Modeling the Shapes of Cells

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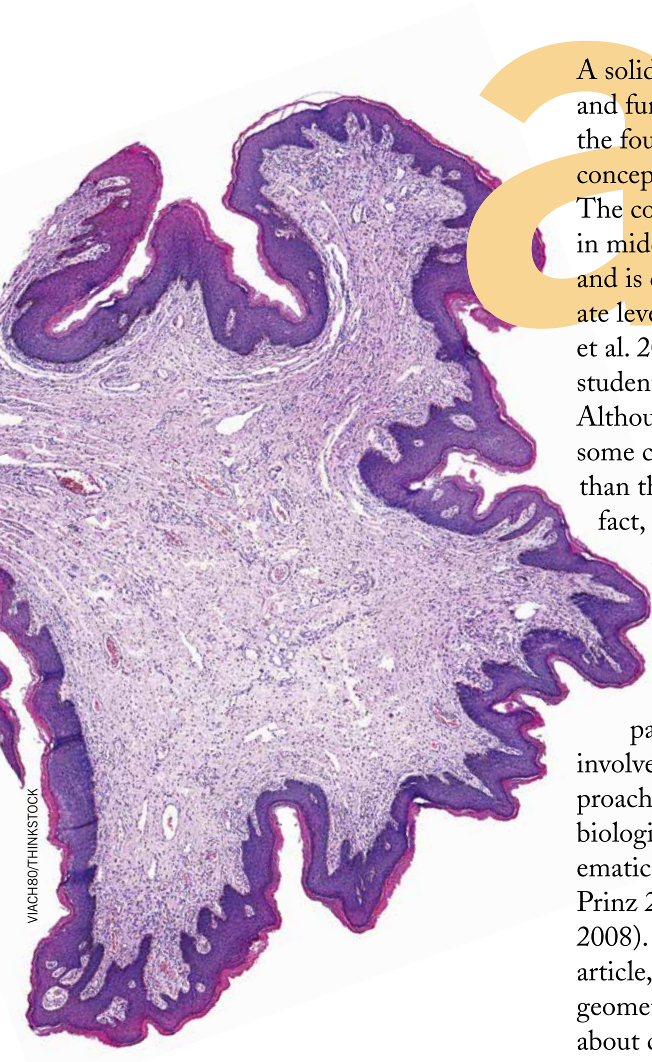

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Edited by S. **Asli Özgün-Koca**, aokoca@wayne.edu, Wayne State University, Detroit, Michigan, and **Marilyn Howard**, marilyn-howard@utulsa.edu, University of Tulsa, Oklahoma. Readers are encouraged to submit classroom-tested activities through **http://mtms.msubmit.net**.

A solid understanding of the structure and function of cells can help establish the foundation for learning advanced concepts in the biological sciences. The concept of the cell is introduced in middle school life science courses and is continued at the undergraduate level in college (NRC 2012; Reece et al. 2014). Cells are introduced to students as resembling blobs or cubes. Although these descriptions apply to some cells or to cell cultures, more than these shapes exist in nature. In fact, a wide range of beautiful and intricate cell shapes are found.

The cause of a cell's three-dimensional structure remains a mystery. Research in understanding the mechanisms that pattern the architecture of cells involves a highly interdisciplinary approach that combines the physical and biological sciences and applied mathematics (Marshall 2013; Voeltz and Prinz 2007; Rafeelski and Marshall 2008). The activities presented in this article, in which students learn about geometric shapes in mathematics and about cells in life science, are appropriate for middle-grades classes.

The mathematical concept of surface-area-to-volume ratio plays a key role in determining the biochemical processes and ultimately the three-dimensional geometric shape and size of a cell. The use of manipulatives has been researched (Moore 2013; Pepper, Phillips, and Wan 2014) and endorsed (NCSM 2013). Although manipulatives are frequently used in teaching mathematics, they are rarely used in teaching science concepts. Cell shape offers a unique opportunity to use manipulatives in ways that link mathematics to essential science concepts.

The surface-area-to-volume ratio of a cell reflects its shape and function. Using manipulatives to build three-dimensional models, students explore mathematical concepts to reinforce the basic concepts and gain a deeper understanding of cell structure to help answer the question, "How does the shape of a cell affect the surface-area-to-volume ratio?"

