Racing toward Algebra and Slope

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A battery-powered vehicle is the engine that drives students' understanding of the meaning of slope.

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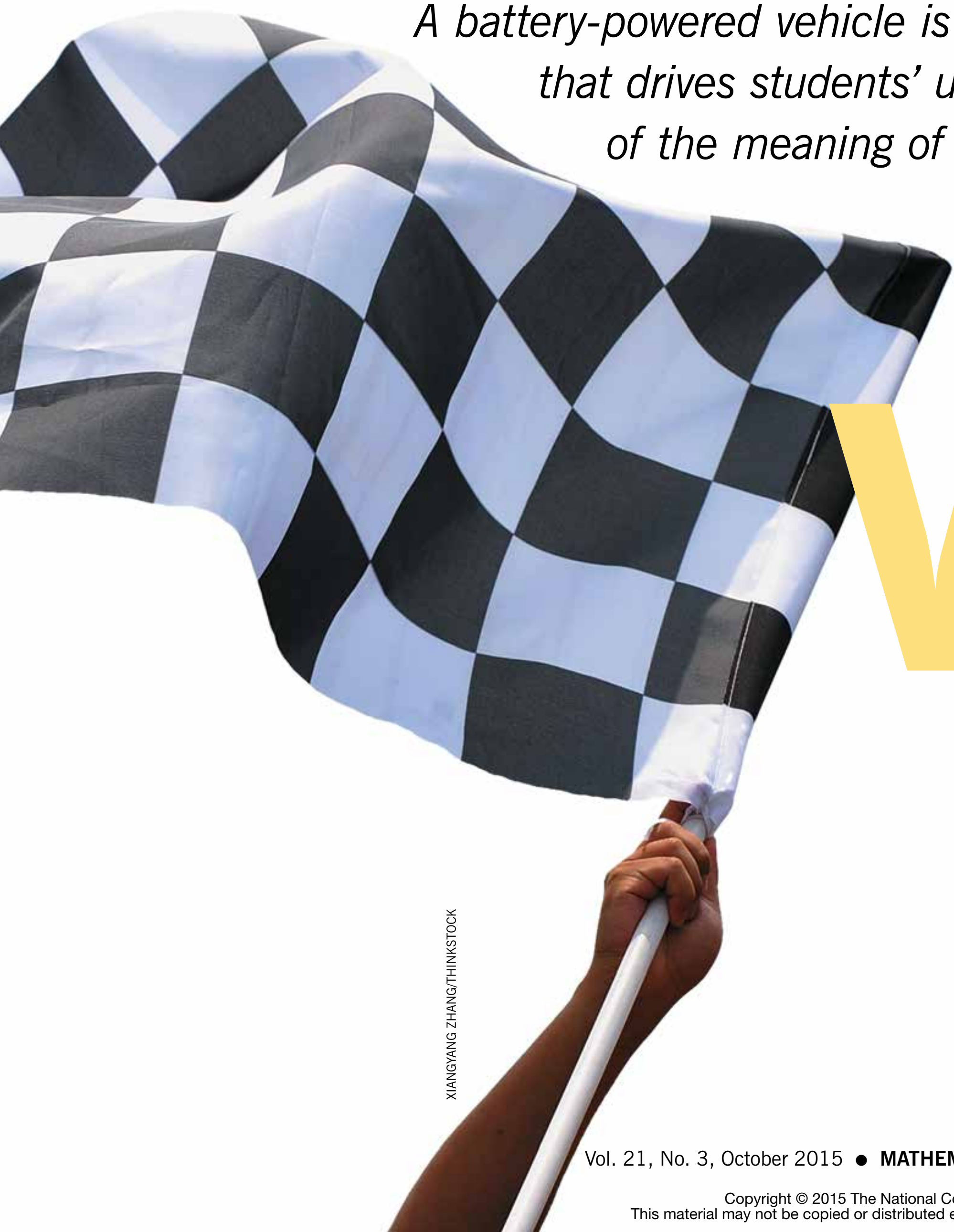


XIANGYANG ZHANG/THINKSTOCK

Why should we let science teachers have all the fun? Throughout our teaching careers, we have worked with science teachers, experiencing physics and chemistry data modeling activities as part of the science curriculum. We have observed students learning mathematical content in their science classes by collecting and analyzing real data to develop a conceptual understanding of various functional relationships. We have also worked with science teachers to bridge connections between physics and chemistry topics when working with the five families of functions: linear, quadratic, trigonometric, inverse, and exponential. As math teachers, one thing has become clear: Science teachers and students are having fun. What offends us is

that they are having fun teaching mathematics content!

