

RETURN OF THE Tug-of-War





In this updated version of a Marilyn Burns task, students use algebraic reasoning to determine the outcome of a contest involving rebel fighters, storm troopers, and (of course) Chewbacca.

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One of my favorite problems to share with students and teachers is Marilyn Burns's "Mathematical Tug-of-War" (Burns 2015) (see **fig. 1**). Long before the release of the Common Core State Standards (CCSSI 2010), the Mathematical Tug-of-War was engaging students in the type of reasoning and problem solving described by the Standards for Mathematical Practice (SMP).

I cannot remember when I was first introduced to the Mathematical Tug-of-War, but over the years I have shared it with many students and countless teachers. I have often used the problem in its original form, but I also like to rewrite it to reflect popular culture or current events. For example, some versions from the past included characters from *The Simpsons* or from the Harry Potter books.

For my most recent version of tug-of-war, I used characters from the newest Star Wars movie, *The Force Awakens*. I knew that many of the sixth graders in the math class I was co-teaching once a week were fans of the franchise. (See the **activity sheet** on page 48.)

To prepare for the lesson, I looked for images to represent the characters in the tug-of-war. Instead of teams of acrobats and grannies (as in the original version of the problem), Round 1 involved five rebel fighters (Rey, Finn, Princess Leia, BB-8, and R2-D2) and four storm troopers, with each team being

of equal strength. For Round 2, one team consisted of Rey, Finn, and one storm trooper. On the other side was Chewbacca, playing the role of Ivan in the original version of the task. Again, both sides were equal in strength. The final round, and the pull for which students needed to figure out the outcome, involved Chewbacca, Rey, Finn, and Princess Leia on one side and the four storm troopers on the other.

SETTING UP THE PROBLEM

To provide students with opportunities to make sense of the problem and motivate them to persevere in solving it (SMP 1), I first showed

them a picture of a tug-of-war to make sure the context was familiar to them. I then asked for volunteers to demonstrate a “mathematical tug-of-war.” Several hands shot up, and I had no problem getting enough volunteers to represent Round 1. As the volunteers came to the front of the room, I hung a picture of either a rebel fighter or a storm trooper around their necks and directed them to make two teams. I also gave them a short length of rope to further represent the tug-of-war.

At this point, we paused, and I asked all students, those involved in the tug-of-war and those still seated, to describe what they saw. Students agreed that Round 1 showed that the team of the five rebel fighters and the team of the four storm troopers were equal in strength (see **fig. 2**).

For Round 2, I called one last volunteer to the front of the room to represent Chewbacca. I then directed the students representing Rey, Finn, and one storm trooper to make one team and the student representing Chewbacca to make the other team (see **fig. 3**).

We again paused and described what we learned from Round 2. Students said things like “Chewy is really strong—as strong as three people!” From this comment, I knew that some students were equating the strength of all of the other characters, both rebel fighters and storm troopers, so I knew that this was something I would likely have to address as students worked on their solutions.

For the final round, I rearranged the students so that we had Chewbacca and three rebel fighters (Rey, Finn, and Princess Leia) on one side and the four storm troopers on the other (see **fig. 4**).

We once again took a good look at the two teams to make sure that all students understood what happened in Round 3.

Fig. 1 The original Mathematical Tug-of-War (Burns 2015) involved acrobats, grandmas, and a dog named Ivan.

A Mathematical Tug-of-War

Use the information given to figure out who will win the third round in a tug-of-war.

Round 1: On one side are four acrobats, each of equal strength. On the other side are five neighborhood grandmas, each of equal strength. The result is dead even.

Round 2: On one side is Ivan, a dog. Ivan is pitted against two of the grandmas and one acrobat. Again, it's a draw.

Round 3: Ivan and three of the grandmas are on one side and the four acrobats are on the other.

Who will win the third round?

Write an explanation of your reasoning.



Fig. 2 In Round 1, five rebel fighters were equal to four storm troopers.

