

palette of problems

Matt B. Roscoe and Stephan Pelikan

1. When you purchase a punch card for access to the Grizzly Pool at the University of Montana, you have three options: 10 punches for \$31.75; 25 punches for \$75.00; or 50 punches for \$131.25. How much will you save per punch by purchasing the card with 50 punches when compared with 10 punches? How much will you save per punch by purchasing the card with 50 punches compared with 25 punches?

2. Golden Double Stuf Oreo® cookies are on sale for \$1.50 off the regular price of \$3.49. What percentage of the original price does the buyer pay on checkout?

3. The “Big Burn” was the largest forest fire in U.S. history. It burned about 3 million acres in Washington, Idaho, and Montana in August of 1910. How many square miles are equivalent to 3 million acres? (Hint: An acre is equal to 43,560 ft.².)

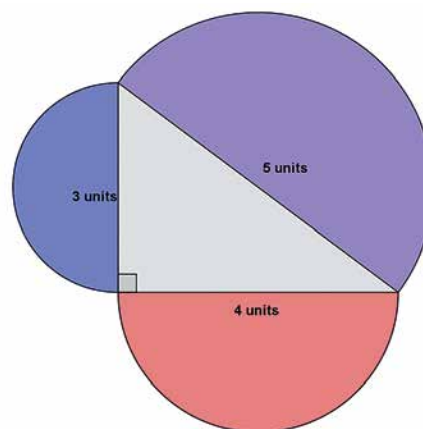
4. Use the information in the previous problem. Among the 50 states, the state with the smallest area is Rhode Island, which covers some 1,545 square miles. Determine the size of the area burned in the “Big Burn” of 1910 as a percentage of the area of Rhode Island.

5. Suppose that the TV weatherperson predicts a 30% chance of rain on Saturday and a 30% chance of rain on Sunday. Assuming that the weather predictions are accurate and that the events are independent, what is the probability that it will rain at least one of those days?



IGOR GONCHARENKO/THINKSTOCK

6. A semicircle is constructed on each of the legs of a 3-4-5 right triangle. Find the area of each semicircle. How are the areas of the semicircles related?



7. Mr. Smith gave an exam to his honors math class. The exam was worth 225 points, and it contained 50 questions. Some of the questions were worth 3 points, and some questions were worth 8 points. How many of each question were on the exam?

8. Joe and his two friends went to Red Robin® for lunch. The total for food and drinks was \$29.50. When the bill came, they noticed they were charged 5% tax. What is the total cost of the bill if they also left a \$6.00 tip?

Prepared by **Matt B. Roscoe**, matt.roscoe@umontana.edu, University of Montana, Missoula, and **Stephan Pelikan**, pelikan@math.uc.edu, University of Cincinnati, Ohio. Teachers, student groups, or mathematics clubs are encouraged to submit single problems or groups of problems to the editor, **Matt B. Roscoe**, at matt.roscoe@umontana.edu. Published problems will be credited. MTMS thanks the following people who submitted problems and solutions this month: Problems 7 and 8 were sent in by Terri Kurz and Jagar Carrillo, Arizona State University; problem 9, by Elizabeth Tague, Orange, New Jersey.

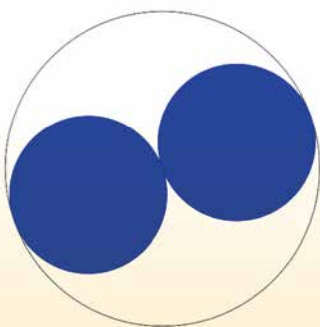
9. Irene loves to keep up with Hollywood news. She always buys the current *People*® magazine when she is shopping. However, she is wondering how much she could save by paying for a subscription. The magazine costs \$4.99 at the newsstand. If Irene orders a 58-week subscription, it will cost her \$112.86. How much would Irene save by paying for a 58-week subscription rather than buying the magazine at the newsstand each week?

10. What is the digit in the 1's place of 3^{2015} ?

11. The sum of the odd integers between 1 and 99 (inclusive) is 2500. What is the sum of the even integers between 2 and 100 (inclusive)?

12. The faucet can fill your bathtub to the top in 9 minutes with the faucet completely open (the water on full blast). A full tub can be completely drained in 21 minutes. By mistake, you leave the drain unplugged when you have the faucet completely open. How long until the tub overflows?

13. The figure below shows a large circle with two identical smaller circles inside and tangent (meeting the circle at a single point) to it. The smaller circles are also tangent to each other at the center of the larger circle. If you pick a point at random from inside the larger circle, what is the probability that the point lies inside one of the two smaller circles?



14. On a math quiz, Ben was supposed to find $X + Y$. Instead, he calculated $X - Y$, so of course what he got, 100, was wrong. Lucky for him, his answer came out to be exactly $1/4$ the correct answer, so his teacher gave him $1/4$ of the points as partial credit. What is X ?

15. If $a > b > 0$, which of the two continuous fractions is larger:

$$\frac{1}{1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{a}}}} \text{ or } \frac{1}{1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{b}}}}$$

(Hint: Use the fact that $1/x > 1/y$ if and only if $x < y$ for positive x and y .)

16. Suppose that $x^2 + y^2 = 1$. Explain why $xy \leq 1/2$. Can you find values of x and y so that the equality is true?

(Answers on page 191)

more4U

The solutions to the Palette of Problems are online at **www.nctm.org**. Online solutions are available to NCTM members only.