Professional Noticing: Learning to Teach Responsively

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In the buzzing activity of the mathematics classroom, teaching in a way that effectively responds to and furthers students' thinking can be quite challenging. Given that teachers' instructional decisions will directly influence students' learning, it is extremely important to develop the sorts of practices that lead to productive in-the-moment teaching choices.

To explore teachers' instructional decisions and the potential to influence students' learning, consider the algebraic task and student response in **figure 1**. What are the important mathematical features of this work? What might we suppose that Jennifer understands about this particular mathematical idea? What next instructional step might be productive?

Teachers have the opportunity to engage in professional noticing in every instructional situation; however, this practice is more complex than merely observing students and their work. Jacobs, Lamb, and Philipp (2010) defined professional noticing as "a set of three interrelated skills: *attending* to children's strategies, *interpreting* children's understandings, and *deciding* how to respond on the basis of children's understandings" (p. 172). To emphasize each of the three interrelated skills, it is useful to slow down one's instructional thought process and consider them separately yet progressively. Beginning with attending, what has Jennifer done in response to the task? Take a moment to consider the salient mathematical features of Jennifer's response before reading on.

As we analyze the features of Jennifer's response, we see that she has initially correctly translated the problem into numbers and symbols; however, she has incorrectly applied the use of parentheses. Further, Jennifer appears to have chosen and substituted a specific value for Y and solved rather than constructed the more generalized

Fig. 1 What can a teacher infer from Jennifer's response?

Using numbers and symbols, write an expression for the following:
The product of 7 times some number *Y* multiplied by the difference of 8 and 2

Jennifer's response:

$$7 \times Y \times 8 - 2$$
$$7 \times (\cancel{Y} \times 8) - 2$$
$$7 \times (2 \times 8) - 2$$
$$7 \times (16) - 2$$
$$\begin{array}{r} {}^{4}16 \\ \times 7 \\ \hline 112 \end{array} \qquad 112 - 2 = 110$$

Edited by **Despina Stylianou**, dstylianou@ccny.cuny.edu, and **Jennifer Eli**, jeli@math.arizona.edu. Readers are encouraged to visit **http://mtms.msubmit.net** to submit manuscripts that take research findings and translate them into practical outcomes, strategies, or tips that directly inform teachers' classroom practice.

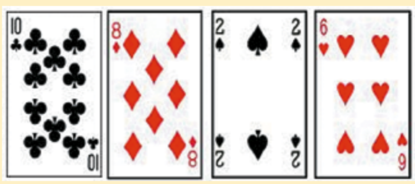
Fig. 2 Variations on a game involving playing cards provide a structured pathway for advancing Jennifer's mathematical development from her initial understanding to the target standard.

Initial Understanding

5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

Making 24 Card Game

Object of the game: Create an expression for 24 using 4 playing cards drawn from a deck. All operations (×, +, –, ÷), parentheses, and/or brackets may be used.



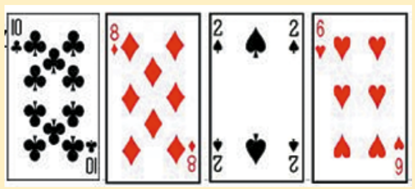
Sample solutions:
$(10 - 8) \times 6 \times 2$
$8/(2 - 10/6)$
$(8 - 2) \times (10 - 6)$
$(10 + 8 - 6) \times 2$

Mathematical Development

5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.

Translating Expressions Card Game

Object of the game: Draw 4 cards from the deck and use these cards to choose which of the two phrases will yield the greatest solution.



Phrase 1: The sum of the larger 2 cards divided by the difference of the smaller 2 cards.
Phrase 2: The product of the smaller 2 cards minus the sum of the larger 2 cards.

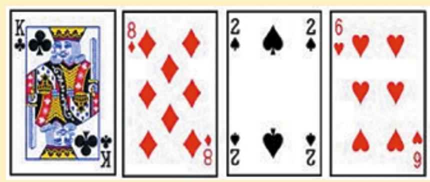
Sample solutions:
Phrase 1: $(10 + 8)/(6 - 2)$
Phrase 2: $(6 \times 2) - (10 + 8)$

Target Standard

6.EE.2.a Write expressions that record operations with numbers and with letters standing for numbers.

Making 24 Card Game with Variables

Object of the game: Similar to the Making 24 card game. In this instance, face cards are introduced as variables. Create an expression for 24 using 4 drawn cards, and then determine the value of the face card.



Sample solutions:
$2K + (8 + 6)$; $K = 5$
$(K - 6) \times (2 + 8)$; $K = 8.4$

expression requested by the task.

Central to this discussion of professional noticing is the focus on students' progressive mathematical development. Regarding Jennifer's work and the second interrelated component of professional noticing, how might we interpret her work in the context of algebraic development? How would we describe what she understands about algebraic concepts? Consider these questions for a moment before continuing.

