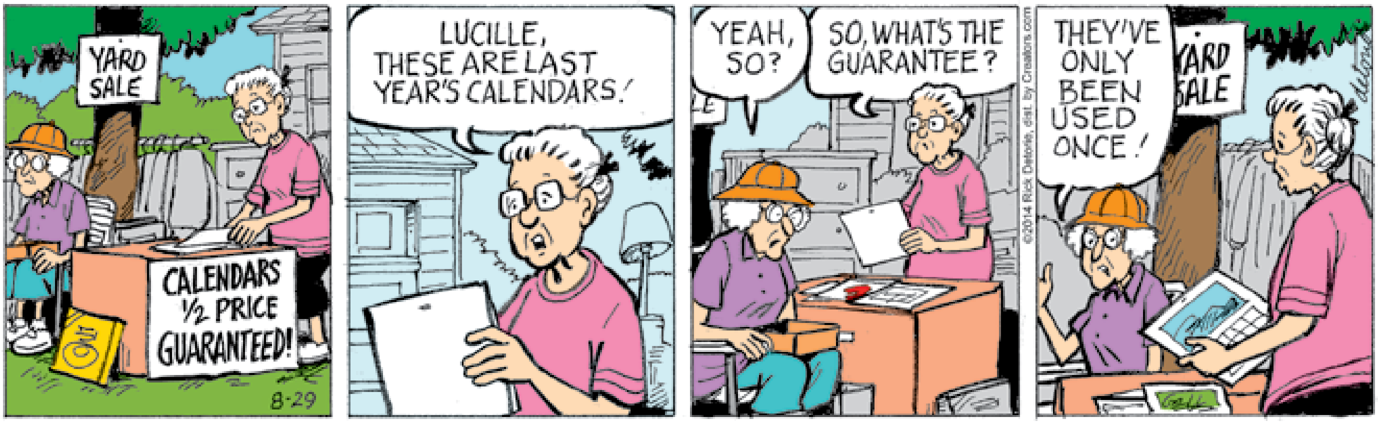
cartoon corner

Name ______________________________

ONE BIG HAPPY by Rick Detorie



CALENDAR CONUNDRUMS

1. a. There is something mathematically special about every date on the calendar. For example, the date December 3, written 12/3, is a “multiple date” because 12 is a multiple of 3. Find all the “multiple dates” during December.

b. December 8, written 12/8, is a “composite date” because both numbers are composite numbers. Find all the “composite dates” during December.

2. a. Pick any 2 × 2 array of numbers on a calendar. Multiply the numbers along the diagonals as shown in red below. Then subtract the smaller product from the larger. Repeat for several other 2 × 2 arrays of numbers. What do you notice about the differences of the products?

Sun	Mon	Tues	Wed	Thurs	Fri	Sat
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19

3 × 11 = 33
4 × 10 = 40
40 – 33 = 7

b. Write an argument that proves that your conclusion in question 2a applies for any 2 × 2 array of numbers on a calendar.

3. The cartoon claims that the calendar being sold was used only once. Do you think the 2015 calendar could be used in another year? If so, when is the first such year?

4. The formula below

$$W = d + 2m + \lfloor (3m + 3) \div 5 \rfloor + y + \left\lfloor \frac{y}{4} \right\rfloor - \left\lfloor \frac{y}{100} \right\rfloor + \left\lfloor \frac{y}{400} \right\rfloor + 2$$

can be used to determine the day of the week, W, on which any date falls.

d: day of the month (from 1 to 31)
m: month number (March = 3, April = 4, . . . , December = 12, January = 13, and February = 14)
y: year (For dates in January or February, use the previous year.)

When an expression is contained within these symbols,

$\lfloor \quad \rfloor$

complete the calculation inside them and, for positive values, discard the fractional portion and use only the whole-number part of the quotient. When you obtain a value for W, divide it by 7. The remainder will tell you the day of the week for your date, in which 0 = Saturday, 1 = Sunday, 2 = Monday, and so on.

a. On which day of the week did July 4, 1776, fall?

b. What day of the week were you born?

CHALLENGE

5. The year 2002 is a palindrome because it reads the same forward and backward. The number 20022002 is also a palindrome. Find three factors (*x, y,* and *z*) of 20022002 such that xyz = 20022002, and that each of those factors is a palindrome consisting of at least two digits.

from the December 2015/January 2016 issue of mathematics teaching in the MIDDLE SCHOOL

Edited by **David B. Spangler**, McGraw-Hill Education, Chicago, Illinois; **Katie A. Hendrickson**, code.org, Seattle, Washington; and **Hoyun Cho**, Capital University, Columbus, Ohio. Classroom teachers interested in field-testing or submitting a cartoon should contact Cartoon Corner at **mtms@nctm.org**. The cartoons must include the date and the newspaper syndicate that holds the copyright.

