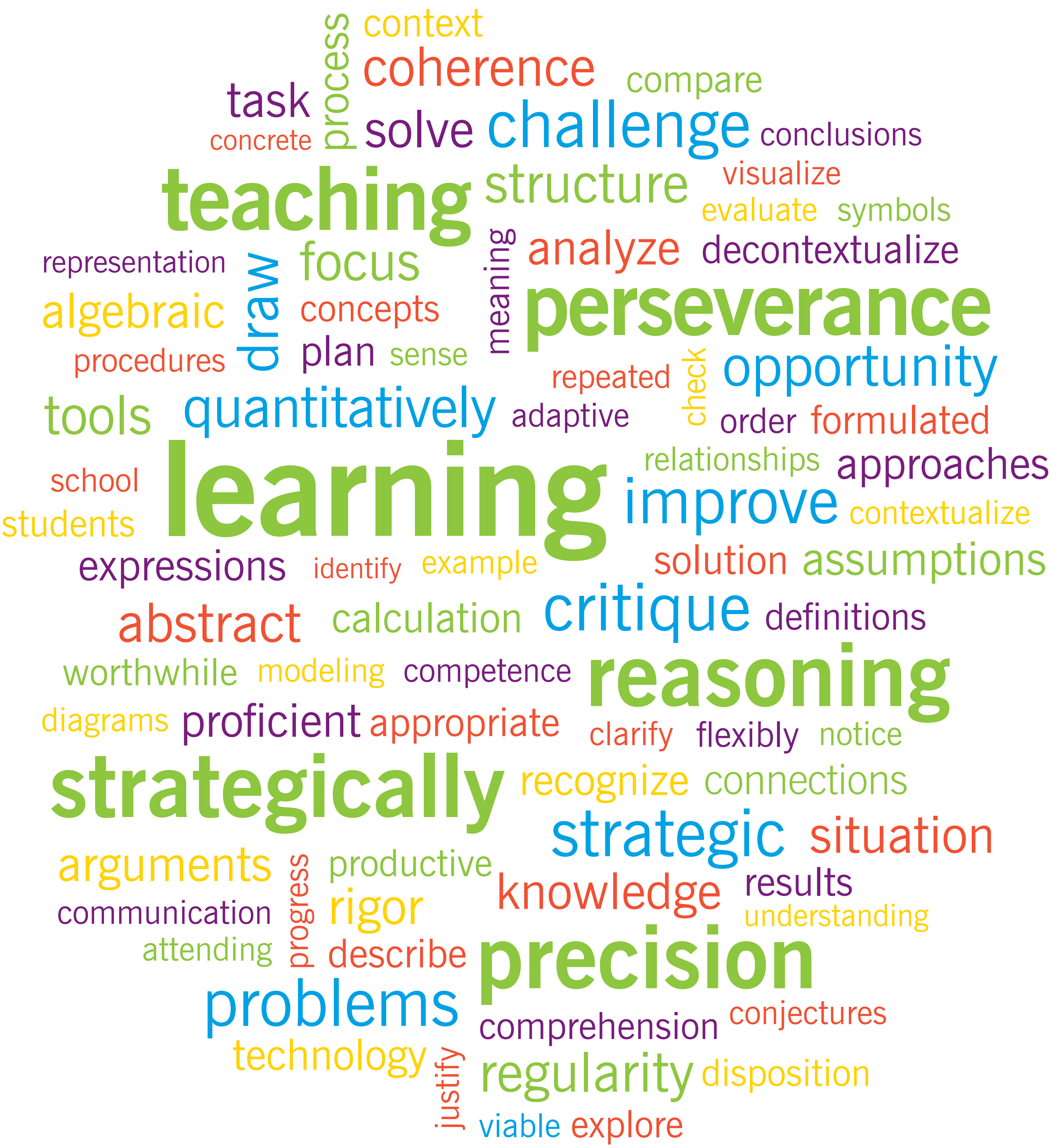
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Debunking Myths about the Standards for Mathematical Practice

Teachers can benefit from productive and manageable suggestions to align instruction to the intention of the Common Core's Standards for Mathematical Practice.

Victor Mateas

The adoption of the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010) by a majority of states has caused a shift in the expectations for student learning, with implications for teaching. CCSSM aims to bring greater focus, coherence, and rigor to mathematics content; it has also introduced a new kind of standard focused on the way that students think about that content in the form of the Standards for Mathematical Practice (SMP) (see **fig. 1**).