The three activities described extend a unit that investigates the perimeter-area relationship using tiles (Learning Resources 2006). Investigating the surface-area-to-volume ratio can be done using snap cubes. Each activity can be completed in a class period, activity 1 and 2 in mathematics class and activity 3 in science class.

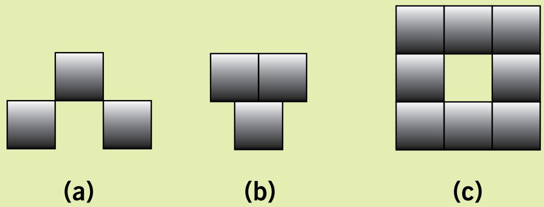
ACTIVITY 1: AREA AND PERIMETER

This activity reviews perimeter change with a fixed area, which may be a prerequisite concept for middle-grades students (Anderson et al. 2005).

Students in small groups are instructed to arrange 12 unit square tiles to design different two-dimensional shapes. Tiles must be joined edge to edge; those with a partial overlap or shapes with holes are not allowed (see **fig. 1**). Students draw each figure on grid paper, then find and record the perimeter. Possible student outcomes are represented in **figure 2**.

Students should conclude that although the area (number of tiles) is being held constant, rectangles that are longer and thinner have larger perimeters. The students are encouraged to think of nonrectangular shapes by posing a question: "Using a 4×3 rectangle, what happens to the perimeter when we shift every other column down 1 unit?" Students work together to design and record several nonrectangular models and determine the perimeters. See **figure 3**'s examples. Students should conclude that nonrectangular figures with more or longer appendages (arms) have larger perimeters.

Fig. 1 These shape configurations are not allowed.



ACTIVITY 2: VOLUME AND SURFACE AREA

Students are directed to use 24 unit cubes (snap cubes or pop cubes) to construct solid figures by following the rules analogous to activity 1 (i.e., the cubes must be aligned face to face, with no overlapping faces or holes in the shapes). Keeping the

Fig. 2 Students use 12 tiles to configure these possible shapes and record their perimeters.

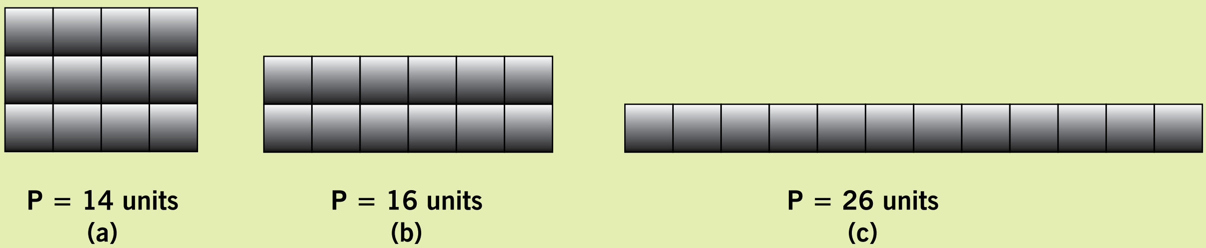


Fig. 3 Figures (a) through (d) and a sample data table show both area and perimeter computations.

