Flipping Out: Calculating Probability with a Coin Game

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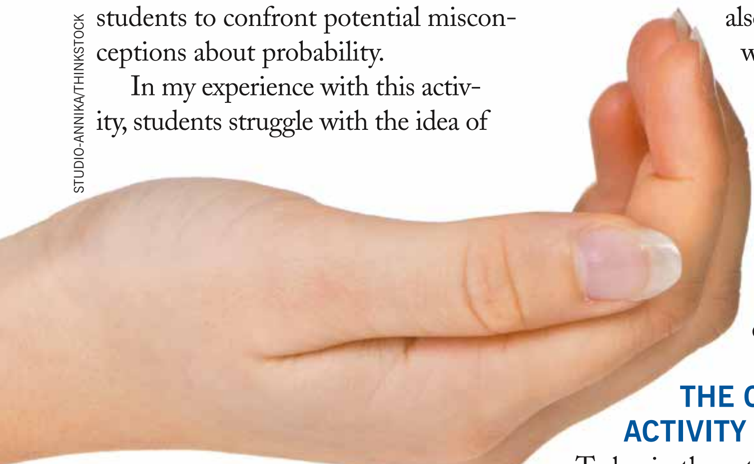
This probability activity builds on students' experience with the common practice of coin flipping. In particular, the activity addresses the grade 7 Common Core State Standards for Mathematics (CCSSM) in probability and statistics (CCSSI 2010). Students calculate the probability of simple and compound events by using an organized list or probability tree. Additionally, the activity provides the opportunity for students to confront potential misconceptions about probability.

In my experience with this activity, students struggle with the idea of *representativeness* in probability (Kustos and Zelkowski 2013). Therefore, this student misconception is part of the classroom discussion about the activities in this lesson. Representativeness is related to the (incorrect) idea that outcomes that seem more random are more likely to happen. This misconception is illustrated by middle school students who believe that a winning lottery ticket of 1, 2, 3, 4, 5 is less likely to occur than a winning ticket of 11, 8, 4, 13, 7. However, Kustos and Zelkowski (2013) also indicate that students who have had experiences in addressing the idea of representativeness through simulation are able to dispel this misconception as it relates to calculating the probability of an event.



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THE COIN-FLIPPING ACTIVITY

To begin the activity, each student has one coin (or uses a random number generator). In pairs, both students flip their coins. Heads is denoted by H; tails, by T. Player 1 will get 1 point if both flipped coins land either HH or TT. Player 2 will get 1 point if the flipped coin combination is HT or TH. I have found that a few students comment that the rules of the game are not fair because the outcomes listed for player 2 are more likely to happen (the idea of representativeness). During the introductory part of the lesson, I choose not to address this comment because students will have the opportunity to develop their understanding of this situation while participating in the activity. Each pair of students continues flipping their coins and keeping track of the count until one player reaches 20 points.

Once students have finished one game, we discuss the question of fairness by reviewing the way that the probability of an event occurring is calculated. Then, students work with their partner to determine whether or not the game is fair. Most determine fairness by creating an organized list; a few students draw a probability tree; and almost none creates the table as shown in the solutions.

As students share answers, we compare the various methods they used for analyzing fairness. Students justify their thinking by answering such questions as these:

- "When does it make sense to make an organized list, instead of a probability tree?"

- "Why might someone prefer to make a probability tree?"

If no student has made a table, I also introduce students to this method for calculating probability. The majority of the students choose to write the probability of the events occurring as fractions, instead of decimals.

As a class, we record how many times player 1 (HH or TT) won the game and how many times player 2 (HT or TH) won the game. If player 1 and player 2 had an equally likely chance of winning, we would expect that about one-half the time player 1 is the winner; about one-half the time player 2 is a winner. Whether or not this is the case, collecting these data from students leads to a discussion about what we expected to happen, based on the probability of an event occurring, and what actually happened.

This discussion addresses another CCSSM standard, which states that students should compare probability from a model to the observed frequencies. Students should also be able to give a reasonable explanation for the observed frequencies not matching the model (CCSSM 2010, 7.SP.C.7). Many times, students are able to call on another example of an unlikely event occurring to explain why one-half the games may not have been won by player 1 and one-half by player 2. (A typical student comment might be, "I know a family that has 4 girls and no boys, which is unlikely but still happens.") Another interesting piece of data to collect is how many coin flips in total were needed before a winner could be declared (this is addressed in question 5d on the **activity sheet**).

