palette of problems solutions

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(*continued from pp. 142–143*)

ANSWERS

1. 12.5%
2. 3000
3. 9/45 = .2
4. Fewest: 0; greatest: 1
5. 16 bouquets; 2 roses, 4 carnations, and 3 peonies
6. 40 points
7. $a + b + c = 30$
8. 4/5 for both the 10th and 100th term
9. 2/5
10. 17 lb., 17 lb., 21 lb., 22 lb., 23 lb.
11. 2, 6, 12, 20, 30, 42
12. $a = 62°$; $b = 10°$; $c = 28°$; $d = 34°$; $e = 46°$
13. 12 square units
14. 41 pairs
15. 32 in.^{2}
16. $a = 1$ and $b = 1/4$

SOLUTIONS

1. The jack-o'-lantern's face comprises 40 squares on the grid. Each square represents 2.5 percent of the whole face ($40 \times 2.5\% = 100\%$). Each eye is 1/2 of a square, making 1 square total. The nose is 1/2 of 2 different squares, also making 1 square total. The mouth is 3 squares total. In sum, the eyes, nose, and mouth comprise 5 squares, or 12.5 percent, of the jack-o'-lantern.

2. $x = 750$ and $y = 2250$, so $x + y = 3000$

3. There are ${}_{10}C_2 = 45$ pairs you might pick: 9 ways to pick the smaller of two consecutive integers and only one way to pick the second integer of the pair. The probability is 9/45 or .2.

4. The digit 0 is written 11 times, and the digit 1 is written 21 times. All other digits are written 20 times.

5. For all the bouquets to be the same, Mr. Freesia will need to distribute each type of flower evenly among the bouquets.

Flower	Possible Number of Bouquets
Roses	1, 2, 4, 8, 16, 32
Carnations	1, 2, 4, 8, 16, 32, 64
Peonies	1, 2, 3, 4, 6, 8, 12, 16, 24, 48

Since the greatest common factor of 32 (the number of roses), 64 (the number of carnations) and 48 (the number of peonies) is 16, the largest number of bouquets that Mr. Freesia can make is 16. Because $32 \div 16 = 2$, each bouquet will have 2 roses; $64 \div 16 = 4$, so each bouquet will have 4 carnations; and $48 \div 16 = 3$, so each bouquet will have 3 peonies.

6. Rashida has scored 16 points after 4 games, which means that she has scored 4 points per game on average. After 10 games, Rashida should score $10 \times 4 = 40$ points. Alternatively, we could double the given number of games and number of points to note that, after 8 games, Rashida would score 32 points. Then, because we need to add one-half of 4 to 8 to get 10, we can add one-half of 16 to 32 to find the number of points after 10 games.

7. There are multiple ways to solve this problem; consider the total sum of the three expressions for which we know the value:

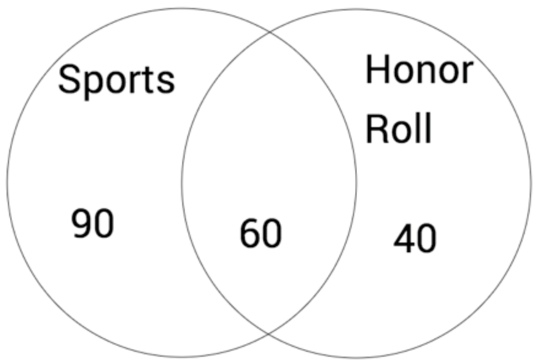
$$(a + b) + (a + c) + (b + c) = 21 + 16 + 23$$
$$2a + 2b + 2c = 60$$
$$a + b + c = 30$$

8. The sequence is

$$3, 5, (5 + 1)/3 = 2, (2 + 1)/5 = 3/5, (3/5 + 1)/2 = 4/5,$$
$$(4/5 + 1)/(3/5) = 3, (3 + 1)/(4/5) = 5, \ldots$$

The five terms of the sequence, 3, 5, 2, 3/5, 4/5, repeat. Both the 10th term and the 100th term are 4/5.

9. This Venn diagram summarizes the information that was given.



We see that 60 of the 150 students playing sports are on the honor roll and 60/150 = 2/5

10. Because there are 5 dogs total, and the mean weight is 20 pounds, the sum of the weights of all 5 dogs must be 100 pounds. The median weight is 21 pounds, which means that if we were to put the dogs in order from lightest to heaviest, the one in the middle would weigh 21 pounds. Also, the mode is 17 pounds, and we know that only one pair of dogs share the same weight, meaning that there must be 2 dogs that each weigh 17 pounds. So far, we know the weights of 3 dogs: 17, 17, and 21 pounds. The total weight of these 3 dogs is 55 pounds. Thus, the fourth and fifth dog (the two biggest dogs) must have a combined weight of 45 pounds, and each of them must weigh more than 21 pounds. The only way this is possible, if each weight is an integer, is if 1 dog weighs 22 pounds and the other dog weighs 23 pounds. Therefore, the dogs weigh 17, 17, 21, 22, and 23 pounds, respectively.