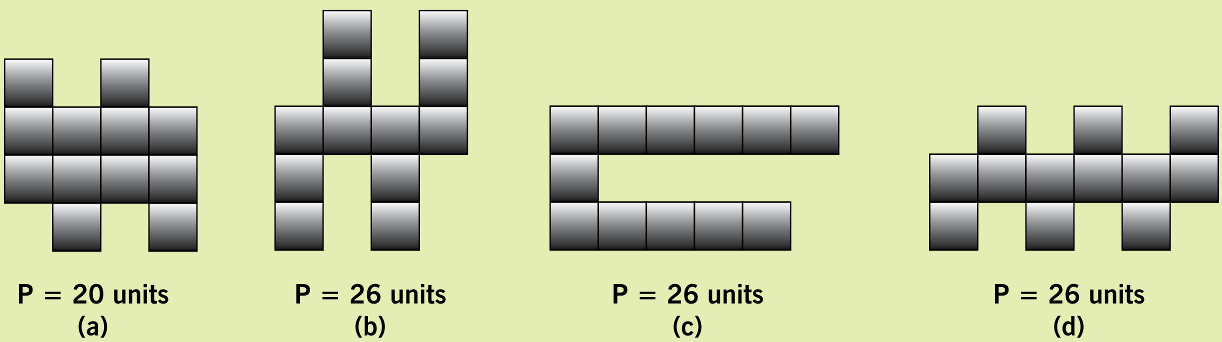
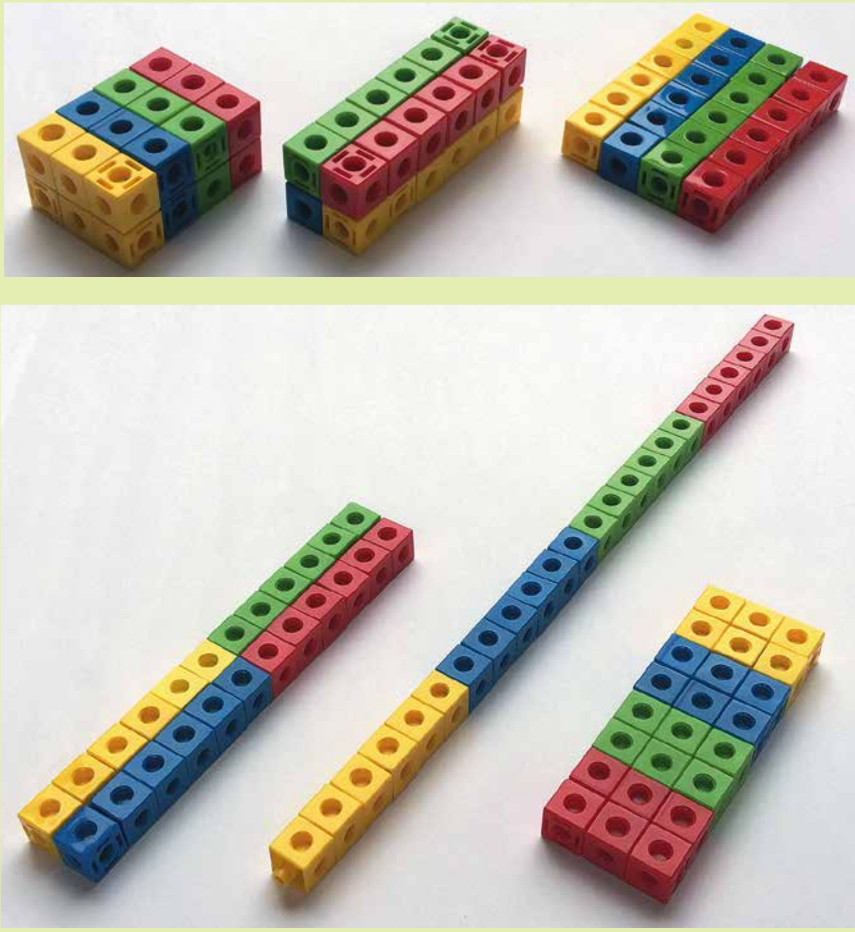


Figure	Area in Square Units	Perimeter in Units	Ratio of Perimeter to Area (P/A)	Ratio in Decimal Form
(a)	12	20	20/12	1.67
(b)	12	26	26/12	2.17
(c)	12	26	26/12	2.17
(d)	12	26	26/12	2.17

Fig. 4 Shapes made from 24 cubes can have different surface areas.



with the volume held constant, a structure that is long and flat has the largest surface area. Students are then encouraged to think of concave shapes by answering this question: "Using a $4 \times 6 \times 1$ model, what happens if we move a corner cube up to start a second layer?" Students may conjecture that for each cube moved from a corner to the second layer, there is a gain of 2 square units of surface area. Students continue moving cubes from the first to the second layer. The faces of the cubes in the second layer should not be touching each other (only the edges of the cubes in the second layer can touch). Some possible figures are presented in **figure 5**.

Students are then prompted to identify mathematical patterns in the data set. A table is an effective tool to organize data and illustrate the pattern. Since a maximum of 8 cubes can be moved to the second layer with those cubes not sharing a face, the domain of the function is 1 cube through 8 cubes. Sample data are presented in **table 1**.