SOLUTIONS

1. a. 12/1, 12/2, 12/3, 12/4, 12/6, 12/12

b. 12/4, 12/6, 12/8, 12/9, 12/10, 12/12, 12/14, 12/15, 12/16, 12/18, 12/20, 12/21, 12/22, 12/24, 12/25, 12/26, 12/27, 12/28, 12/30

2. a. The difference of the products is always 7.

b. Sample argument: Let x = the number in the upper-left-hand corner. Then the other factor in its product is $(x + 8)$. The two factors of the other product are $(x + 1)$ and $(x + 7)$. Thus, the products are $x^2 + 8x$ and $x^2 + 8x + 7$, respectively. The difference of those products is 7.

3. The 2015 calendar will work again in 2026. Calendars repeat themselves 11 years later, unless you are starting with the year after a leap year. There are exactly 52 1/7 weeks in a non-leap year because 365 ÷ 7 = 52 1/7, meaning that any given non-leap year will start and end with the same day of the week. (For example, if January 1 in a non-leap year is a Monday, then December 31 will also be a Monday). Each leap year contains an extra day, so if January 1 is a Monday, December 31 will be Tuesday. In 2015, January 1 was on a Thursday, so in 2016, January 1 is on a Friday. In 2017, January 1 will occur on a Sunday (because 2016 is a leap year); 2018: Monday; 2019: Tuesday; 2020: Wednesday; 2021: Friday (because 2020 is a leap year); 2022: Saturday; 2023: Sunday; 2024: Monday; 2025: Wednesday (because 2024 is a leap year); 2026: Thursday. For those years immediately after leap years (such as 2017), the calendar repeats itself every 6 years.

4. a. In the formula

$$W = d + 2m + \lfloor (3m+3) \div 5 \rfloor + y + \left\lfloor \frac{y}{4} \right\rfloor - \left\lfloor \frac{y}{100} \right\rfloor + \left\lfloor \frac{y}{400} \right\rfloor + 2,$$

$d = 4$, $m = 7$, and $y = 1776$. So:

$$\begin{aligned} W &= 4 + 2(7) + \lfloor (3 \cdot 7 + 3) \div 5 \rfloor + 1776 \\ &+ \left\lfloor \frac{1776}{4} \right\rfloor - \left\lfloor \frac{1776}{100} \right\rfloor + \left\lfloor \frac{1776}{400} \right\rfloor + 2 \\ &= 4 + 14 + \lfloor (21+3) \div 5 \rfloor + 1776 \\ &+ \lfloor 444 \rfloor - \lfloor 17.76 \rfloor + \lfloor 4.44 \rfloor + 2 \\ &= 18 + \lfloor 24 \div 5 \rfloor + 1776 \\ &+ 444 - 17 + 4 + 2 \\ &= 18 + 4 + 1776 \\ &+ 444 - 17 + 4 + 2 = 2231 \end{aligned}$$

$$2231 \div 7 \to 318 \text{ R5},$$

so July 4, 1776, was on a Thursday (because the remainder is 5).

b. Answers will vary.

CHALLENGE

5. 959 × 949 × 22. Hint: it may help to start with the prime factorization of 20022002, which is 2 × 7 × 11 × 13 × 73 × 137.

FIELD-TEST COMMENTS

This cartoon provided practice in number play and patterns for my sixth-grade prealgebra students. In problem 2, students wanted to describe the answer of 7 as proportional, so we discussed the difference between constant change and proportional change.

Problem 3 led to a lengthy conversation and many suggestions, including 2380, since that number refers to 365 years later. As a class, we listed the years from 2015 to 2050. Students calculated that 365 days is equivalent to 52 weeks with 1 extra day, whereas leap years have 2 extra days. Beginning with 2015 on a Thursday, we added the 1 weekday for each year. The cheers for 2020 were quickly squelched when students realized that it was a leap year and that the 2015 calendar could not be reused. Next, one student continued the pattern to see whether or not the 2016 calendar could be reused; he then realized that he just had to multiply 4 × 7 since it had to be a leap year calendar to account for the 7 extra days. Other patterns emerged as the students checked 2017's calendar "reusability."

Problem 5 encouraged a lively discussion about the definition of a palindrome. The first two proposed answers were 1 × 2002 × 10001 and 2 × 1001 × 10001, with these students arguing that one-digit numbers are palindromes. Students were not satisfied with *Merriam-Webster*'s definition: "A number (as 1881) that reads the same backward or forward." This definition did not mention single digits. I suggested that students try to find another set of factors with at least two digits each. After completing the prime factorization, 22 × 949 × 959 was discovered.

Judy Kraus
Hyde Park Middle School
Las Vegas, Nevada

Ed. note: On the basis of the comments above, we adjusted the directions to exclude one-digit palindromes.

