solve it student thinking

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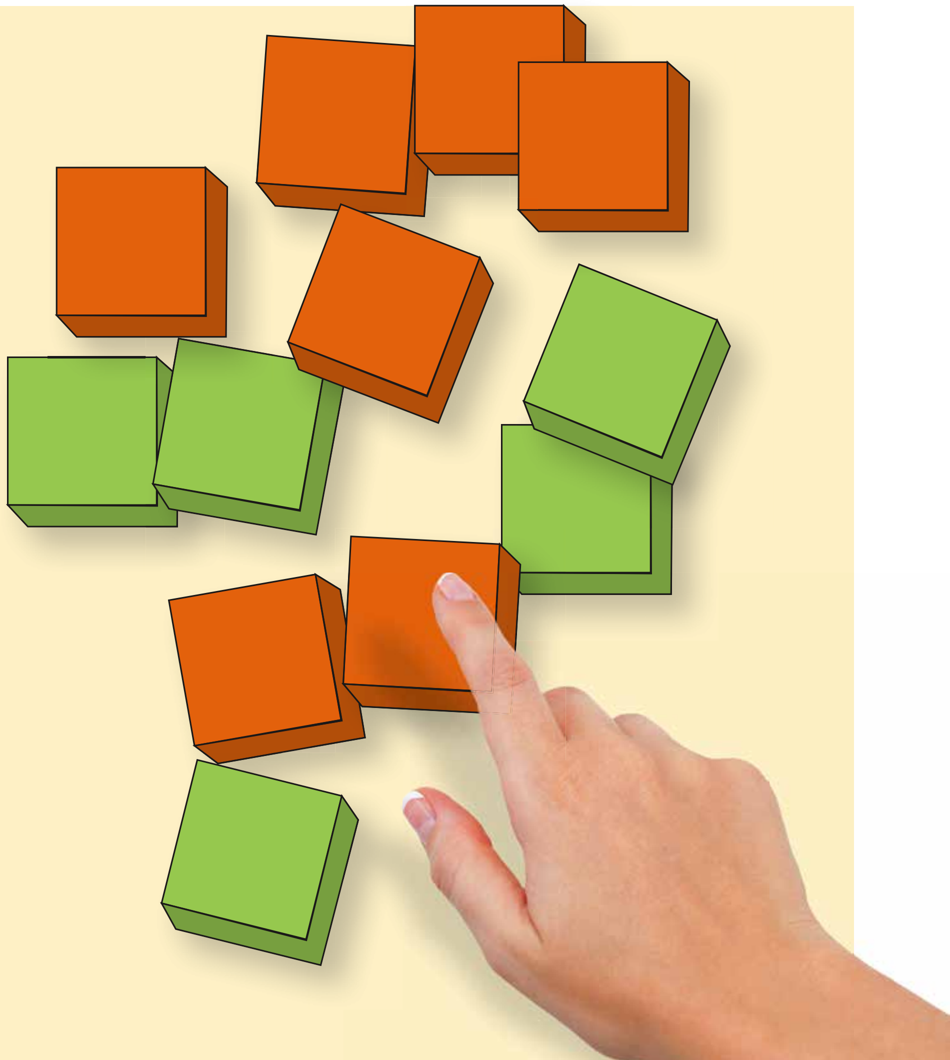
big solutions to little problems

This Solve It task appeared in the March 2015 issue:



DENYS PRYKHODOV/THINKSTOCK

Taking a Chance

Your school is having a fund-raiser. Your class contributes by manning a game-of-chance booth. Since it is a fund-raiser, you want to design a game in which the player has a 1 in 3 chance of winning. Your plan is to use up to 20 color tiles (10 green and 10 orange) and have players draw 2 tiles from the bag. To win, the player must draw 2 orange tiles.

1. Decide how many of each color tile should be put in the bag.
2. Explain how you decided how many tiles of each color to use.

CCSSM: 7SP.5; 7SP.6; 7SP.8a; 7SP.8b

This department shares creative solutions to the problems presented in Solve It. Edited by **Jo Ann Cady**, jcady@utk.edu, and **Sherry L. Bair**, slbair@yahoo.com.