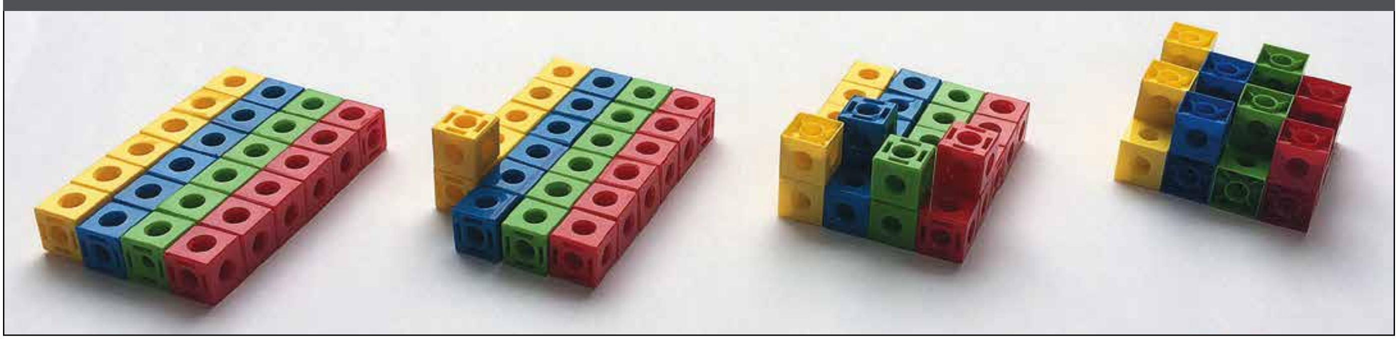
volume constant, students find the surface area and calculate the surface-area-to-volume ratio for each figure (Anderson et al. 2005). In activity 1, the figures with the maximum edges exposed produced the largest perimeter. In activity 2, students want to expose the maximum faces to answer, "If a cell has a volume of 24 cubic units, how will the shape of the cell affect the surface area? Do all figures with a volume of 24 cubic units have the same surface area?" Some possible configurations are presented in **figure 4**.

Students should conclude that

ACTIVITY 3: CELL SHAPES AND FUNCTIONS

This activity integrates mathematics and science by applying mathematical concepts learned in activities 1 and 2 to understand the three-dimensional shapes of cells and the relationship to specific function. Students research a variety of animal or plant cell shapes and their functions using textbooks

Fig. 5 Surface area is increased when cubes are moved to create a second layer.



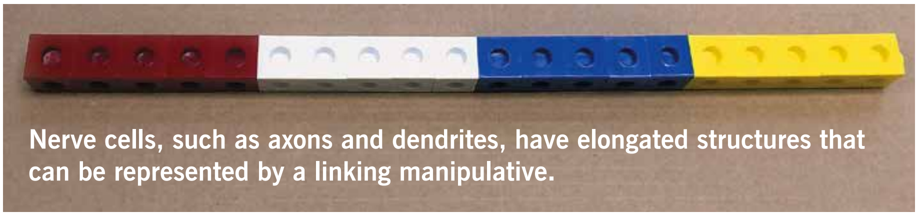
or online resources. Each group of students selects one cell type and prepares a poster presentation explaining the shape and function of the cells by using the mathematical data (surface-area-to-volume ratio) as evidence.

Neurons, the cells of the nervous system, look very different from other cells and from one another. Neurons have highly complex extensions and are designed both in shape and function to carry information reliably and quickly over long distances (communication). Since the function is to conduct electrical impulses across the membrane's surface, the thin wire-like shape minimizes the volume and maximizes the surface area, producing a larger surface-area-to-volume ratio. The linear model represents the axon of a nerve cell. (Teachers can search online for images of neurons, for classroom use.)

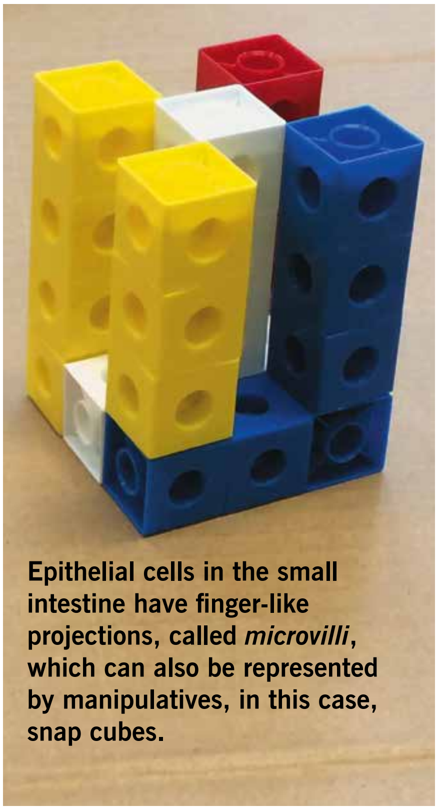
Epithelial cells lining the outer surface of the small intestine, for example, are specialized for absorption. The cells have small finger-like extensions called *microvilli,* resulting in a 20-fold to 30-fold increase in absorption, thus greatly increasing the efficiency of nutrient absorption during the digestive process. These cells have relatively larger surface area and smaller volume, thus a larger surface-area-to-volume ratio. (Teachers can search online for images of microvilli, for classroom use.) Other cell shapes that can be explored are the disk-shaped red blood cells, cuboidal-shaped glandular and fat cells, columnar epithelial cells, simple squamous epithelial skin cells, and spindle-shaped smooth muscle cells.