Jennifer appears to have some understanding that parentheses may be used to structure numerical expressions, although they were misapplied in this instance. Interpreting these features, Jennifer's strategy appears to reflect an emerging understanding of the Common Core State Standards for Mathematics (CCSSM) standard 5.OA.1: Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. However, the demands of the task are likely targeting 6.EE.2a: Write expressions that record operations with numbers and with letters standing for numbers (CCSSI 2010).

Considering Jennifer's work and the third interrelated component of professional noticing, how might one decide on an appropriate next instructional step in the context of algebraic development? What sorts of teaching moves might be most effective? Contemplate these questions for a moment.

To decide on appropriate instructional tactics to advance Jennifer's mathematical thinking, the Early Equations and Expressions learning trajectory (Confrey, Maloney, and Nguyen 2011a) provides a productive pathway. For this particular example, variations on a game involving playing cards may be used to provide one possible structured pathway for mathematical development (see **fig. 2**).

Table 1 These five practices can be used to coordinate productive mathematical discourse and professional noticing.

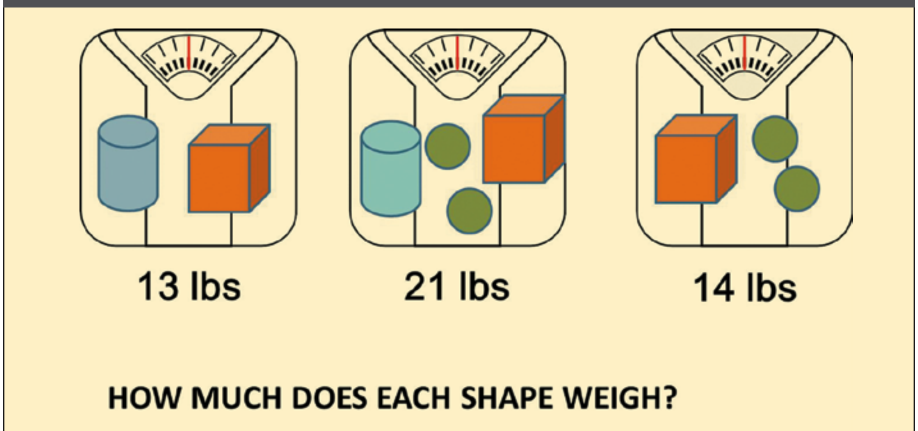
	Professional Noticing		
Five Practices for Productive Discussions	Attend	Interpret	Decide
Anticipate	Teachers *anticipate* by adopting a student's perspective and considering the strategies, questions, and difficult points that may arise as students complete the chosen task.		
Monitor	Teachers *monitor* the work of the students by giving attention to and interpreting their written work, their math talk, and their interaction with manipulatives and multiple representations. These interpretations provide deeper insight into students' thinking.		
Select	Teachers *select* particular student examples and work that will contribute to a broad and developmental understanding of the chosen mathematical goal or concept. These selections are based on the manifest features of student strategies to which teachers have attended and interpreted.		
Sequence	Teachers *sequence* the selected works by drawing on their knowledge of mathematical development (e.g., learning trajectories) to construct a purposeful order for student sharing.		
Connect	Teachers *connect* students' strategies through discussion. Teachers seek to identify and explore connections between the different selected examples as well as encouraging connections to the larger goal of the lesson. Skillful orchestration of connecting discussions once again draws on a teacher's capacity to attend to and interpret contours of fluid discussions and make decisions that enhance the connections among the presented mathematics.		

PROFESSIONAL NOTICING INFORMED BY TRAJECTORIES

Professional noticing in the context of the preceding example may be considered trajectory-based. When engaging in professional noticing, multiple trajectories might guide teachers' practice. CCSSM provides the basis for one such trajectory. Indeed, the underlying trajectories of CCSSM have been the focus of considerable study (Confrey et al. 2011b), and this work provides a cognitive model for thinking about and acting on the features of Jennifer's work.

This application of noticing is built on an understanding of students' development along a particular mathematical pathway or trajectory. Such trajectories may support professional noticing because teachers' understanding of the common development of particular mathematical concepts can inform their interpretation of students' mathematics and how to respond instructionally.

Fig. 3 The Scale and Shape problem was presented to students.