"Mathematical modeling of the physical world should be the central theme of physics instruction" (Hestenes 1987). Science teachers know this, but because of standardized state testing, many math teachers and some science teachers feel the need to focus on memorizing facts that will be on a multiple-choice test (Volante 2004). As a result, instead of tackling physical representations of mathematics in science and math classes, many students only experience modeling in their science classes, if at all. This is especially troubling for English language learners who may benefit from concrete experiences to understand a specific situation. A concrete experience, such as the ones we are proposing, will give all students an anchor with which to connect to more abstract symbolic representation (Garrison and Mora 1999).

In mathematics classrooms, helping students understand slope can be a daunting task. From simple rate problems to derivatives, the idea of the rate of change requires a depth of understanding well beyond "rise over run." To better understand this overarching concept that permeates all secondary level mathematics, students need to be given opportunities to connect the abstract concept of slope with physical situations. Giving students a concrete situation with which to talk about an abstract concept can help them understand it at a different level (Ball 1988).

GIVING THE HISTORY (AND THE CREDIT)

This series of activities began as a weeklong science lesson workshop led by Larry Dukerich at Arizona State University (ASU). He used battery-powered vehicles (BPV) that moved at a constant speed from different starting points to motivate students' understanding of mathematics and physics. We attended the workshop and brought this lesson back to our classes. The feedback from students was amazing.

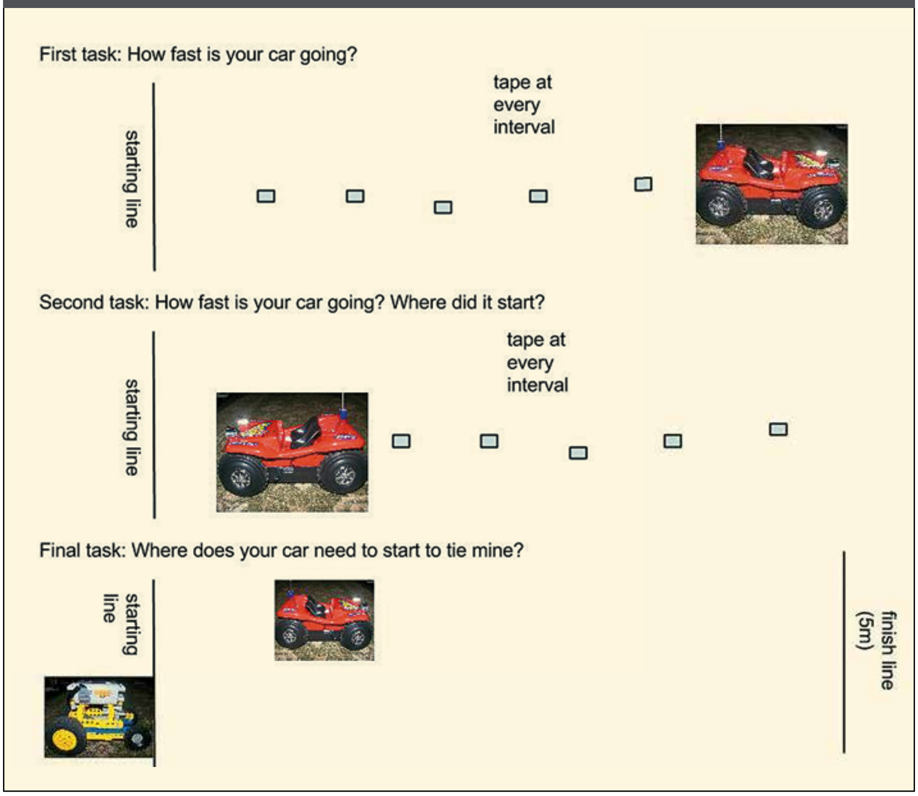
Because this feedback came from students in algebra 1 and AP Calculus AB classes, we found that this idea was appropriate in helping students at many different levels. The focus of this article will remain on implementing these activities in an algebra classroom. However, it is important to note that characteristics of these activities can be adapted to assist in developing students' understanding of instantaneous rate of change, as well.

