activity sheet 1

Name ______________________________

DISCOVERING NIFTY NINES

1. *Use your calculator to convert the following fractions to decimals. Be sure to use correct notation in expressing each decimal numeral.*

Examples: $\frac{7}{9} = 0.\overline{7}$ $\frac{4}{99} = 0.\overline{04}$

a. $\frac{8}{99} =$

b. $\frac{52}{99} =$

c. $\frac{23}{99} =$

d. $\frac{6}{999} =$

e. $\frac{41}{999} =$

f. $\frac{347}{999} =$

Stop to compare and discuss your findings with your group members. To verify your findings, you may wish to try (on scratch paper) a few more of your own fractions with denominators of 99, 999, 9999, and so on.

YOUR FINDINGS

Given a fraction with any numerator over one of these denominators of 9s, you can predict the decimal representation without actually using division. How can this be done? Demonstrate your understanding by responding to the following questions.

2. Explain how you can determine the exact length of the repetend, that is, the number of digits written under the “bar” in the decimal numeral. Stop to compare and discuss your findings with your group members.

3. Explain how to predict the exact digits of the repetend in the decimal numeral. Stop to compare and discuss your findings with your group members.

from the October 2016 issue of mathematics teaching in the MIDDLE SCHOOL

activity sheet 2

Name ______________________________

USING NIFTY NINES

Let's turn things around. Suppose you are given the decimal numeral instead of the fraction. You should now be able to predict the fraction using the "nifty nines" strategy, and without even using your calculator. Give it a try.

1. Convert each of these repeating decimals to a lowest terms fraction (without using a calculator).

a. $0.\overline{54} =$

c. $0.\overline{013} =$

b. $0.\overline{0002} =$

d. $0.\overline{9} =$
(*Ah…an interesting one!*)

Stop to compare and discuss your findings with your group members.

2. Write a simple rule for converting a repeating decimal numeral to a fraction when the decimal's "repetend" begins in the tenths place (immediately to the right of the decimal point, as in the examples above). Stop to compare and discuss your findings with your group members.

activity sheet 3

Name ______________________________

RAISING, OR "SHIFTING" THE BAR

Suppose the repetend does not begin immediately after the decimal point, such as in the rational number $0.1\overline{5}$. Let's use a jazzy little manipulation to convert this decimal to a fraction. Remember, stop to compare and discuss your findings with your group members.

Let's begin by looking at an example.

Convert $0.1\overline{5}$ to a simple fraction:

$$\begin{aligned}
0.1\overline{5} &= 0.1\overline{5}\cdot 1 && \text{(multiplicative identity property)}\\
&= 0.1\overline{5}\cdot\left(10\cdot\frac{1}{10}\right) && \text{(property of multiplicative inverses)}\\
&= (0.1\overline{5}\cdot 10)\cdot\frac{1}{10} && \text{(associative property for multiplication)}\\
&= 1.\overline{5}\cdot\frac{1}{10} && \text{(simplifying)}\\
&= 1\frac{5}{9}\cdot\frac{1}{10} && \text{(pattern from nifty nines!)}\\
&= \frac{14}{9}\cdot\frac{1}{10} && \text{(mixed number to improper fraction)}\\
&= \frac{14}{90} && \text{(multiplying)}\\
&= \frac{7}{45} && \text{(and simplifying)}
\end{aligned}$$

Using the same process, without documenting quite so many steps.

Convert $0.3\overline{5}$ to a simple fraction:

$$\begin{aligned}
(0.3\overline{5}\cdot 10)\cdot\frac{1}{10} &= 3.\overline{5}\cdot\frac{1}{10}\\
&= 3\frac{5}{9}\cdot\frac{1}{10}\\
&= \frac{32}{9}\cdot\frac{1}{10}\\
&= \frac{32}{90}\\
&= \frac{16}{45}
\end{aligned}$$

Convert $0.41\overline{6}$ to a simple fraction:

$$\begin{aligned}
(0.41\overline{6}\cdot 100)\cdot\frac{1}{100} &= 41.\overline{6}\cdot\frac{1}{100}\\
&= 41\frac{2}{3}\cdot\frac{1}{100}\\
&= \frac{125}{300}\\
&= \frac{5}{12}
\end{aligned}$$

Do you understand these examples? Stop to compare and discuss your findings with your group members.

1. Now it's time to try some on your own. Using the models in the right-hand column above, convert the following to fractions in lowest terms. Show your steps clearly and accurately on a piece of notebook paper.

a. $0.0\overline{3}$

b. $0.0\overline{12}$

c. $0.6\overline{5}$

d. $0.11\overline{8}$

e. $0.2\overline{9}$

(Ah…an interesting one!)

