Social Justice and PROPORTIONAL

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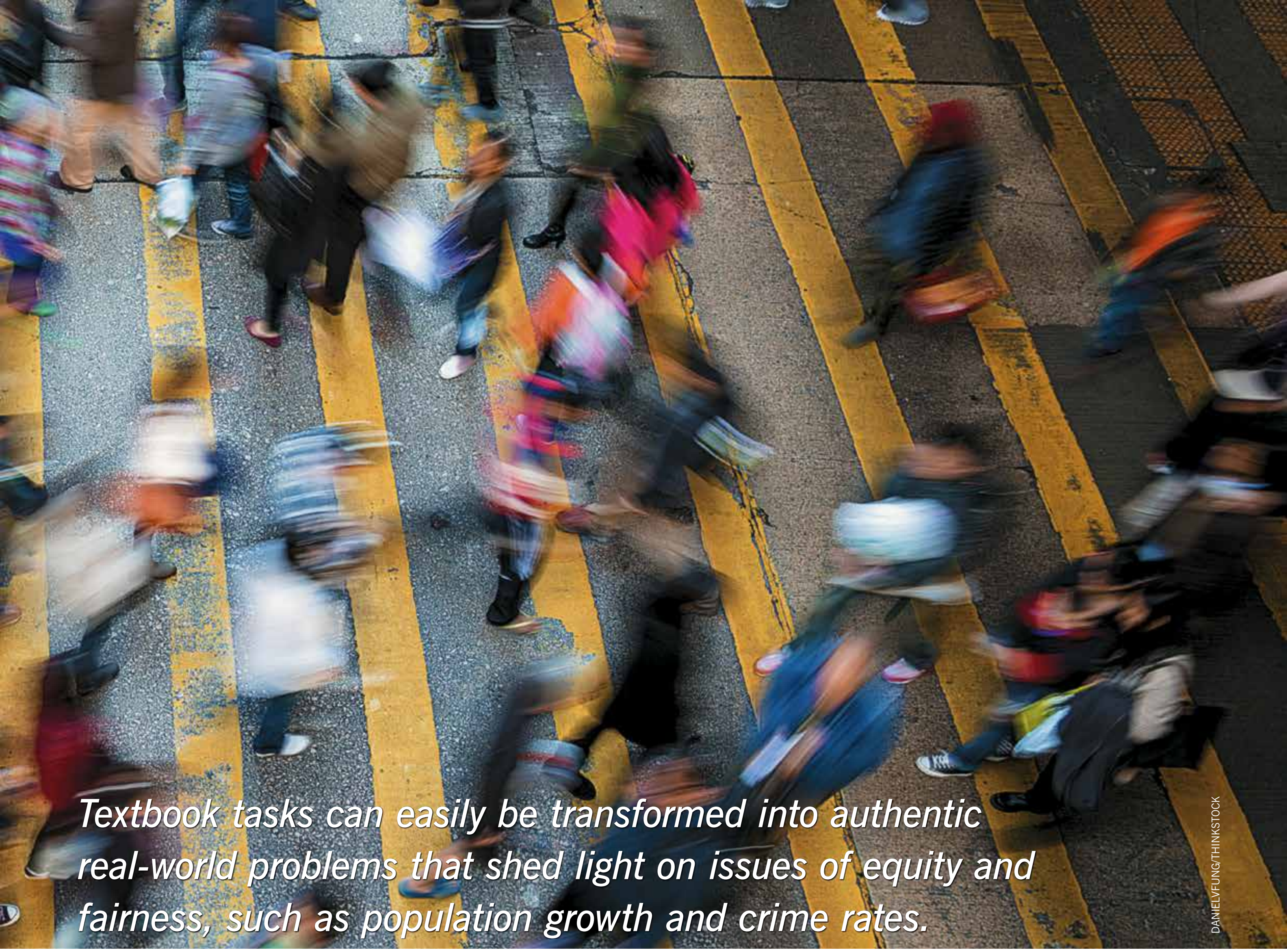
Being a mathematically literate citizen requires daily applications of proportional reasoning, especially when answering such questions as these: "Is the larger package a better buy?" "Is Chicago more dangerous than Detroit?" Many of these applications, like the second question above, are related to social justice issues found in society. Mathematical story problems seen in textbooks are often contrived and only superficially deal with the real world and its complex issues. For example, the following Mixture problem is typically used when teaching ratios:

> Juice mixture A has 3 parts water and 1 part fruit concentrate, while juice mixture B has 5 parts water and 2 parts fruit concentrate. Which mixture is "fruitier?"

Although this problem is certainly interesting and provides a convenient context for comparing ratios, it is not a problem that a student would genuinely be interested in solving. It also does not provide the student with



Textbook tasks can easily be transformed into authentic real-world problems that shed light on issues of equity and fairness, such as population growth and crime rates.

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REASONING

new information about the world that he or she lives in.

The textbook approach to story problems no longer being sufficient can be seen from the Common Core State Standards for Mathematics (CCSSM), which includes mathematical modeling, or "apply[ing] the mathematics [students] know to solve problems arising in everyday life, society, and the workplace" (CCSSI 2010, p. 7). This statement is one of eight essential mathematical practices at all grade levels. Ratio and proportional reasoning tasks abound that have connections to real-world situations. The document *6–7, Ratios and Proportional Relationship* reinforces the importance of proportional reasoning in the mathematics curriculum. It emphasizes its many applications, such as finding trigonometric ratios and instantaneous rates of change in calculus, finding concentrations in chemical solutions, calculating tips, or analyzing descriptive statistics (Common Core Standards Writing Team 2011). Although extensive, this list of topics excludes a host of real-world situations in which the need for reasoning proportionally arises naturally, such as population growth, crime rates, incarceration rates, immigration rates, and other topics that provide valuable information about the world we live in.

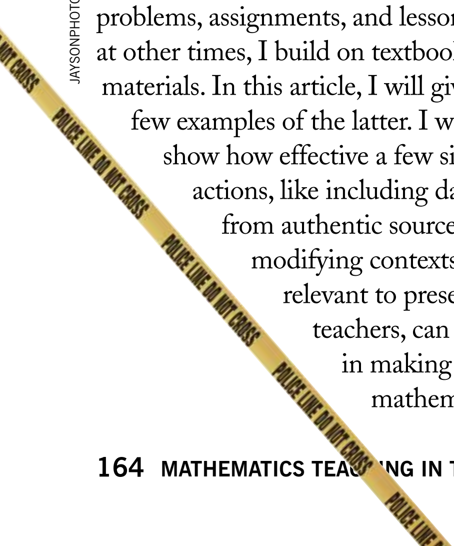
In the two-part mathematics content course for preservice K–grade 8 teachers that I teach, I frequently use authentic real-world contexts related to social justice. These contexts infuse the curriculum but do not replace

Table 1 A textbook problem was modified to infuse the local context.

Original Problem	Modified Problem	Mathematical Content	Modification	Benefits
Does a population growth from 1,000 to 2,000 people have the same implications (social, economic, and so on) for a community as a growth from 100,000 to 101,000 people? Explain.	The population of Tacoma grew from 193,556 in 2000 to 198,397 in 2010, whereas the population of Olympia grew from 42,514 in 2000 to 46,478 in 2010. Both cities grew by 4,000–5,000 people. Does this growth have the same impact on both cities? Explain.	• Additive and multiplicative comparisons • Growth rates • Possible percentage change	Real population numbers were used, which made the mathematical demands of the problem slightly higher. The names of actual cities in the vicinity of our university were used.	Context is familiar and therefore more relevant. Questions are raised about why the growth of the other city was faster.

mathematically rich problems without a realistic context. The social justice contexts introduce mathematical modeling through "messy" real-world problems that allow multiple strategies, typically both numerical and algebraic, and do not always use "nice" numbers. Items like the Mixture problem mentioned above allow for the introduction of other tools, like tables and diagrams, which typically do not occur naturally with real-world data. Both types of problems are needed in teacher education programs. In particular, social justice contexts help preservice teachers see mathematics and the world in a different light, which in turn will help them become better-informed teachers of mathematics.