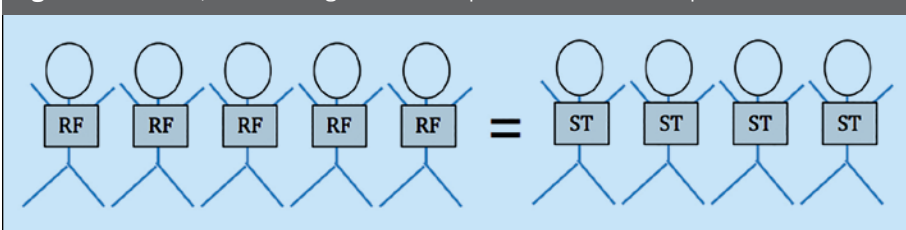


Fig. 3 In Round 2, two rebel fighters and one storm trooper were equal in strength to Chewbacca.

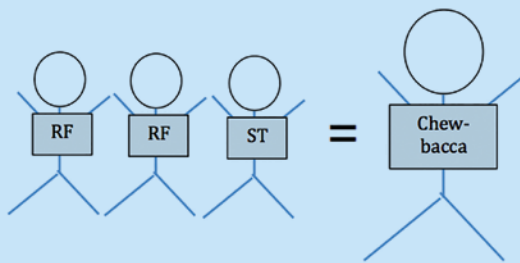
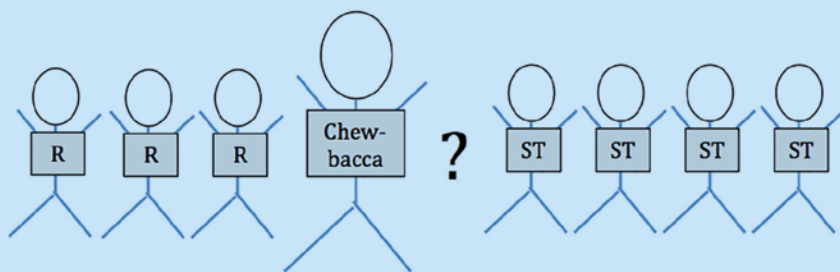


Fig. 4 In Round 3, three rebel fighters and Chewbacca took on four storm troopers.



QUIET THINK/WORK TIME

Once I was certain that everyone understood the context and knew that their task was to determine the outcome of Round 3, I sent volunteers back to their seats and gave students the **activity sheet**. I instructed them to first work quietly on their own to consider the problem and start to work on a plan for determining which team would win Round 3. I suggested that even if they were not sure how to begin, they could start by deciding which team (the rebel fighters or the storm troopers) was stronger. That way, when it was time to begin working with their group, all students would have something to contribute to the conversation.

PUTTING STUDENTS IN GROUPS

After about five minutes of quiet work time, I passed out playing cards to make working groups. I put students in groups of two or three, thinking that groups of four might

not allow for all students to have input into the group's solution strategy and to engage in SMP 3. Before they began to work together toward a final solution, I told each student to share his or her current thinking. I suggested that they start by saying, "I think that . . . because . . .," even if they still weren't 100 percent sure who would win. This allowed everyone to contribute to the discussion because the expectation was to share thinking, not present a solution.

As soon as the students got into their groups, I was pleased to hear mathematical conversations begin. Most groups quickly decided that the team with Chewbacca and the three rebel fighters was stronger than the team with the four storm troopers, although many did not yet have a clear rationale other than the fact that Chewbacca was "really strong."

A handful of students initially ignored the information provided by Round 1, that the five rebel fighters

were equal in strength to the four storm troopers. Instead, they used the information from Round 2 to erroneously decide that since Chewbacca was equal to two rebel fighters and one storm trooper, the storm troopers and rebel fighters were all equal in strength. Although this would allow them to arrive at the correct solution to the question of which team would win Round 3, it did not allow for a precise answer that indicated how much stronger the team of rebel fighters and Chewbacca was compared with the team of storm troopers.

STUDENTS' STRATEGIES: ASSIGNING VALUES

Several groups decided to give the characters a value, or "strength points," whereas others determined the outcome by using what I call "intuitive substitution." Among the students who awarded a point value, most of them started by giving each rebel fighter 1 point and were eventually able to use that value to decide that each storm trooper would be worth $1\frac{1}{4}$ points because five rebel fighters were equal in strength to four storm troopers. (One group needed a bit of help with this idea, so I pressed them to consider what the values of the two teams would be if each character were worth only 1 point as they originally thought. They determined that each storm trooper would have to be worth more than each rebel fighter, and at first they thought that the storm troopers would be worth $1\frac{1}{5}$ points. I had them add these, and when they determined that four $1\frac{1}{5}$ s did not equal 5, they figured out that each storm trooper was worth $1\frac{1}{4}$ points.) Once the values were established, they determined that Chewbacca was worth $3\frac{1}{4}$ points; using this information, students were able to easily find that the rebel fighters and Chewbacca would win Round 3 by adding the

values of all of the participants and determining that $6 \frac{1}{4} > 5$ (see **fig. 5**).