Students could attempt to solve the Taking a Chance problem either experimentally, by using tiles, or theoretically. An experimental approach helps students understand the different outcomes when drawing two tiles out of a bag. This approach also helps students develop an intuition to the probability of each outcome, given the number of orange and green tiles. The results of an experiment will not always match the theoretical probability, so students who initially employ this approach will have to use a theoretical argument to justify the number of orange and green tiles that results in a probability of drawing two orange tiles equal to one-third.

After working on this task, a reader inquired as to whether the tile drawn first was put back into the bag. The situation of drawing two tiles at the same time is the same as drawing one tile, not replacing it, and then drawing a second tile.

None of the students whose work is discussed below solved the problem experimentally. To calculate the combined probabilities, the students made an initial guess about the number of orange and green tiles in the bag. Then they multiplied the probability of getting an orange on the first draw by the probability of getting an orange on the second draw.

The theoretical probability of drawing an orange tile on the first draw can be calculated by

$$\frac{\textit{number of orange tiles}}{\textit{total number of tiles}}.$$

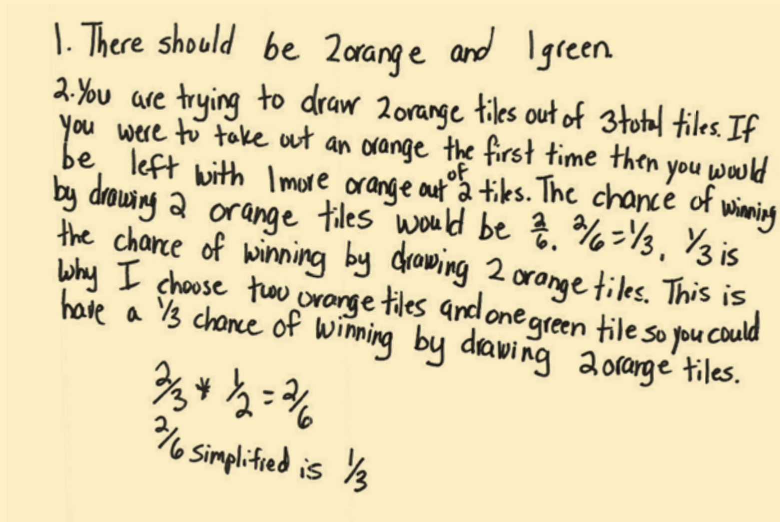
The theoretical probability of also drawing an orange tile on the second draw can be calculated by

$$\frac{\textit{number of orange tiles} - 1}{\textit{total number of tiles} - 1}.$$

Since an orange tile was removed from the bag on the first draw, the number of orange tiles in the bag will be one fewer when drawing for the second time. Consequently, the total number of tiles in the bag will also be one fewer on the second draw. Students might theorize that the number of orange tiles will need to be larger than the number of green tiles to draw two orange tiles for a win.

Many of the younger students chose to use a guess-and-check strategy, and most were able to find a correct solution of 3 tiles in the bag, 2 orange and 1 green, using this method. Maddie, an eighth-grade student in Kate Carl's "Puzzles, Patterns, and Predictions" math elective class, justified her answer in **figure 1a**.

Fig. 1 Maddie (**a**) and Daniel (**b**) found a valid solution: 2 orange tiles and 1 green tile.



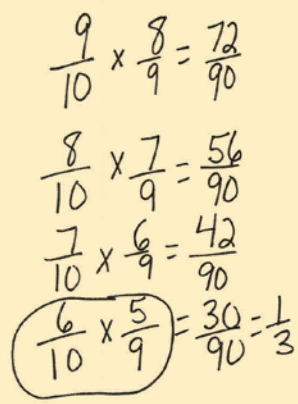
1. There should be 2orange and 1green

2. You are trying to draw 2orange tiles out of 3total tiles. If you were to take out an orange the first time then you would be left with 1more orange out of 2 tiles. The chance of winning by drawing 2 orange tiles would be $\frac{2}{6}$. $2/6 = 1/3$. $1/3$ is the chance of winning by drawing 2 orange tiles. This is why I choose two orange tiles and one green tile so you could have a $1/3$ chance of winning by drawing 2orange tiles.