The SMP differ drastically from the Standards for Mathematical Content in what they describe and how they are organized (see **table 1**). The ideas behind the SMP have a history in the mathematics education community (Cuoco, Goldenberg, and Mark 1996; NCTM 2000; NRC 2005); however, requiring these ideas as standards to be taught and assessed is new. Therefore, it becomes important to consider what teaching aligned to these standards looks like. The SMP present both an opportunity and a challenge to rethink the kind of teaching practice that will develop mathematically proficient students. This article will shed light on some myths regarding the SMP and provide suggestions for teaching practice.

This article is the result of a National Science Foundation-funded research project that created professional development (PD) materials for the SMP (see EDC 2016). These materials were then tested with more than 400 middle school and high

Fig. 1 Mathematical practices described in the SMP focus on the way that students think about mathematical content.

SMP 1: Make sense of problems and persevere in solving them.
SMP 2: Reason abstractly and quantitatively.
SMP 3: Construct viable arguments and critique the reasoning of others.
SMP 4: Model with mathematics.
SMP 5: Use appropriate tools strategically.
SMP 6: Attend to precision.
SMP 7: Look for and make use of structure.
SMP 8: Look for and express regularity in repeated reasoning.

Table 1 Differences between the Standards for Mathematical Practice and the Standards for Mathematical Content present a challenge to rethink the kind of teaching practice that will develop mathematically proficient students.

	Standards for Mathematical Practice	Standards for Mathematical Content
What are they?	Descriptions of ways of thinking that mathematically proficient students use.	Descriptions of what students should understand and be able to do mathematically.
How are they organized?	The same set of eight standards is to be used at all grade levels, from kindergarten through grade 12. They develop over time, broadening in meaning as students encounter new content and becoming more sophisticated as students develop cognitively.	A different set of standards at each grade level; grouped by cluster within a grade level and by domain across grade levels. (High school standards are not organized by grade level but by conceptual category related to mathematical domains.)

school teachers and district leaders from a range of districts across seven states. Written artifacts, interviews with teachers and PD facilitators, and observations of PD sessions revealed common misconceptions about the SMP. These misconceptions can make implementing the standards difficult and overwhelming. This article aims to debunk five myths about the SMP and provide suggestions to help teachers align instruction to the SMP in ways that are productive, manageable, and true to the intentions of CCSSM.

Myth 1: Every lesson must incorporate all eight SMP.

The SMP describe mathematical ways of thinking, and not all those ways are appropriate for every task or lesson. Although some mathematical practices, such as SMP 1 and SMP 3, can pervade most mathematical work, others are more applicable to certain situations. In fact, trying to force opportunities for all eight mathematical practices to occur in a lesson inherently brings a lack of focus and "waters down" the opportunities for the mathematical practices that do make sense. Incorporating all the SMP throughout a unit, as opposed to each lesson, is more realistic. It also ensures that a lesson designed to address a particular subset of the SMP will present genuine opportunities for students to engage in those mathematical practices.

For example, in a seventh-grade unit on ratios and proportional relationships, different lessons might lend themselves to different mathematical practices. A beginning lesson on computing unit rates (content standard 7.RP.A.1) can ask students to interpret the unit rate on the basis of a problem's context and to consider the units involved (SMP 2). In lessons on identifying the constant of proportionality from various representations (content standard 7.RP.A.2.B), students can be asked to explain correspondences among tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships and how the same constant of proportionality may appear in different representations (SMP 1). In later lessons using proportional relationships to solve multistep ratio and percentage problems (content standard 7.RP.A.3), students can model real-world scenarios involving interest, taxes, sales, percentage error, and so on (SMP 4).

Suggestion 1: When planning, consider which lessons have genuine opportunities for students to use particular mathematical practices, addressing all eight within the span of a unit, or across several units, rather than in each lesson.

Myth 2: Students can engage in only one mathematical practice as they work on a task.

Although Myth 1's concern is that all the mathematical practices must be involved at all times, this second myth, which suggests that students are capable of focusing on only one mathematical practice at a time, is untrue, as can be seen by looking at the work of student A in **figure 2,** who was engaged in several mathematical practices. The student developed an equation to model the postage amounts that can be made (SMP 4). Student A also thought about the structure of the numbers involved in the problem (SMP 7), realizing that once a particular postage amount is possible, all multiples of that amount are possible as well.

