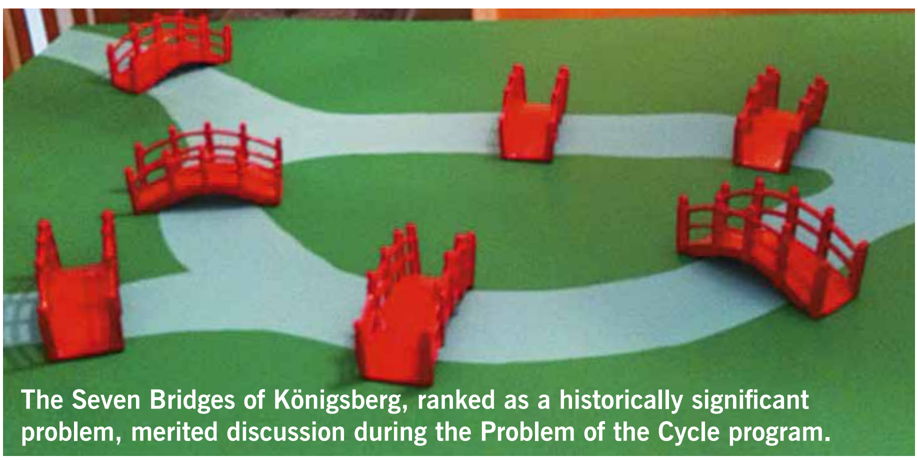
A Historical Twist on Problems of the Week

Alessandra King

Bertrand Russell explained that "Mathematics, rightly viewed, possesses not only truth, but supreme beauty" (1919, p. 60). It also has the ability to engage, mesmerize, and fascinate. We witnessed its power during our school's Problem of the Week program last year. "Mathematics is one of the greatest cultural and intellectual achievements of humankind, and citizens should develop an appreciation and understanding of that achievement, including its aesthetic and even recreational aspects" (NCTM 2000, p. 4). In addition to NCTM, many professional organizations such as the National Research Council (NRC) and the National Council for Accreditation of Teacher Education (NCATE) have underlined the importance of the history of mathematics in school curricula, which is a position supported by research (Dutta 2015; Lingard 2000).

Problem solving is central to mathematics because it provides "a context for learning and applying mathematical ideas" (NCTM 2000, p. 256). A series of Problems of the Week with a historical foundation connects all these strands. It provides students with a richer and deeper understanding of mathematical concepts; it shows them how and why such concepts were developed through years and sometimes centuries of hard work, partial success, sacrifice, trials, excitement, adversities, and delight; and it places mathematics in a clear and practical human context, demonstrating its astonishing usefulness.



The Seven Bridges of Königsberg, ranked as a historically significant problem, merited discussion during the Problem of the Cycle program.

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With its wealth of elegant concepts and proofs, colorful characters and interesting stories, such a foundation may increase students' interest. A positive attitude toward mathematics can also result.

LEARNING OBJECTIVES

The primary objective of our yearlong Problem of the Week program is to focus attention on, and foster interest in, mathematics through a series of exciting, historically important problems that can raise students' appreciation for and understanding of mathematical thinking. Exposing all our middle school students to challenging problems is in itself an objective of this project, as they all will benefit from engaging in problem-solving experiences (NCTM 2000).

Another objective, which aligns with the NCTM Standards for Grades 6–8, is a stronger emphasis on geometry. As geometry is an area that can sometimes be overlooked in our school curricula and an area in which U.S. students need practice, a significant portion of the year is reserved for this topic. Finally, this activity clarifies mathematics thinking and sustains conversations about mathematical ideas and strategies. This process allows students to explore problem solving from multiple perspectives and "analyze, compare, and contrast the meaningfulness, efficiency, and elegance of a variety of strategies" (NCTM 2000, p. 268).

PROGRAM DESIGN AND ACTIVITIES

Some of the schools—generally high schools, but also middle schools and college math departments—in our area run a Problem of the Week, or POW, program. This contest is designed to challenge the participants' critical-thinking abilities through fascinating problems, puzzles, and brainteasers. Such contests are usually open to the general population; in our case, middle school students and faculty. (In the past, however, parents, students, and faculty in other divisions and departments have also participated.) The basic structure of a POW contest is as follows: A new problem is posted on a (physical or electronic) bulletin board on a designated day of the week; on the same day the following week, the solution is posted. Those who have submitted correct answers receive a small award or recognition.

I teach in a small independent all-girls school in the suburbs of Washington, D.C. The middle school comprises about 160 students in grades 7 and 8. Since we have a six-day schedule (A to F days), our program was called Problem of the Cycle (POC), and the problems were presented and solved on F days (not Fridays).

Although following essentially the same plan of a regular POW contest, ours differed in some important aspects: First, the problems and the solutions were presented and discussed in morning assembly with the entire middle school community. Second, the contest was organized by teams: In our school, the students are divided into two houses (White and Blue). Although each contestant could enter her solution individually, it was the aggregated number of correct results that decided the winning team for that cycle. (In case of a tie, the tie-breaker would be the highest number of entries.) This result would then be entered in the running contest that the two houses have during the school year and would contribute to the final score for each quarter. (The winning house receives a day out of dress code as a reward.)