I used this cartoon with a sixth-grade math class near the end of the first semester. This class had high and low achievers, ESL students, and special education students. Students worked in groups of two or four based on their seating arrangements and were able to quickly complete problems 1a, 1b, and 2a.

Problem 1 provided a nice review of factors and multiples, which are vocabulary words that many students still struggle to understand. These questions also offered an opportunity to revisit prime and composite numbers. Most students were fascinated to learn that the difference of array products in problem 2 was always 7. However, several students had difficulty understanding problem 2b. If I were to use this cartoon again, I would insert another question before problem 2b asking the students to explain why the difference is always 7. I think this would make writing the argument in problem 2b easier.

For problem 3, only one student thought that the calendar could be used again. His answer was only one year off! All other students stated that the calendar could be used only once, and several made remarks about time only going forward and not repeating. I found this to be interesting, especially because I know how much time the elementary teachers in my school spend on calendar math. I am going to spend some time with my classes discussing the calendar and when it will repeat. We will talk about birthdays and how the day of the week of the students' birthdays changes from year to year.

None of my students attempted problem 4. Order of operations is a topic we spend a significant amount of time on in sixth grade but they have never worked with brackets or expressions this lengthy. Instead, I asked the students to calculate their ages in years, months, and days (from "Other Ideas"). I was gearing up to teach the NCTM "Presidential Data" lesson (from the March 2014 issue of *MTMS*) to go along with President's Day. In this activity, students determine the age of each U.S. President on his inauguration day. Many students want to merely subtract the years, and they forget to consider the month of birth, so calculating their own ages was good practice for this.

Carol Fears
Landstuhl Elementary/Middle School
Armed Forces Post Office, Area Europe

My students enjoyed looking at the calendar to complete this activity. They found the pattern of the 7s in problem 2 to be very interesting. The second part of problem 2 offered a good springboard for us to discuss argument writing, and we were able to do some interesting work on comparing representations—verbal, symbolic, numerical examples, and so on. I liked working through this cartoon in small groups, to encourage conversations about number relationships and vocabulary.

Problem 3 presented a big challenge. We ended up moving on to problem 4 and looking at what that formula represented and how it was actually just a big order of operations problem.

Allison Lashley
DC Preparatory Academy
Washington, D.C.

I gave this cartoon to my seventh-grade students as an extra-credit assignment over a school break. This task served as a good review of the vocabulary (composite, multiple, and so on) that they had learned in sixth grade. All students easily completed the tasks in problems 1 and 2.

I was surprised to see that many

WE NEED YOU TO TEST DRIVE CARTOON CORNER

Cartoon Corner thrives on readers just like you. If you and your students enjoy this department, sign up to be a field tester and receive cartoon activities before publication. The process is easy:

1. Volunteer by sending an email to Cartoon Corner at **mtms@nctm.org**.
2. Use the Cartoon Corner activity in your classroom.
3. Within about three or four weeks of receipt of the activity, write a two- to three-paragraph description about what worked, what did not work, and what your students learned. Any descriptions of *aha!* moments experienced, surprises found, or ideas for further exploration are especially welcome.

You will be helping the larger community of math teachers with not only your expertise but also your students' problem-solving acumen. Your name and school affiliation will be listed in the journal, as well.

students did not know how the days of the year shift; therefore, many did not have mathematical answers for problem 3. Although all students attempted to use the formula in problem 4, several students had trouble understanding the complexity of the directions. I used this problem as a teachable moment to review how to evaluate algebraic expressions and how to use the order of operations. I will consider using this cartoon with my sixth-grade students after we complete a unit on algebraic expressions.

Students questioned whether the date 12/24 would count as a correct answer to problem 1a, since 24 is a multiple of 12. This led to a discussion about the implied definition of a "multiple date."

Margaret Merwin
St. Mary's Academy
Hoosick Falls, New York

OTHER IDEAS

Extend this Cartoon Corner task with these ideas:

- Have students determine their age in days, weeks, and hours. Remind them to account for leap years.
- Ask students to find something mathematically special about their birthday or other dates that you select. You may want them to work in groups to find something mathematically special about every date during a particular month or during the entire year.
- Have students use the formula in problem 4 to find the day of the week of a historical day of their choosing.
- Mention that

  $$\lfloor \quad \rfloor$$

  is known as the "floor," or "greatest integer," function. In particular,

  $$\lfloor x \rfloor$$

  is the largest integer not greater than *x*.
- The formula used in problem 4 is called "Zeller's Rule."For more information, see http://mathforum.org/dr.math/faq/faq.calendar.html.
- The formula used in problem 4 applies the concept of modular, or clock, arithmetic. The October 2012 Cartoon Corner (pp. 144–45), "Cupcake Leftovers," addressed this concept.
- Ask students to describe when a leap year occurs.
- Ask students to investigate how long until a leap-year calendar can be reused.