EXPLANATION AND FACILITATION

We begin the lesson by showing students the battery-powered vehicles and asking them how they could determine the speed of the vehicle, with minimum human error. Students are guided toward the idea of taking data points and using a best-fit line to average out the potential error. After a procedure is established, volunteers from each group come up to the front of the class and practice on the "pace car" (a glamorized teacher car) to help minimize confusion once they begin recording data on their own. In their groups, students use a stopwatch and masking tape to mark where the car is

every two seconds. They then measure the distance from the starting line to each point and place these data in a table. Students are given a sheet to help organize their data and prompt their thinking (see the **activity sheet**). The accuracy of the measurements is very important, so at the conclusion of

Fig. 1 A word and image depiction of tasks helps students to visualize the task at hand.



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this series of lessons, students repeat this procedure at least once and average their data points to reduce error. At this point, they graph the distance-versus-time data and draw the line of best fit.

After comparing their data and analyzing the line of best fit, students' discussion of speed and what its corresponding units represent should help them develop an understanding that their car's speed is the slope of the best-fit line. This connection strengthens their conceptual understanding of rate of change. At this point, it is important to discuss the concept of the origin as a location versus a quantity. We establish the origin as the starting place, and students are instructed to start their car from a different position while keeping the starting line in the same place (see **fig. 1**).

Each group is given a unique set of instructions (e.g., "Start 200 cm from the starting line and point your car toward the starting line"; "Start 50 cm before the starting line and point your car away from the starting line"; and so on). Because of varying battery power, each group has a unique vehicle. After gathering data from the new starting point and finding the best-fit line, students prepare a whiteboard presentation of their results. After a brief summary from each group, students discuss the similarities and differences among their results.

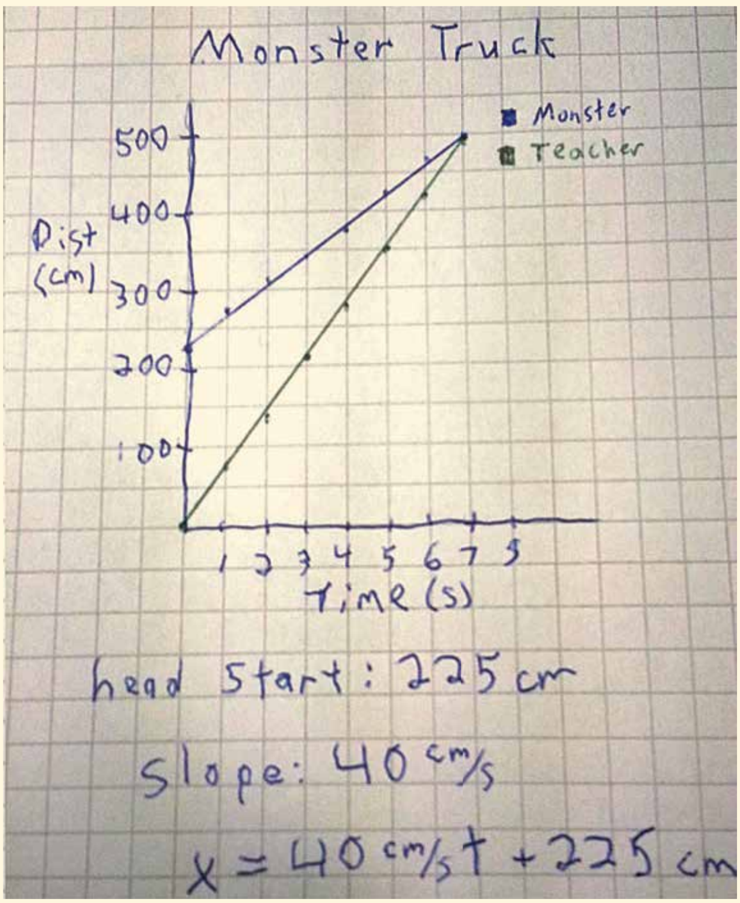
During this time, the teacher's role is to facilitate discussion. Ideally, students' discussions will naturally flow through the differences in the instructions that each group was given. If this discussion does not naturally develop, ask such questions as these:

- Whose car was going away from the starting line and how could you tell?
- Who started before the starting line? Who started after?
- How can you determine the starting point by looking at the graph?
- Whose car went the fastest? The slowest? How do you know?