SOURCE: VAN DE WALLE 2007

PROFESSIONAL NOTICING IN A DISCOURSE-RICH CLASSROOM

Although analyzing student work retroactively is beneficial for developing the interrelated skills of professional noticing, much of the work of teaching requires in-the-moment noticing of classroom interactions. A knowledge of trajectories continues to be a valuable tool to draw on as teachers plan for and facilitate productive discussions. Underlying productive discussions is the selection of appropriately rich tasks. Such tasks require students to think critically; they also

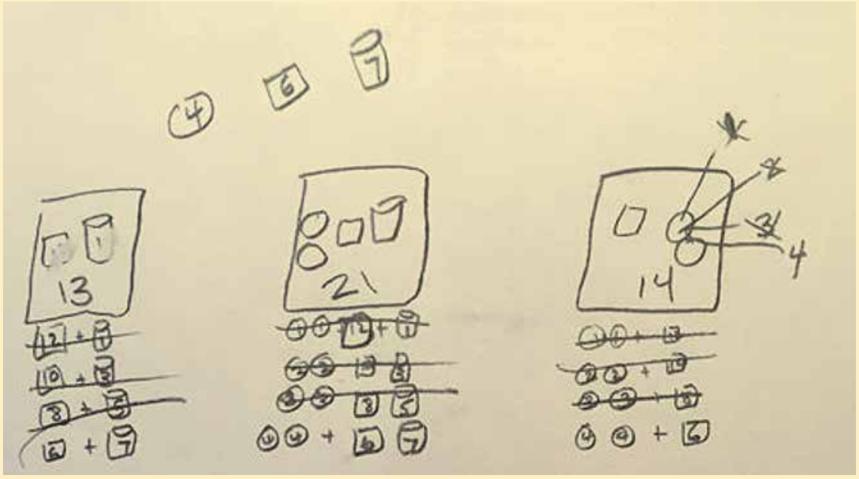
may be considered from multiple perspectives and solved using a variety of strategies.

Once a task has been identified, students may construct viable arguments while considering and critiquing the arguments of their peers. Students who are engaged in productive activity must wrestle with ideas and listen to others. While they are negotiating situations that provoke disequilibrium, they also construct mathematical meaning. Regarding the manner in which teachers may initiate and lead such discussions, Stein and colleagues (2008) outline five practices for orchestrating productive mathematical discourse. These practices correspond to the component skills of professional noticing (see **table 1**).

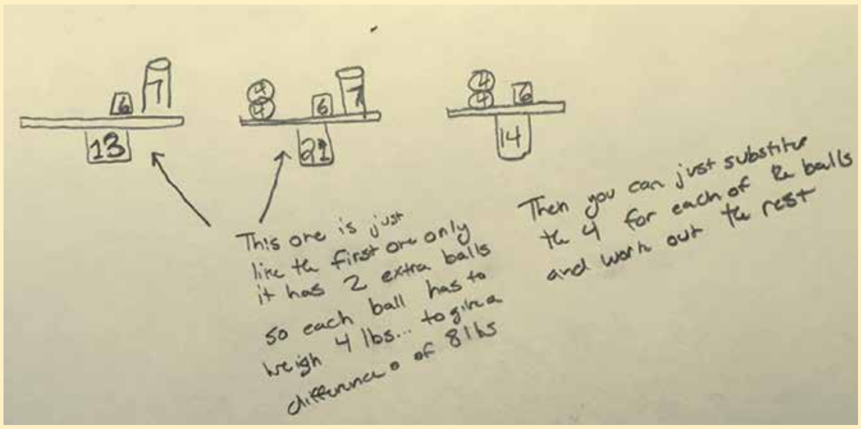
Returning to the assertion that productive discourse occurs in the context of rich mathematical tasks, scale and shape problems (Van de Walle 2007) provide sound contexts for students to explore and discuss algebraic ideas (see **fig. 3**). In this particular example, pairs of students worked on such a problem (see **fig. 4**).

For this particular task, the teacher might anticipate that students would either use a guess-and-check strategy or deductive comparison between the scales. However, as the teacher monitors the student work, he or she might see that groups 1 and 3 used some manner of a guess-and-check strategy; group 3's work may be interpreted as more systematic in that Robin and Cathy considered addends that would equal 13, whereas Adrienne and Jim began testing progressively increasing values for the disc. Given this difference, the teacher might decide to select all three groups to share in the following order: group 1, group 3, group 2. This sequencing (based on the interpreted features of the response) provides the opportunity to examine and discuss the task through

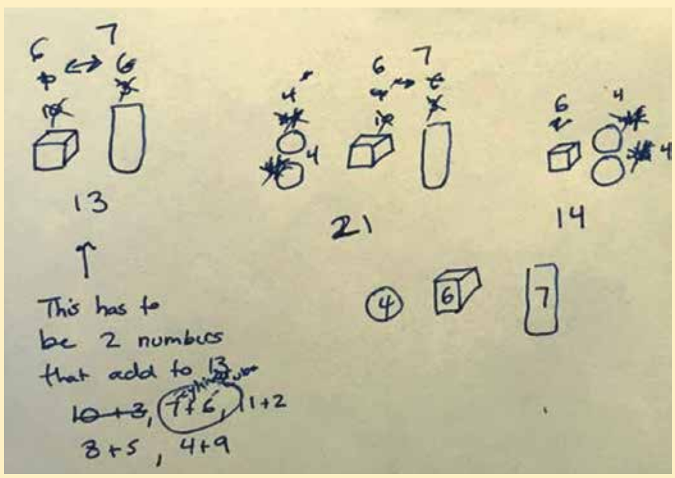
Fig. 4 Teachers select and sequence student responses to the Scale and Shape problem for a class discussion.