11. The first time through the loop, number = 1. The code will print 2 (that is, it will calculate 1 * 1 + 1 and print the result. The second time through the loop, number = 2. The code will print 6 (it will calculate 2 * 2 + 2). Continuing, the entire printout will be 2, 6, 12, 20, 30, 42.

12. To find the missing angles, remember that supplementary angles sum to 180 degrees and that the angles in a triangle sum to 180 degrees. Using supplementary angles, each of the four angles around point *X* is a right angle. Look first at triangle *AEC*:

$$a + (c + d) + 56 = 180; \text{so } a + (c + d) = 124$$

because

$$a = c + d, \text{a} = 62 \text{ and } c + d = 62.$$

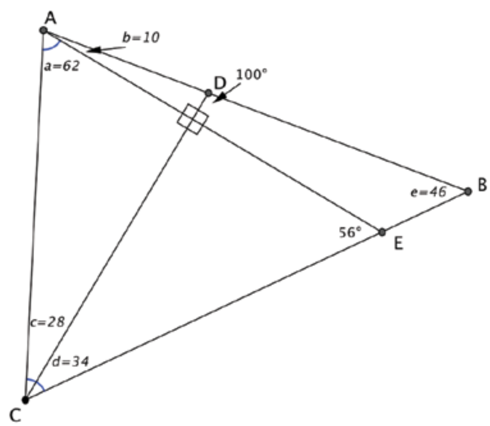
Now look at triangle *AXC*:

$$62 + 90 + c = 180, \text{so } c = 28.$$

Because we already found that $c + d = 62$, that means that $d = 34$. Now look at triangle *AXD:* Notice that angle *ADX* is supplementary to angle *BDX*; so angle *ADX* measures 80 degrees. Applying the triangle sum property, $b + 80 + 90 = 180$, so $b = 10$. Last, look at the large triangle *ACB*: Using what we have learned about *a, b, c,* and *d,* we know that

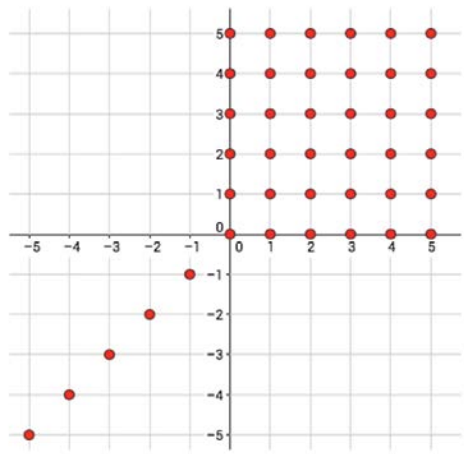
$$62 + 10 + 28 + 34 + e = 180.$$

Solving the equation for e, we find that $e = 46$.

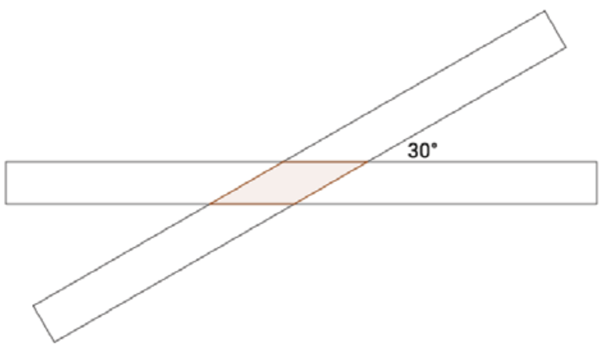


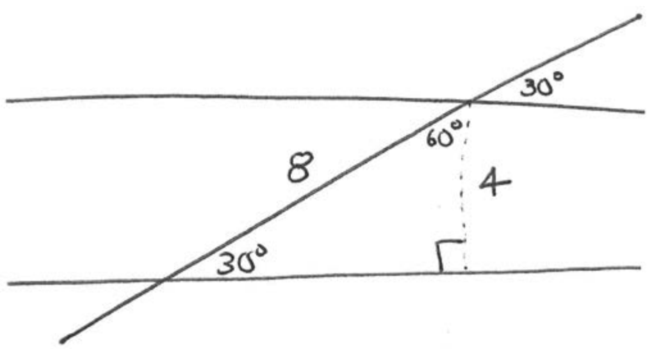
13. *ABC* is isosceles and comprises two "back-to-back" 3-4-5 triangles. The height of *ABC* is 4 above a base of length 6.

14. This is a graph of all pairs of integers (x, y) that satisfy $x + |y| = y + |x|$. There are 41 such pairs.



15. The overlap is a parallelogram. Each side of the parallelogram (rhombus) is 8 inches long since it is the hypotenuse of a 30-60-90 triangle with the shortest side length 4. The area is 4 in. × 8 in. = 32 in.^{2}.





16. We know that $a/4 = b/a$ and that $b - a = -1/2 - b$. Using the second equation to substitute for b in terms of a in the first equation gives $a^2 - 2a + 1 = 0$, showing that the only possibility is that $a = 1$. Then either equation tells us $b = 1/4$.