

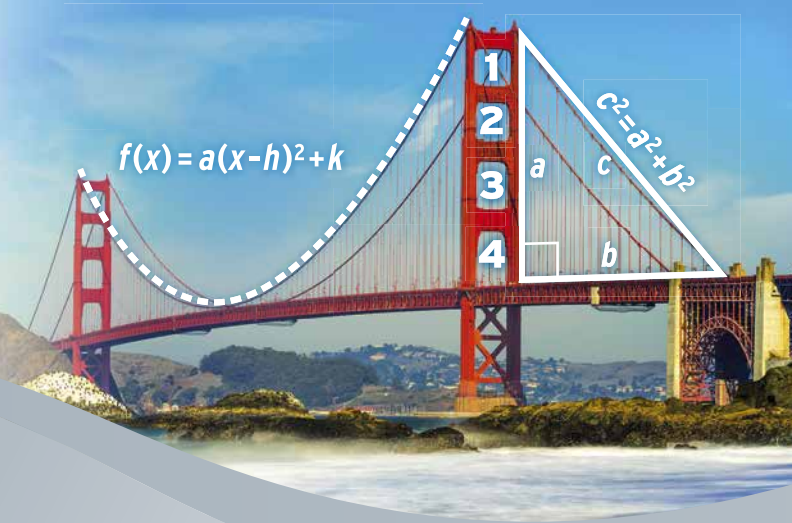


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PREMIER MATH EDUCATION EVENT

# 2016 NCTM ANNUAL MEETING & EXPOSITION

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## FEATURES

### 208 12 Math Rules That Expire in the Middle Grades

*Karen S. Karp, Sarah B. Bush, and  
Barbara J. Dougherty*

Turn away from overgeneralizations and consider alternative terminology and notation to support student understanding.

### 216 Fred Applegate's Money-Making Scheme

*Annette Ricks Leitze and Kristen L. Soots*

Tasks requiring a high level of cognitive demand will develop students' problem-solving skills exponentially.

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### 222 Exploring Algebraic Misconceptions with Technology

*Matthew Sakow and Ruveyda Karaman*

Long-standing common student misinterpretations concerning expressions and equations can be addressed by using an app such as iSolveIt: MathScaled.

### 230 Do Twelfths Terminate or Repeat?

*Rebecca Ambrose and Erica Burnison*

While seventh-grade students group fractions, they learn more about decimals, in particular, and division, in general.



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# mathematics teaching in the MIDDLE SCHOOL

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**more4U**

### Look online for these additional items:

- Solutions to the problem in Solve It, p. 196
- Solutions to the Palette of Problems, p. 206
- Solutions to Mathematical Explorations, p. 244
- Solutions to the problems in Math for Real, p. 256

More4U content is an additional benefit for members only.

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## in the next issue

Be sure to look in the December 2015/January 2016 journal for these features:

- In “Stacking Cans: Abstracting from Computation,” George J. Roy, Farshid Safi, and LuAnn Graul discuss how ordering and analyzing stacks of cans will help students connect computation and algebraic reasoning.
- As a preview of coming attractions, namely, the February Focus Issue: Probability and Statistics, read “Journey to Centers in the Core,” in which Randall E. Groth, Kristen D. Kent, and Ebony D. Hitch explore a series of lessons to help students choose appropriate measures of center when analyzing data.