Next, students design a similar game in which 3 coins are tossed, instead of 2. Students write the rules for the game, so that both players have an equally likely chance of winning.

Students will most likely work to find all outcomes by making an organized list or drawing a probability tree. The total number of combinations (8) can still be split evenly between two people, with each person being assigned a total of four outcomes. However, students must focus their attention on the importance of being able to differentiate between the various outcomes, or they will need to write the rules in such a way that the order does not matter. For example, when 3 coins are flipped, how do they differentiate among the outcomes HHT, THH, and HTH? In all three cases, the outcomes include 2 heads and 1 tails. To avoid this, students could write the rules to state that a coin flip resulting in at least 2 heads (HHT, THH, HTH, and HHH) gives 1 point to player 1 and a coin flip resulting in at least 2 tails (TTH, HTT, THT, and TTT) gives 1 point to player 2.

In the first situation, students did not need to differentiate between an outcome of HT or TH because in either situation, the same player was awarded 1 point (this was important in the way the rules were written). However, in the game with 3 coins, it becomes important to differentiate between the outcomes. The most common solution that students devise is to flip the coins one at a time, instead of simultaneously, to differentiate between events. Other solutions are to flip a penny, nickel, and dime or to flip colored chips instead of coins.

Some students will also incorrectly write the rules to state that HHH and TTT will award player 1 a point and all other combinations will award player 2 a point because they are simply parroting the rules of the game at the beginning of the lesson. This incorrect answer is a great opportunity to have students determine through playing the game and calculating probabilities whether or not their game design is fair.

Although there is no one correct answer for designing a fair game, the most straightforward way to write the rules is to use the statements "as least 2 heads" and "at least 2 tails." In some cases, the order in which the events occur does not matter, so there is no reason to differentiate between the outcomes. However, in other cases the order may be quite important (depending on the way the students have written the rules). As an extension, ask students to evaluate the fairness of other games involving coin flipping or using dice and a spinner. Another idea is to ask students to give examples of other compound events in which the order is important compared with the order being inconsequential.

REFERENCES

Common Core State Standards Initiative (CCSSI). 2010. Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf

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Name ______________________________

COIN FLIPPING AND COMPOUND PROBABILITY

Work with a partner to make a team of 2 students. Each team member will have 1 coin to flip. Both team members flip their coins. If both coins show heads (HH) or both coins show tails (TT), player 1 gets 1 point. If the coins show heads-tails (HT) or tails-heads (TH), player 2 gets 1 point.

1. Do the rules of the game seem fair to you? Why or why not?

2. Play until one person gets 20 points. Record the points in the tally box below.

Player 1 (HH or TT)	Player 2 (HT or TH)
Total points:	Total points:

Winner:

3. Think about the game you just finished playing.

a. What are all of the possible outcomes for flipping 2 coins?

b. What is the probability of having each of those outcomes occur?

c. Is this a fair game? That is, does each player have an equally likely chance of winning the game? Explain your reasoning.

4. Since each event is equally likely to occur, what would you expect for the number of tallies in each box from question 1? Did you get what you expected? If not, give a possible explanation for the discrepancy.

5. Now, gather data from all teams in the class.

a. How many games in your class were won by player 1? __________

b. How many games in your class were won by player 2? __________

c. If each of these players had an equally likely chance of winning, what would you expect your answers for 5a and 5b to be?

d. What was the win margin for each team in the class? __________

6. Each team of 2 people will play a new game, flipping 3 coins instead of 2 coins.

a. What are all the possible outcomes for flipping 3 coins?

b. What is the probability of having each of these outcomes occur?

7. Use your probability model from question 6 to determine the—

a. rules for this new game so that player 1 and player 2 each still have an equally likely chance of winning the game in which the order that the coins are flipped does not matter. Write your rules below.

b. rules for this new game so that player 1 and player 2 each still have an equally likely chance of winning the game in which the order that the coins are flipped does matter. Write your rules below.

8. How does using 3 coins make the rules of the game more complicated than using 2 coins?

EXTENSION

9. Design a game in which 1 player will flip 1 coin and the other player will roll a 6-sided die. Write the rules of the game, so that each player has an equally likely chance of winning.

10. List other situations in which the order of events matters.