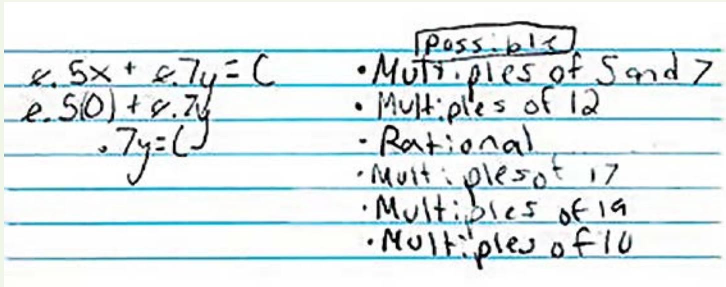
Part of the reason that students use multiple mathematical practices in a single task is because the eight SMP are standards "of mathematical practice": They help codify some (not all) of the major ways that proficient practitioners of mathematics work. By their very nature, these aspects of practice blend and support each other; creating strict boundaries around these aspects is not sensible and rarely feasible. For example, it is no coincidence that *units* appear in more than one of the SMP.

Considering the units of a quantity is key both to quantitative reasoning (SMP 2) and to communicating precisely (SMP 6). When students work on a task, they participate in the practice of mathematics and naturally use whatever thinking seems productive to them, regardless of what the SMP may call that thinking. This is not to say that the mathematical practices described in the SMP should be ignored or that all eight will occur automatically. The SMP provide a blueprint for teachers, drawing attention to different ways that mathematical thinking occurs, which is useful when planning how to build these capacities in students.

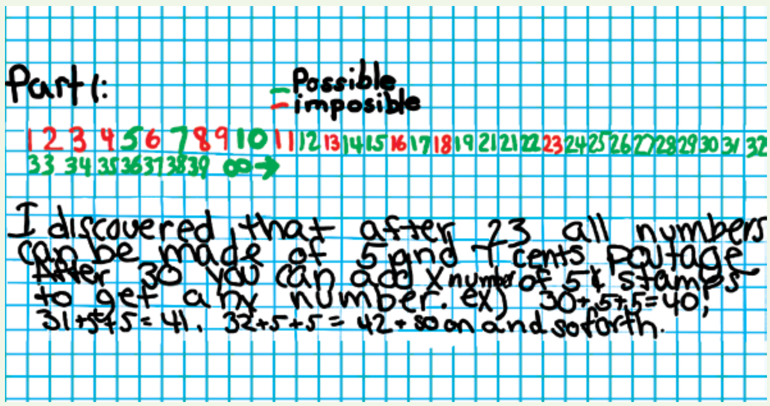
Suggestion 2: When planning, foster a few relevant mathematical practices for each task.

Fig. 2 Samples of student work show how students can engage in several (or no) mathematical practices, given the same task.

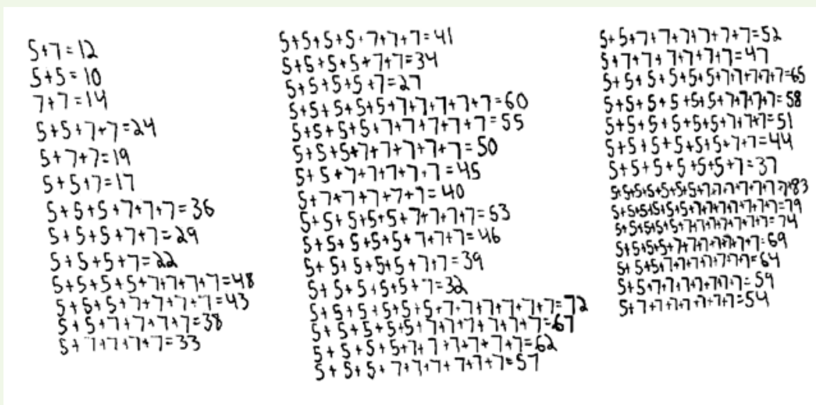
Mathematics Task: Suppose the post office only sold 5 cent stamps and 7 cent stamps. Some amounts of postage can be made with just those two kinds of stamps. For example, one 5 cent and two 7 cent stamps make 19 cents in postage, and two 5 cent stamps make 10 cents in postage. Which amounts of postage is it impossible to make using only 5 cent and 7 cent stamps?