The rich conversation that develops should help students with their emerging understandings. In so doing, the students with varying language abilities will hear different explanations of slope and y-intercept in this specific context. This tangible experience can then be referenced when linear equations in slope-intercept form occur in different contexts later on. The situation will also challenge more advanced students by requiring them to articulate what they already understand.

To spur discussion and help elucidate student understandings, we also tend to ask specific questions based on the group's graph. For example, about the graph in **figure 2** we might ask, "Your line doesn't look as steep

Fig. 2 Student work is useful for prompting such questions as, "Your line doesn't look as steep as mine, why is that?"



as mine. Why is that?" A follow-up question can then be, "How would the graph change if your car were slower?" For the graph in **figure 3**, we might ask, "Those lines look parallel. What does that say about the cars' speeds?" and "If they were parallel, when would the pace car pass your car?"

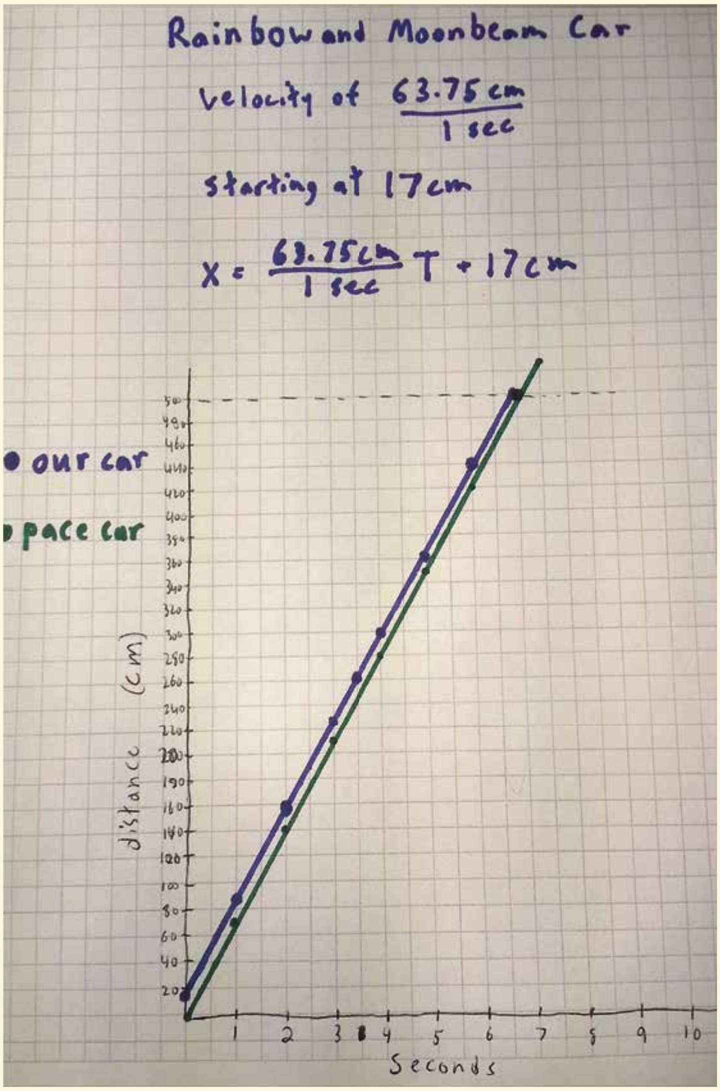
The preceding activity and discussion are used to prepare students for the culminating challenge: to determine where they need to place their car to tie the pace car in a 5 meter race (see **fig. 1**). At this point, students have the data for both their car and the pace car, so they simply need to combine what they understand about the speed in relation to the slope and the starting point in relation to the *y*-intercept to determine what their group's unique starting point must be. This is easier said than done. However, with time and guidance, all groups are successful.