Table 1 A table helped students find patterns in the data

Number of Relocated Cubic Units	Volume in Cubic Units	Surface Area in Square Units	Surface Area/ Volume
1	24	68 + 2 = 70	70/24
2	24	68 + 2 + 2 = 72	72/24
3	24	68 + 2 + 2 + 2 = 74	74/24
4	24	68 + 4(2) = 76	76/24
8	24	68 + 8(2) = 84	84/24
n	24	68 + n(2)	n/24



Nerve cells, such as axons and dendrites, have elongated structures that can be represented by a linking manipulative.



Epithelial cells in the small intestine have finger-like projections, called *microvilli*, which can also be represented by manipulatives, in this case, snap cubes.

POTENTIAL STRENGTHS AND DIFFICULTIES

Activity 1's perimeter and area task allowed students to review their background knowledge of formulas for calculating perimeter and area and finding perimeter-to-area ratio. Flat tiles were easy to manipulate, to construct different shapes and then to draw on a grid. Students concluded that with surface area being constant, the perimeter length depends on the shape. Through mathematical representations such as figures and tables, students quickly learned that the relationship between perimeter and area is dynamic.

In activity 2's volume and surface area task, students had difficulty working with snap cubes and had a harder time building the models. Offering a few sample models and giving students time to build the models before doing calculations would help the students' progress.

Students calculated the volume of a 24-unit cube with 2 unit × 3 unit × 4 unit dimensions. However, they had difficulty calculating surface area. Several of them calculated surface area by using the formula $2(l \times h \times w)$, instead of calculating the surface area of each face and adding them. A review of the mathematical formulas and process of calculating surface area and volume will avoid confusion.

For an extension, or for a separate activity, provide an in-class example by selecting a specific cell shape and walking the class through the steps to compare with the snap-cube model. Calculating the surface-area-to-volume ratio and explaining the relationship among the model, cell shape,

and the function will be helpful. Students can research the following:

1. Research animal and/or plant cell shapes and their functions.
2. Select one type of cell that has a specific shape and function.
3. Compare the shape of the cell you selected and a figure that you built that most resembles the shape of the cell. Explain the relationship between mathematical data (surface-area-to-volume ratio) and the cell's function.

MATHEMATICAL MODELS OF CELL SHAPES

Mathematics is an important tool for understanding the patterns and relationships that exist in nature. Instead of starting with the different cell shapes that are somewhat irregular, students can focus on the mathematical concepts of surface-area-to-volume ratio by using cubes. Showing the natural connections between disciplines can enhance students' comprehension and appreciation of mathematics and science. Constructing models using manipulatives, measuring space in two and three dimensions, and calculating ratios and proportions are key explorations for students to experience before they are asked to relate the data to different cell structures found in living organisms. Through this activity, students will understand the relationship between structure and function and see a meaningful connection among existing disciplines.

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The reproducible activity sheets and the solutions to Mathematical Explorations are online at **www.nctm.org**. Online solutions are available to NCTM members only.

activity sheet 1

Name ______________________________

AREA AND PERIMETER

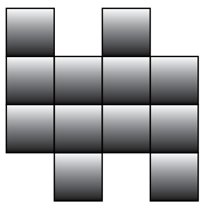
1. Use 12 tiles. Keeping in mind that each tile is a square unit, make as many different rectangles with the tiles as possible, each with an area of 12 square units.

2. Sketch the figures on a separate piece of graph paper, and label each figure with its perimeter. Number the figures, beginning with 1.