In developing a social-justice–based curriculum, I sometimes create original problems, assignments, and lessons; at other times, I build on textbook materials. In this article, I will give a few examples of the latter. I will also show how effective a few simple actions, like including data from authentic sources and modifying contexts to be relevant to preservice teachers, can be in making the mathematics curriculum more realistic and thought-provoking.



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PROPORTIONAL REASONING PROBLEMS RELATED TO SOCIAL JUSTICE

Example 1: Population Growth and Crime Rates

The difference between additive comparisons (i.e., finding differences) and multiplicative comparisons (i.e., finding ratios), and the knowledge of when each is appropriately used, is one of the most important topics in proportional reasoning instruction (Lamon 2011). The examples in **table 1** and **table 2** were modified from the textbook used for the course (Sowder, Sowder, and Nickerson 2010). Both were already interesting problems addressing additive and multiplicative comparisons, and were almost authentic, except for using made-up numbers. Modifying them was simple: All that was needed in this case was to find regional information that would fit the demands of the problems.

The two cities in the modified problem in **table 1** were chosen so that students would experience the same absolute growth during the same time period while having different starting populations. This information kept the mathematical demands of the modified problem similar, although not quite identical to those of the original problem, where it was easier to see different impacts of the growth (100 percent versus 1 percent). Conveniently, one of the cities featured in the modified example was Tacoma, where students attended classes; the other, Olympia, was only thirty miles away. These locales made the question relevant to preservice teachers, the vast majority of whom grew up in the region.

This problem was discussed during class. Most preservice teachers correctly used both additive and multiplicative comparisons to argue that both answers to the question were correct, claiming that there were cases in which the starting population of the city was not relevant (e.g., when building a fixed number of housing units), and others in which it was (e.g., considering demands on the electric grid). Because the two cities were familiar to them, preservice teachers were also curious about the different growth rates of the two populations. They speculated that the city with a larger growth rate

Table 2 Local crime rate numbers were added to localize this problem.

Original Problem	Modified Problem	Mathematical Content	Modification	Benefits
In 2000, City A, with a population of 55,489, reported 214 violent crimes. In the same year, City B, with a population of 185,217, reported 639 violent crimes. Which city has the worse crime rate? Explain your answer in 1 sentence.	There were 471 violent crimes in King County in 2010, and there were 335 violent crimes in Thurston County in 2010. King County had an estimated 1,931,249 inhabitants in 2010, and Thurston County had 252,264 inhabitants in 2010. If we consider only violent crimes to determine community safety, which county was safer in 2010 and why? (For comparison, Pierce County had 1,134 violent crimes and 795,225 inhabitants in 2010.)	• Additive and multiplicative comparisons • Ratios and percents	Real crime data from nearby counties were used, including information about the county that houses our campus, which was not needed for solving the problem, but was provided for comparison.	Crime in one's community is typically a relevant and pressing topic and will generate meaningful discussions. This was especially true with our county's data, added for comparison, as its crime rate is much higher than in the surrounding counties.

Table 3 This problem analyzed and compared the racial and ethnic composition of the Congress.

Original Problem	Modified Problem	Mathematical Content	Modification	Benefits
Find a distribution of 1,000 delegates that is representative of the U.S. population.	Compare the racial and ethnic composition of the actual U.S. Congress with the one that would equitably represent the U.S. population.	The original problem required solving proportions. The modified problem required comparing ratios or percentages.	Population figures were updated, context was changed from hypothetical to actual, and writing was emphasized.	Fairness is investigated in a real rather than abstract context, making the conclusions more relevant, and the impact of mathematics more clear, as numbers effectively show the lack of diversity in the U.S. Congress.

was more appealing to new residents.