Student A



Student B



Student C

Myth 3: The mathematics task alone determines which mathematical practices students will use.

Although it is common to tag tasks to particular content standards or topics, an essential part in developing standards-based lessons, doing so for the SMP is a little more challenging. The SMP and the Standards for Mathematical Content both set expectations for students; however, the two sets of standards are different in what they ask of students. For the most part, the Standards for Mathematical Content determine the topic and the SMP describe the mathematical thinking. Therefore, evidence of the mathematical practices lies in a student's thinking and approach, not in the text of the task a student is given. Tasks can, however, set up opportunities for students to engage in certain mathematical practices, depending on what the problem is asking students to do. Linking possible mathematical practices to a task is helpful for planning, while acknowledging that the roles of the student and teacher in the enactment of the task can alter which, if any, of the mathematical practices will be ultimately used.

For example, **figure 2** shows three samples of student work for the same task. This task offers opportunities for students to engage in several of the mathematical practices. Student A modeled the postage that can be made using an equation (SMP 4) and thought about the structure of numbers to decide that multiples of possible postage will also be possible (SMP 7). Student B stated a conjecture, based on the data, and then constructed an argument for why all postage greater than 23 will be possible (SMP 3). However, the work of student C does not display any of the mathematical practices. Although the student shows some perseverance in doing so many calculations, his or her work entails little sense making or strategy. However, with some questioning on the part of a teacher, this student could in time evaluate progress and reorganize his or her possible postage values (SMP 1), ultimately developing a conjecture and argument (SMP 3) similar to that of student B.

Fig. 3 Students have a conversation about fractions as quantities (SMP 2).

(1) Sam: How do you do 2/5 + 1/2?

(2) Dana: It's just 3/7, isn't it?

(3) Anita: But 3/7 is less than 1/2 , so it can't be that!

(4) Sam: So . . . how do you do it?

(5) Dana: But we're just adding: 2 + 1 is 3, and 5 + 2 is 7, so it *should* be 3/7.

(6) Anita: We already know that 2 fifths plus 1 fifth is 3 fifths [writes 2/5 + 1/5 = 3/5]. It's not 3 tenths. You can't just add everything you see.

(7) Sam: So…how *do* you do it?

(8) Dana: [To Anita] Oh, right, I get it. It's like when we were saying, "2 cats plus 1 cat, 2 grapes plus 1 grape, 2 fifths plus 1 fifth."

(9) Sam: Yeah, I get it, *too*, but how do we do 2 *fifths* plus 1 *half*?! It's not just 3 of something, but what *is* it? We're adding two different things. Like 2 cats and 1 grape; 2 feet and 1 inch. Or, maybe like 2 thousand and 1 hundred. We *can* add them, but they're not *3* of something.

Note: The conversation is not meant to be an accurate illustration of how typical middle school students talk but does accurately illustrate the type of thinking that typical middle school students can use. This conversation comes from PD materials (see http://www.mathpractices.edc.org) that have been reviewed by teachers, teacher educators, and mathematicians.

Suggestion 3: When planning a task, anticipate how students might think about a problem. Also try to anticipate which mathematical practices they might use, recognizing that these practices are opportunities to encourage growth in student thinking, not certainties determined by the task.

Myth 4: Only specialized tasks can be used to develop mathematical practice.

It is easy to see how a nonroutine problem, like the one in **figure 2**, may have students use mathematical practices while solving it. However, almost any problem, even a procedural problem, can also develop mathematical practice. The key is in the approach, largely a result of the nature of the teaching, not of the task. For example, a task like 2/5 + 1/2 (content standard 5.NF.A.1) could be solved using a simple, efficient algorithm, which is one goal of teaching fraction addition. However, this problem could also be presented at the beginning of students' study of fraction addition, before students know the algorithm. At this stage, the problem becomes a question that can allow students to discuss the meaning of each fraction as a quantity (SMP 2). **Figure 3** shows an example of this kind of conversation. Alternatively, students can use a number line or bar diagram as tools to visualize and add the fractions (SMP 5). Or the (untrue) equation $2/5 + 1/2 = 3/7$ could be presented to students who are then asked to explain why the equation is true or untrue (SMP 3). The conversation in **figure 3** shows examples of two explanations that students might use; in line 3, Anita argues that 3/7 cannot be the solution because it is less than 1/2; in line 6 the counterexample, $2/5 + 1/5 = 3/5$, is given.