Stop to compare and discuss your findings with your group members.

Name ______________________________

NIFTY NINES REVISITED: WHY ALL THOSE 9S?

Let's try to figure out why this "Nifty Nines" technique works. That is, let's show that a repeating decimal number between 0 and 1, of the form $x = 0.\overline{d_1 d_2 \ldots d_n}$ (each d_n representing a digit in the nth decimal position), can be represented as a common fraction with a denominator comprised only of 9s. As you work through the arithmetic example in the left-hand column, you should also (before moving down to the next step) algebraically "generalize" the process in the right-hand column. Remember, stop to compare and discuss your findings with your group members.

Arithmetic Approach

Let's begin with the following equation:

$x = 0.\overline{13}$

Fill in the blanks to complete each statement below.

1. Multiply both sides of this equation by 10^2, and simplify the right-hand member.

$$\_\_\_\_\_ \bullet x = \_\_\_\_\_ \bullet 0.\overline{13}$$
$$= \_\_\_\_\_\_\_\_\_\_$$

(Hint: Multiplying by 10^2 merely shifts the digits two places to the left.)

2. Now, we will subtract x from both sides of the equation.

$$10^2 \bullet x - x = \_\_\_\_\_\_\_\_\_\_ - x$$

3. Factor x out of the left-hand member.

$$(\_\_\_\_\_)x = \_\_\_\_\_\_\_\_\_\_ - x$$

4. Simplify. (Hint: Remember that $x = 0.\overline{13}$). Also, let's just express $(10^2 - 1)$ as 99 from now on.

$$(\_\_\_\_\_)x = \_\_\_\_\_\_\_\_\_\_$$

(Ah . . . it *is* a whole number now.)

Algebraic Approach

Let's begin with the following equation:

$x = 0.\overline{d_1 d_2 \ldots d_n}$

Fill in the blanks to complete each statement below.

1. Multiply both sides of this equation by 10^n, then notice the simplified representation of the right-hand member.

$$\_\_\_\_\_ \bullet x = \_\_\_\_\_ \bullet 0.\overline{d_1 d_2 \ldots d_n}$$
$$= d_1 d_2 \ldots d_n.\overline{d_1 d_2 \ldots d_n}$$

(Hint: Multiplying by a positive nth power of ten merely shifts the digits n places to the left. You should see why we have an expression that includes only subscripted ds.) Remember, stop to compare and discuss your findings with your group members.

2. Now, we will subtract x from both sides of the equation.

$$10^n \bullet x - x = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ - x$$

3. Factor x out of the left-hand number.

$$(\_\_\_\_\_\_\_)x = \_\_\_\_\_\_\_\_\_\_ - x$$

4. Simplify the right-hand number. (Hint: Remember that $x = 0.\overline{d_1 d_2 \ldots d_n}$.)

$$(\_\_\_\_\_\_\_\_\_)x = \_\_\_\_\_\_\_\_\_\_$$

(Does this expression appear to be a whole number?)

Name ______________________________

Arithmetic Approach

5. Solve for x by dividing both sides by $(10^2 - 1)$.

$$\frac{\quad}{99} = \frac{\quad}{99}$$

Do you see that the left-hand member of this equation is now just *x*? We now have the following:

$$x = \frac{\quad}{99}$$

6. If $x = 0.\overline{13}$ (the decimal that we started with), then by substitution we now have the following:

$$0.\overline{13} = \frac{\quad}{99}$$

Algebraic Approach

5. Solve for x by dividing both sides by $(10^n - 1)$.

$$\frac{\quad}{(10^n - 1)} = \frac{\quad}{(10^n - 1)}$$

Do you see that the left-hand member of this equation is now just x? We now have the following:

$$x = \frac{\quad}{(10^n - 1)}$$

Remember, stop to compare and discuss your findings with your group members.

6. If $x = 0.\overline{d_1 d_2 \ldots d_n}$ (the decimal number that we started with), then by substituting, we now have this:

$$x = 0.\overline{d_1 d_2 \ldots d_n} = \frac{\quad}{(10^n - 1)}$$

7. Finally, let's consider the right-hand member of the equation. Numbers of the form $(10^n - 1)$ are actually whole numbers comprised only of the digit 9 (e.g., 9, 99, 999, 9999, and so on). Right?

Since $(10^n - 1)$ is a whole number comprised of only 9s (9, 99, 999, 9999, and so on), explain what can be concluded about the fraction (in step 6) that represents our original repeating decimal number $x = 0.\overline{d_1 d_2 \ldots d_n}$.

Remember, stop to compare and discuss your findings with your group members.