Group 1: Adrienne and Jim



Group 2: Sarama and Kyle



Group 3: Robin and Cathy

increasingly systematic strategies culminating in Sarama and Kyle's deductive comparison. Note that both the selection and sequencing in this example are built on some knowledge of trajectory because the teacher is making informed judgments about the development of students' mathematical understanding. In this sense, we may consider discourse-oriented noticing related to trajectory-oriented professional noticing.

PRACTICE TIPS AND FINAL REMARKS

The following are tips to keep in mind as teachers begin to practice the component skills of professional noticing.

1. Before noticing, identify the mathematical goals and try to anticipate how students might respond.

When planning lessons, teachers should have a robust sense of the most important mathematical ideas that are to be developed in a particular lesson. For example, with Jennifer, the desired instructional goal dealt with writing expressions, which included variables. Further, during the planning process, teachers should try to envision the different ways in which students might respond to the lesson activities or tasks, envision a range of such hypothetical responses, and organize them into a pathway or trajectory that illustrates progress toward the identified goal. This hypothetical pathway of development will provide a roadmap as teachers attend to, interpret, and decide on the mathematical thinking of their students.

2. When attending, take a moment to get the story straight.

There is often so much happening in a classroom in a given moment that it can be quite easy to compress the attending portion of professional noticing and jump quickly into interpretations or even move straight to decisions. For example, in Jennifer's initial response (see **fig. 1**), the aim is to examine each individual line rather than just the answer. We find it useful to take a moment to consider students' actions and words so that we might construct a mental, mathematical play by play of what just occurred. It can also be beneficial to ask another student to summarize the actions and words of his or her peer, but teachers should be mindful in such instances because students may superimpose some manner of interpretation onto the original student's actions and words.

3. When interpreting, stay connected to the mathematical evidence.

It is likely impossible to separate mathematical interpretations from the broader contextual knowledge that teachers have of their students. Moreover, use of such knowledge is likely appropriate. For example, knowing a student's recent failure to make the soccer team the prior afternoon might inform the manner in which mathematical struggle is interpreted the following morning. Nevertheless, it may be tempting to let external, contextual factors have an outsized influence on interpretations of students' mathematics. Rather, we suggest that teachers try to connect interpretations to the evidence gained when attending. In the Shape and Scale problem (see **figs. 3** and **4**), groups 1 and 3 both used a guess-and-check strategy; however, interpreting the two strategies (based on the evidence) illustrates a key difference in the students' thinking. Similarly, if, for example, we interpret that a student has a fundamental misconception of equality, what mathematical actions or statements did we attend to in the preceding moments that led us to this interpretation?

4. When deciding, go for information or goal-focused instruction.

Sometimes, the events to which teachers have attended do not provide sufficient grounds for a solid interpretation of students' mathematical thinking. In these instances, decisions may be aimed at gathering additional information about students' strategies or mathematical thinking. For example, deciding to pose questions (e.g., "How were you thinking about that?" and "Why did you write that particular expression?") aimed at eliciting additional student activity (to which a teacher might attend) is quite appropriate. Conversely, if teachers feel that their interpretation of a particular student's mathematical thinking is sufficiently robust, then we encourage them to briefly recall the mathematical goals of the experience (identified in the planning process). From this, teachers should ask themselves the following question: What action might I take to help this student move closer to an identified mathematical goal? In the case of Jennifer (see **fig. 2**), the teacher might enact a sequence of progressively more sophisticated card games. More generally, appropriate teaching actions might include co-constructing, with the student, a representation of the mathematics at hand. Perhaps another appropriate action may be to relieve the student from the obligation of the current task and pose a different task. Although there may be instances when teachers elect to pursue unplanned yet related mathematical ideas, generally, instructional decisions should directly connect to some point on a pathway or trajectory that concludes with the identified goals of the lesson.

Professional noticing may be applied in a variety of mathematical contexts. Although standards (e.g., CCSSM) provide a framework for the

development of mathematical content, they often fail to comment on instructional practices; thus, it is left to the professional judgment of teachers to determine the instruction that will best meet the needs of students. This affirms the importance of responsiveness in the classroom and how critical it is for teachers to make in-the-moment decisions, informed by knowledge of the common developmental pathways of mathematical thinking. These decisions, built on attending to and interpreting students' strategies, have much power to positively affect students' mathematical understanding.

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