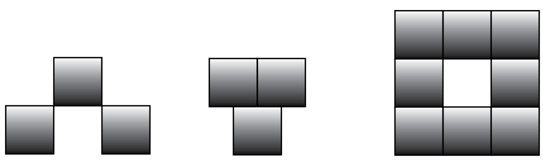
3. Complete the table below.

Figure Number	Area in Square Units	Perimeter in Units	Ratio of Perimeter to Area (P/A)	Ratio in Decimal Form
1				
2				
3				

4. Use 12 square unit tiles and make 6 *different* nonrectangular shapes. See the figure below for an example. When creating nonrectangular shapes, notice that the tiles are joined edge to edge.



The following figures are not allowed: Tiles that only touch at the corners (a), a tile that overlaps two other tiles (b), and tiles that produce a figure with an open space (c).



(a) (b) (c)

from the October 2015 issue of mathematics teaching in the MIDDLE SCHOOL

Name ______________________________

5. Sketch and number the figures on a piece of graph paper, and label each one with its perimeter.

6. Use the table below to enter the data and complete the calculations.

Figure Number	Area in Square Units	Perimeter in Units	Ratio of Perimeter to Area (P/A)	Ratio in Decimal Form
Example from question 4	12	20	20/12	1.66

7. Begin with a 4×3 rectangle. What happens to the perimeter when you shift 1 tile to another column?

8. Which nonrectangular figure has the smallest perimeter? Which nonrectangular figure has the largest perimeter?

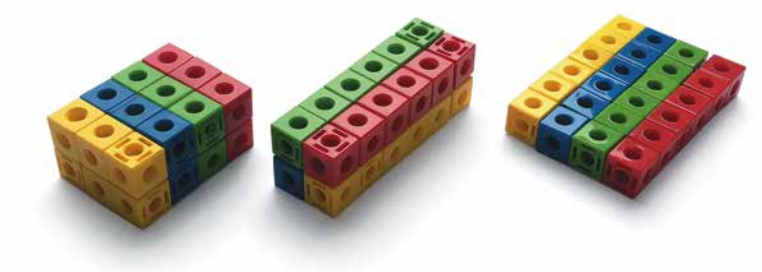
9. Is there any relationship between the number of exposed edges and the length of the perimeter?

activity sheet 2

Name ______________________________

VOLUME AND SURFACE AREA

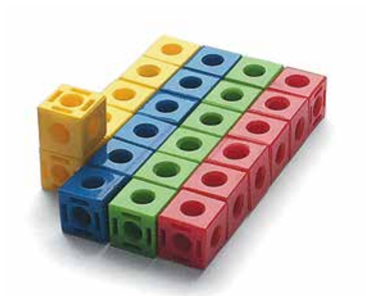
1. Use 24 cubes and make as many different rectangular prisms as possible with a volume of 24 cubic units. Each cube has a volume of 1 cubic unit. Sample prisms are shown below.



2. Use the table below to enter the data and complete the calculations for each shape you build.

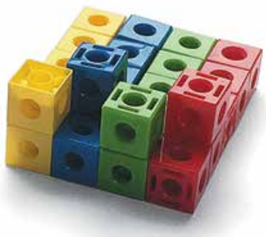
	Number of Units						
Figure	**Width**	**Length**	**Height**	**Surface Area (SA) in Square Units**	**Volume (V) in Cubic Units**	**SA/V Ratio**	**SA/V Ratio in Decimal Form**
$3 \times 4 \times 2$	3	4	2	52	24	52/24	2.16

3. Use a $4 \times 6 \times 1$ shape and move 1 corner cube from the first layer to the second layer. See the figure below for an example. Notice that the volume stays the same. By how much did the surface area change from the $4 \times 6 \times 1$ prism's surface area?



Name ______________________________

4. Using the $4 \times 6 \times 1$ rectangular prism, continue to move cubes to the second layer. The faces of the cubes in the second layer should not be touching (only the edges of the cubes can touch). Two examples are shown below. Calculate the new surface area by moving 2, 3, 4, 5, 6, 7, and 8 cubes from the first layer. Record your findings in the table below.



Number of Cubic Units Moved	Surface Area (in Sq. Units)	Surface Area/Volume Ratio
0	68	68/24
1		
2		
3		
4		
5		
6		
7		

5. Which shape has the smallest surface area? Which shape has the largest surface area?

6. Identify mathematical patterns you notice in the data set.