I have used the problem in **table 2** on many occasions, sometimes as a homework assignment to introduce the unit on proportional reasoning. Perhaps because crime rates are reported regularly in the news, most preservice teachers immediately know that considering only the number of crimes without regard to population is problematic. When this problem was assigned as homework, all but one preservice teacher correctly used different facets of proportional reasoning to give the answer. However, this problem can still raise good questions about calculating rates. For example, one preservice teacher, reflecting what a middle school student might ponder, asked:

> Am I assuming one inhabitant did one crime? Can one inhabitant do more than one crime? Can a noninhabitant commit a crime? Is the smaller ratio safer or is it safer to have a smaller number of crimes?

Many students showed conceptual understanding of ratios by using their own strategies rather than familiar algorithms, like the preservice teacher who concluded that "[In King County], 1 in 4,100 people were victims of violent crime while in Thurston County, 1 in 653 people were victims. It was

Fig. 1 Census data were mined to find out the racial makeup of the U.S. House and Senate.

Below are some data from a recent census:

Race	Population
White or European American	223,553,265
Black or African American	38,929,319
Asian American	14,674,252
American Indian or Alaska Native	2,932,248
Native Hawaiian or other Pacific Islander	540,013
Hispanic or Latino	50,477,594
Total	308,745,538

Note that the list is not complete (for example, individuals of two or more races are not included), and that the categories are not exclusive (the Hispanic/Latino category has overlap with all the others).

Based on these data and the knowledge that there are 435 members of the U.S. House of Representatives and 100 members of the U.S. Senate, what should be a "fair" racial makeup of the House and Senate?

(a)

Racial Composition of the 112th Congress (including Delegates in the House)		
	U.S. House	**U.S. Senate**
White	361	96
Black	44	0
Hispanic	25	2
Asian	7	2
American Indian	1	0

Write a paragraph, as if for a newspaper, about your findings regarding the racial makeup of the U.S. House and Senate, using the mathematical information from this activity sheet and reasoning mathematically. Make an argument either for or against the statement that the U.S. House and Senate are representative of the U.S. population.

(b)

more than five times more dangerous!"

Both examples provided contexts that were familiar and relevant to preservice teachers, especially as the cities and counties whose statistics were used are ones that many of them grew up in. Both show the importance of using rates when considering the impact of certain events (e.g. crime, population growth, number of cars) on a population.

Example 2: Race and Congress

The problem found in **table 3** was adapted from the Selecting Delegates exploration in the Comparing and Scaling unit of the *Connected Mathematics Project* (Lappan et al. 2006). The original problem provided a table with the populations of each state, broken up by race/ethnicity and region. Students were tasked with choosing 1,000 delegates to fairly represent each region, and each race and ethnicity given in the table. This problem was mathematically rich and addressed fairness, but only abstractly, because the delegates were chosen not for a real function but for an imaginary function. Its modification included updating population information (see **fig. 1a**), and considering a very real example: the U.S. Congress, especially Senate, with its pronounced racial and ethnic inequality (see **fig. 1b**).

This, like the previous problems, was open-ended enough for the preservice teachers to be able to use a variety of approaches when solving it, although the most common approach was to find and compare percentages. Special attention was paid to writing because preservice teachers will often find an answer to a problem but not think about its meaning or implications. As expected, preservice teachers were dismayed by the discrepancy between the results (see **fig. 1**), especially by the underrepresentation of Hispanics in both the U.S. House and Senate, and called for efforts to increase the diversity of elected representatives. Although there was not enough time to pursue these recommendations further, they would have provided an excellent entry point to related mathematics topics of redistricting and voting.