The contest between the two houses and the day-out-of-uniform reward constituted excellent motivation for many students; others were encouraged to participate by the lack of personal competition, as the answers were submitted privately in a blue or a white box. In general, participation in this program, although always optional, increased as time went on. By the end of the year, all students were involved at various levels of engagement. In addition, the solution of the POC was presented to the assembly by a student who was chosen on the basis of her solution's creativity, elegance, insight, or possibility for generalization. The other various strategies were also briefly mentioned and celebrated, and the students got to experience firsthand how each problem could be approached with different methods. The students were enthralled to see one of their classmates lead the assembly on mathematical topics, and many vied for that opportunity.

BUILDING ON HISTORICAL PROBLEMS

The defining aspect of our POC contest was that our leading problem was always historically important. Then, for a few cycles, we would study either its extensions or related problems. For example, one segment of the project was dedicated to the Seven Bridges of Königsberg problem. As for all the leading problems, there was a board, placed in an accessible corner of the hall, that the students could refer to and play with. The following few cycles were dedicated to its extensions (what happens if the people of Königsberg could afford to build another bridge? Is there a way that the network could be traversable with the additional bridge?) and to some simple application problems of graph theory.

For the segment on number theory, the students were introduced to the story of young Gauss adding the first 100 integers in a matter of seconds after discovering the underlying

Problems of the Week

Visit the sites below to access the historical problems discussed in the article.

The Königsberg Bridge problem:

- http://www.mathsisfun.com/activity/seven-bridges-konigsberg.html
- http://mathworld.wolfram.com/KoenigsbergBridgeProblem.html

Lune of Hippocrates:

- http://mathworld.wolfram.com/Lune.html
- http://jwilson.coe.uga.edu/emt668/EMAT6680.2000/Obara/Emat6690/Lunefolder/Quad%20Lune.html

Rice and Chessboard problem:

- http://britton.disted.camosun.bc.ca/jbchessgrain.htm
- http://nrich.maths.org/1163

Gauss and the sum of 100 integers:

- http://mathandmultimedia.com/2010/09/15/sum-first-n-positive-integers/

Pythagorean theorem:

- http://jwilson.coe.uga.edu/EMAT6680Fa2012/Smith/6690/pythagorean%20theorem/KLS_Pythagorean_Theorem.html

pattern. The students first solved that very problem themselves, and in the next few cycles concentrated on other arithmetic series and pattern recognition tasks. Other famous problems included the Pythagorean theorem and the rice on the chessboard problem, with forays into functions, exponential functions, and pattern recognition.

The Pythagorean theorem supplied the foundation for possibly one of the most interesting investigations of the year. Students thought that they already knew all there was to know about it when in fact they were about to discover a vast mathematical treasure. We set out first to understand and "prove" that the theorem was true. All too often, students viewed the Pythagorean theorem as a useful, if abstruse, algebraic formula until we studied its extensions: Would the area relationship hold if we consider other polygons built on the legs and the hypotenuse of the right triangle? And how about circular, complex shapes? What is the fundamental concept here? We ended the segment by solving the problem of the Lune of Hippocrates.

PROGRAM EVALUATION

Because of the nature of this activity, there was no plan for individual students' assessment. However, the program itself was received very positively by the school community, students and teachers alike. In fact, one of the most successful, if rather unusual, features of this project was its very communal nature: The problems were first proposed during middle school morning assembly and so were the solutions. This public sharing drew attention to mathematics as an exciting endeavor, and it became not only a hallmark of some assembly days but also a universal, intellectually bonding experience for the community. The students and the teachers of other subjects were fascinated by the problems and the stories and often offered positive, constructive feedback that was instrumental in creating a more enriching experience.

ENCOURAGING COMMUNITY

The students enjoyed this collective approach to problem solving, both the collaborative part of introducing the problem and sharing the solutions in assembly as well as the independent part of struggling with the challenge and submitting their answer independently. They liked to be involved in "complex investigations" and "working for several days on a single problem or its extensions" (NCTM 2000, p. 256). They learned to value different approaches and solutions to the same problems; they appreciated that one among them was singled out to take the lead in assembly; they cherished the opportunity to present their solution in public; and they loved the stories behind the problems. The students' eager participation, keen involvement, thoughtful contributions, and enthusiastic comments throughout the school year provided the energy for this journey and made it memorable, both for them and for the school community.

Successful POC experiences show that this valuable idea can be easily replicated, extended, or altered in myriad ways to suit different needs, schedules, or age groups while maintaining its appeal. One year, we ran a similar POC program based on short story problems: Doubling Pennies, Liars and Truthtellers, the Missing Dollar, the Monty Hall Problem, Birthday Probabilities, and so on. (Great resources for these problems are the Math Forum website at http://mathforum.org, and one of my favorite books, *The Man Who Counted: A Collection of Mathematical Adventures,* written by Júlio César de Mello e Souza under the pen name of Malba Tahan.)

Other series of POW can focus on logic puzzles (such as Sudoku, Ken-Ken, and so on) and logic matrices for any age group; Martin Gardner brainteasers; "Math Magic" algebraic

tricks; or current events (including sports), depending on the age and interests of the audience. This versatile activity can, with some creativity, be adapted to many school environments.

ACKNOWLEDGMENTS

It would not have been possible for me to create and manage this project without the help of a few key people. I would like to thank Middle School Director Tony Shawe for offering teachers and students the time and opportunity to explore mathematics together during assembly. I am also grateful to my colleagues, whose patience, interest, and curiosity for this project were essential for its success. Finally, and most important, I want to thank my students, as their enthusiasm and engagement throughout this journey have supported and encouraged my efforts.

Ed. note: Bertrand Russell's full quote is too lovely not to share in its entirety: "Mathematics, rightly viewed, possesses not only truth, but supreme beauty—a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show" (Russell 1919, p. 60).

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