Realizing that almost any problem can be used in a way that fosters mathematical practice is important because it means that one is not limited by his or her curriculum (although some may help more than others). Furthermore, using typical tasks in ways that encourage mathematical practices can ensure that the thinking described in the SMP is not only for days when "special problems" are used but is part of the everyday culture of the classroom.

Suggestion 4: When planning, think about which teaching practices can leverage existing tasks to promote mathematical practices.

Myth 5: Mathematical practice can be taught separately from mathematical content.

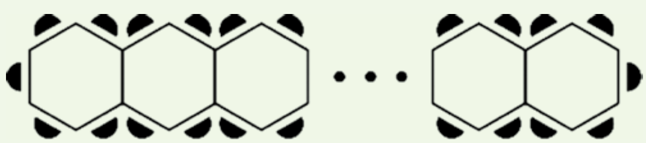
When teaching mathematical content, a topic is sometimes isolated and taught alone so students might attain a particular understanding or develop a particular skill. However, the same logic does not apply to mathematical practice: It is impossible to think mathematically without thinking mathematically about something. CCSSM states,

> The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. (CCSSI 2010, p. 8)

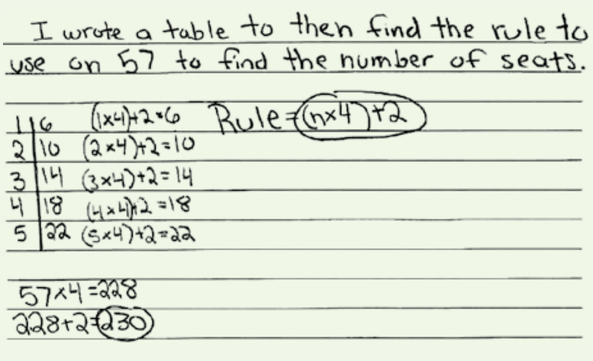
Engaging with the subject matter means that one cannot teach or use

Fig. 4 Below are examples of SMP 8 in numerical and geometric contexts.

A big party is being planned and everyone will sit at hexagon-shaped tables. The tables will be put together in one long line as shown below.

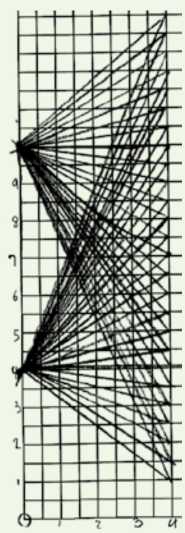


If there are 57 tables and each side of the table fits only one person, how many guests can be seated? Write an expression to represent the number of guests that can be seated at 57 tables.



(a)

Two vertices of a triangle are located at (0, 4) and (0, 10). The area of the triangle is 12 square units. Where is the third vertex located?



(b)

It is impossible to think mathematically without thinking mathematically about something.

the mathematical practices independent of some mathematical content.

Even if mathematical practice and content are considered together, spending a few lessons focused on this combination may be sufficient to master the content, but it is not enough to master the thinking called for in the SMP. There are at least two reasons. The first is that each mathematical practice is a way of thinking and can manifest itself differently in different content areas; therefore, many and varied opportunities are needed for students to learn to think mathematically across contexts. For example, looking at **figure 4**, SMP 8 takes very different forms in a numerical context than in a geometric context. In the numerical example, the student used repeated reasoning to calculate the number of seats for several smaller chains of tables before building a general expression and using that to find the number of seats at 57 tables. In the geometric example, the student uses repeated reasoning to see that there are multiple triangles that fit the constraints of the problem; however, the student does not go as far as expressing the regularity. With some intervention on the part of a teacher, the student could exhibit all aspects of SMP 8 by stating that the vertex can lie anywhere on the line $x = 4$ (and with further questioning, could realize that it could lie anywhere on the line $x = -4$, too).