$\frac{2}{3} * \frac{1}{2} = \frac{2}{6}$

$2/6$ Simplified is $1/3$

(a)



$$\frac{9}{10} \times \frac{8}{9} = \frac{72}{90}$$

$$\frac{8}{10} \times \frac{7}{9} = \frac{56}{90}$$

$$\frac{7}{10} \times \frac{6}{9} = \frac{42}{90}$$

$$\frac{6}{10} \times \frac{5}{9} = \frac{30}{90} = \frac{1}{3}$$

(b)

Daniel, a student in Karen Dorneman's class, appeared to understand that the probabilities on the first draw and second draw involved consecutive numbers. He made the computations shown in **figure 1b**. He then provided this explanation:

> I think there are 6 orange cubes and 4 green cubes in the bag. I used guess and test to find the answer. I started with $9/10 \times 8/9$ and made my way down until I got $6/10 \times 5/9$, which equals 1/3. You would start with 6 orange cubes and 4 green cubes and it was dependent, so it would change to 5 orange cubes and 4 green cubes.

Fig. 2 Cameron's work showed several possible tile combinations.

The question is to make a game where you need to pick 2 orange tiles in a row (without replacing them) to win, and there needs to be a 1 in 3 chance of winning. There can only be up to 10 tiles of each orange and green.

So, P(O) x P(O) = 1/3

x/y x z/t = 1/3

I found the algorithm is: (y*t) / 3 = x*z

And that x and z are consecutive and y and t are consecutive. So I found the products of all the consecutive pairs up to 20… and divided them by 3.

1x2=	2	
2x3=	6	÷3= 2
3x4=	12	÷3= 4
4x5=	20	
5x6=	30	÷3= 10
6x7=	42	÷3= 14
7x8=	56	
8x9=	72	÷3= 24
9x10=	90	÷3= 30
10x11=	110	
11x12=	132	÷3= 44
12x13=	156	÷3= 52
13x14=	182	
14x15=	210	÷3= 70
15x16=	240	÷3= 80
16x17=	272	
17x18=	306	÷3= 102
18x19=	342	÷3= 114
19x20=	380	

Then I looked for matches between (the green column and the orange one). ex: 1x2= 2, and 2x3/3= 2. So the 2 answers are:

Answer 1: 2 Orange and 1 Green.
P(O) x P(O) = P(O,O)
2/3 x 1/2 = 2/6 --> 1/3!

Answer 2: 6 Orange and 4 Green.
P(O) x P(O) = P(O,O)
6/10 x 5/9 = 30/90 --> 3/9 --> 1/3

It seems that with a 1 in 3 chance of winning that there won't be fractions like 2/3 and 6/10. But there is!

And the next possible answer, even though it is over 20, is:
21 Orange and 15 Green.
P(O) x P(O) = P(O,O)
21/36 x 20/35 = 420/1260 --> 42/126 --> 7/21 --> 1/3!!!

His guess-and-check strategy restricted the total number of cubes, or tiles, to 10, and he systematically checked the combined probability by reducing the number of orange cubes from 9 down to 6.

Both Maddie and Daniel found correct solutions to the problem, but neither tried to find out if other solutions were possible. They also did not try to find all tile combinations that might work. Cameron, a seventh-grade student in Carl's class, *did* try to find all possible tile combinations (see his explanation in **fig. 2**).

Cameron systematically listed all possible solutions to the problem; Daniel's list contained only the first solution of 10 tiles in a bag. Cameron also began to develop extensions to the original problem and reflected on patterns that he noticed. The reasoning displayed by all three students showed how one problem could challenge and engage a range of students in the same classroom.

REFLECTION ON THE TASK

In reflecting on this task and the student work submitted, we believe it would be beneficial for students to initially explore the problem experimentally using tiles in a bag. This method would allow students to think about the total number of outcomes and how drawing two tiles without replacement influences the distribution of outcomes. It would also help them to think about how the number of orange tiles would change from the first draw to the second draw. Having manipulatives available for students and asking them to simulate the task would also help. We also encourage small-group discussions and whole-class discussions to brainstorm possible solution strategies.

Kate Carl, STEM Academy, Sandwich, Massachusetts; Karen Dorneman, St. Katharine Drexel Regional Catholic School, Holland, Pennsylvania; and Andrea Kowalchik Dwenger, Wilmette Junior High, Wilmette, Illinois, submitted student work for this month's Solve It Student Thinking article. We appreciate their willingness to try these problems with their students and to submit student work samples for us to share with *MTMS* readers. It is through this sharing of ideas that we can all learn about mathematics and how students think about mathematics problems. We welcome submissions of complete student solutions; even partial work on problems is helpful.