer Carol Fears, who suggested this idea.)
- Ask students to calculate the amounts needed for each ingredient when scaling up or scaling down a recipe for a given number of servings. See the Feeding Frenzy activity on the NCTM Illuminations website: <http://illuminations.nctm.org/Lesson.aspx?id=2854>
- Have students make up their own measurement “conversions” similar to those in the Challenge.
- Ask students to make various conversions between Litercakes and the other cakes by converting between measurement systems.