Another reason a mathematical practice cannot be learned in just a few lessons is that the way students think changes "as they grow in mathematical maturity with expertise throughout the elementary, middle and high school years" (CCSSI 2010, p. 8). The types of arguments, structures, tools, and so on that a first grader uses will look very different from that of a twelfth grader. This should not be confused with thinking that younger students cannot engage in mathematical practice—but rather that the ways in which they will do so is different and will change as they mature and as they learn more content.

Furthermore, mathematical practices must be taught because they are not always natural or obvious. For example, using multiple examples until some regularity is found and finally expressed (SMP 8) is not obvious and needs to be developed in student thinking until it becomes a habit that students can use in new scenarios across content domains or grade levels. For this reason, the SMP span K–grade 12 and should be viewed as a spectrum of increasingly sophisticated ways of doing mathematics.

Suggestion 5: When planning, provide students with multiple opportunities to engage in the mathematical practices across content domains and time spans, both within a school year and across grade levels.

PLANNING AND INSTRUCTION THAT SUPPORT THE SMP

The SMP are a different kind of standard and require a shift in thinking about instructional methods. For the mathematical practices to truly become a habit or practice that students will use on their own, students need multiple opportunities, across several years, to engage in the thinking called for in the SMP. It should not be expected that students will master the mathematical practices in a short period of time.

Planning related to the SMP should be done at two levels. Planning at larger scales (e.g., within units, across the school year, across grade levels with the content team) will help identify when particular SMP may best fit in an instructional sequence and ensure that students are getting the multiple and varied opportunities they need to engage in the mathematical practices. Planning at the task (not lesson) level will help focus on how to support students' mathematical thinking. The evidence of the SMP is in student thinking, and the task is the site of interaction between students and mathematics; planning particular supports to help students use the mathematical practices must be done at this level. Planning at the task level should also move beyond identifying prior understandings that students need or possible misconceptions that they might have to anticipating how students might go about working on the particular problem. Although planning on the big scale helps one think about the *when*, planning on the small scale helps one think about the *how* and recognize that a task may lead to several mathematical practices and that all tasks have potential for engaging students in mathematical practice, depending on the teaching methods used.

Finally, instruction should be deliberate and should focus on both mathematical content and mathematical practice. Because some features of mathematical practice are not immediately obvious to students and because they might use different mathematical practices for any given task, being intentional is important when choosing which practices to support students in using. This could

Are There Other Myths to Debunk?

We invite *MTMS* readers to submit additional instances of myths about the Standards for Mathematical Practice and corresponding suggestions that this article does not address.

Join us as we continue this conversation. Post on *MTMS*'s blog at **http://www.nctm.org/SMPmyths**.

Let's Chat about Myths

On Wednesday, September 21, at 9:00 p.m. EDT,

we will expand on the article "Debunking Myths about the Standards for Mathematical Practice" (pp. 92–99), by Victor Mateas.
Join us at **#MTMSchat**.

We will also Storify the conversation for those who cannot join us live. Our monthly chats will always fall on the third Wednesday of the month.

mean offering particular supports as students work on a task or calling attention to particular approaches that students used during a whole-class debriefing of the task.

The SMP offer the chance to reflect on the way mathematics is taught and provide a focus on developing students who are "practitioners of the discipline of mathematics" (CCSSI 2010, p. 8) and not just consumers of mathematics. It requires explicit planning—not just of what content is to be taught but how to teach the content in ways that support the development of mathematical practice. Key to this type of planning is attention to students' mathematical thinking and the incorporation of instructional practices that value, encourage, share, and discuss student reasoning in the classroom. Incorporating the mathematical practices into instruction will not happen overnight; however, beginning to experiment with small changes to one's teaching practice and collaborating with colleagues can help move students toward the vision of mathematical proficiency described in the SMP.

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REFERENCES

Common Core State Standards Initiative (CCSSI). 2010. Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf

Cuoco, Al, E. Paul Goldenberg, and June Mark. 1996. "Habits of Mind: An Organizing Principle for Mathematics Curricula." *The Journal of Mathematical Behavior* 15 (4): 375–402.

Education Development Center (EDC). 2016. Implementing the Mathematical Practice Standards. www.mathpractices.edc.org

National Council of Teachers of Mathematics (NCTM). 2000. *Principles and Standards for School Mathematics.* Reston, VA: NCTM.

National Research Council (NRC). 2005. *How Students Learn Mathematics in the Classroom,* edited by Suzanne Donovan and John D. Bransford. Washington, DC: National Academies Press.

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