Example 3: Immigration

Although realistic and familiar contexts usually seem to make the mathematics easier to comprehend, they can also add confusion. Take the following unmodified example from Bassarear (2012). Given a table showing the United States immigration rate over the decades, from 1820 to 2000, preservice teachers were asked the following:

> Make two separate line graphs, one using the total number and one using the rate per 1000 population. Describe the different impressions by the two graphs. Which graph do

you think gives a "better" comparison of immigration to the United States? Why? (Bassarear 2012, p. 392).

This problem also dealt with additive and multiplicative comparisons, but it was confusing to preservice teachers. The two graphs had different appearances: Although the immigration numbers had been growing steadily since the 1950s and had reached historic highs, immigration rates leveled off at a value much lower than those from the 1890s. With crime rates (see **table 2**), preservice teachers understood that total population size needed to be considered when examining the impact of the number of crimes committed. However, with immigration data, they overwhelmingly chose the first representation, using the total number as more accurate and disregarding the general population growth when examining the impact of the number of U.S. immigrants. Some preservice teachers claimed that the immigration rates were deceptive because they "hid" the raw numbers.

This problem pointed to the importance of having conversations with students about the impact the same number of people or objects can have on populations of different sizes: Just because students understand this idea in one context does not mean that they will follow it in another context, especially one that is more controversial and polarizing, as is the case with immigration. Teachers can choose to focus exclusively on mathematics, and ask what the graphs are telling us about immigration rather than discuss students' personal opinions about the issue.

BENEFITS OF REAL-WORLD CONTEXTS

Preservice teachers appreciated real-world problems that dealt with issues they could relate to. The change to the curriculum that resulted in problem modifications was minimal, as the same mathematical content was addressed. All four examples shared here addressed topics that preservice teachers knew and had opinions about, and in some cases were personally invested in. This resulted in increased interest in the material being studied. In their reflections, most praised the applied nature of the proportional reasoning unit, strongly supporting the use of contextual problems in teaching mathematics. Many saw it as an added benefit: The mathematics learning was not replaced by the real-world issues, but, as a preservice teacher stated, "Using these issues makes us figure out how to build the equation as well as how to see that math really is around us all the time."

PROVIDING SOPHISTICATED MATHEMATICS

Most teachers already modify textbook problems, such as when they use their students' names or change problem settings to resemble ones that are familiar to their students. However, teachers are less likely to adapt problems to include more difficult issues like crime or immigration, possibly because they fear that their students are not ready for or would not understand those issues. Work with K–grade 8 students, especially those from marginalized backgrounds, has repeatedly shown that young students are not only aware of the issues that people and communities face but also ready to use sophisticated mathematics to make sense of them (Simic-Muller, Varley Gutierrez, and Turner 2009; Gutstein 2005; Varley Gutierrez 2013). Preservice and in-service teachers alike should therefore seriously consider incorporating these relevant, authentic topics into their teaching.


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Strategies and Advice

The following list presents a few ideas on how to modify textbook tasks to become real-world problems.

- Ask yourself the following: Is this a question that people would want or need to answer in their daily lives? Does this problem give me any new and relevant information about the world I live in? The answer to at least one of these questions should be yes.
- Replace textbook numbers with actual data found on the Internet. Mathematics textbooks seldom use real-world data, likely because these data are constantly changing. Sometimes all that is needed to make a problem authentic is a number change.
- Require students to create a written argument based on the information given in the textbook problem. This is one of the most effective ways to engage students with real-world problems.
- Use problems that already come with real-world data and complex mathematical questions. If some wonderful and rich mathematical problems fail to be relevant to students or do not speak to their experience, they should be changed.
- Take note of controversial issues, but do not avoid them. Discussions should always be respectful and appeal to the mathematical ideas as much as possible.

Modifying textbook problems to include social justice contexts is not difficult. For example, if a problem asks about a percentage increase of the price of notebooks, why not instead look at food price increase or compare markups in urban and suburban grocery stores? The mathematical content is already present in the textbooks; it is only the plausibility of the context that needs to be added.

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