Then, it is time for the big race. Usually at least one car "wins" the race, with two or three others close in tow, followed by that one group that played around too much and killed the battery. However, everyone is given a second chance, so that they can learn from their mistakes and be successful. On the second race, almost everyone ties the pace car. If students need prompting to help determine where to place their cars, the following questions may be effective to help spur their thinking:

- How long will it take your car to travel 5 meters?

- Should your car be placed ahead or behind the pace car?
- How long will it take the pace car to travel 5 meters? How many meters can your car travel in that amount of time?

Fig. 3 Parallel lines on a graph can prompt a question about what they mean in this context.



To conclude, each group member enters his or her work in a portfolio and responds to pointed questions about the meaning of the point on the graph where the cars intersect and how they decided where to place their car. This presents a nice segue into systems of linear equations.

EXTENDING THE CAR ACTIVITY

This project can be extended to develop students' understanding of quadratic relationships. Using a large ramp with gravity-powered vehicles, such as Matchbox® cars and trucks, students can record data in the same manner as before, graphing their points and finding the line of best fit. Special prompting may be needed for students to consider that these data are not linear. Students may realize that their pieces of tape are not the same distance apart, that the distance between the points on their graph is increasing, or that the graph of their data appears more curved than linear. By looking at

The car project can be extended to develop students' understanding of such quadratic relationships as the "slope of the slope," or second-order differences.

the rate of change between consecutive points and determining that the slope is increasing, students can determine that the rate of change of the slope, or the second-order differences, is constant. Using the slope of the slope, students are able to predict where the car should be at the next second in its travel down the ramp. It is helpful to "conveniently" have a larger ramp prepared with the same slope to test students' estimate.

UNDERSTANDING SLOPE: THE GOAL

The goal of these investigations and conversations is to give students a concrete model they are able to reference when developing their understanding of slope, the vertical intercept, and linear equations in slope-intercept form. By giving students these experiences and requiring them to verbally justify their understanding and conclusions, it is hoped that students will understand these concepts and be able to apply them to new situations (Boaler 1998). We further found that presenting students with these types of learning opportunities provides an entry point into a problem situation. When experiencing challenges, students immediately want to begin gathering data and using logic to predict what kind of mathematical model can best represent their findings.

We also noticed that this activity increased the willingness of reluctant students to work in small-group settings. It is very hard to gather accurate data in groups of only one or two people. This activity showed students who had not yet bought into the idea of group work why working with other students is a good idea. Once these students start working as group members, they usually become more engaged. This activity required multiple hands to gather data and multiple brains to analyze data. As a result, all must work together, which contributed to the learning of all (Cohen 1994).

Ultimately, developing students' ability to use logic to defend their thinking and to evaluate their own work is one of the highest priorities in any math class. The Common Core's standards state, "Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later" (CCSSI 2010, p. 7). In our classrooms, we have found that providing opportunities such as these has increased students' willingness to take risks and to try and learn from their errors. It has encouraged students to communicate their thinking and to reason through the thinking of their peers. Students develop their ability to connect multiple representations of content and to consider multiple solution strategies, making them more efficient problem solvers and mathematical thinkers. Although these activities certainly take time to implement well, it has been our experience that the benefits of doing so are worthwhile.

In the era of the Common Core State Standards for Mathematics, it is important now more than ever that students have opportunities to experience mathematics content in concrete situations. Mathematics instruction that favors "Solve $3x + 2 = 10$ for x" procedures is in need of a makeover. Solving this type of problem does not give students a basis for understanding the meaningless manipulation of symbols to solve for the mysterious "x."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Any thoughts on this article? Send an email to **mtms@nctm.org**.—*Ed.*

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activity sheet

Name ______________________________

Car ______________________________

Data:

Time (sec.)	Distance (cm)		
	Trial 1	Trial 2	Average

Graph:

1. Plot the data of distance versus (average) time.

2. Find the equation for the line of best fit.

3. How did you find the equation?

4. What does the slope mean?

5. How far will the car go in 3.5 seconds?

6. How long will it take for the car to go 5 meters?

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