Other students started with the idea that in Round 1 (five rebel fighters versus four storm troopers), each side had to be equal to 1. With this in mind, they determined that each rebel fighter had a value of $\frac{1}{5}$ and each storm trooper had a value of $\frac{1}{4}$. Students who chose this method were able to use the information from Round 2 to conclude that Chewbacca had a value of

$\frac{13}{20}$. They then added the values of all the participants and showed that $1 \frac{1}{4}$ (the value of Chewbacca and three rebel fighters) was greater than 1 (the value of the four storm troopers).

Using the results of Round 1, one final group decided that, to begin with, each side would be worth 40 points. They explained this choice of starting value by stating that “both four and five can go into 40.” They used tape diagrams to determine that

each rebel fighter had a value of eight (since $40 \div 5 = 8$), and each storm trooper had a value of ten (since $40 \div 4 = 10$). Their strategy is shown in **figure 6**, in which the top bar is partitioned into five sections for the rebel fighters and the bottom bar into four sections for the storm troopers.

As described above, this group was then able to use these values to easily determine that Chewbacca was worth 26 points, and hence the rebel fighters won Round 3. Students assigning values were clearly engaged in SMP 2 and SMP 4 as they considered the problem in the context of a tug-of-war as well as a mathematical inequality.

STUDENTS' STRATEGIES: “INTUITIVE SUBSTITUTION”

A few students were able to correctly find the results of Round 3 without needing to assign any numerical values to the characters. These students used what I call “intuitive substitution.” I use the term “intuitive” because the students had not had any formal instruction on using substitution, so their use of it was strictly because it made sense to them. (An example is shown in **fig. 7**).

An explanation of this group’s work may make it easier to understand:

- Round 1 (R1) is shown by the four red circles representing the four storm troopers versus five green triangles representing the five rebel fighters.
- Round 2 (R2) is shown by the two green triangles (two rebel fighters) and one red circle (one storm trooper) versus the purple cloud representing Chewbacca.
- Round 3 (R3) is shown by the four red circles (four storm troopers) versus one purple cloud (Chewbacca) and three green triangles (three rebel fighters). Chewbacca is equal in strength to

Fig. 5 This sample of student work shows students assigning values.

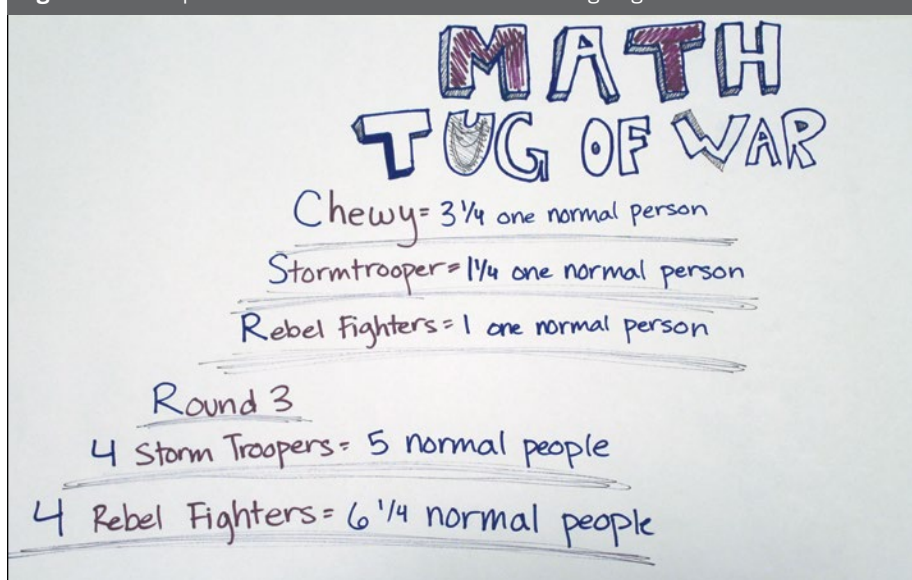
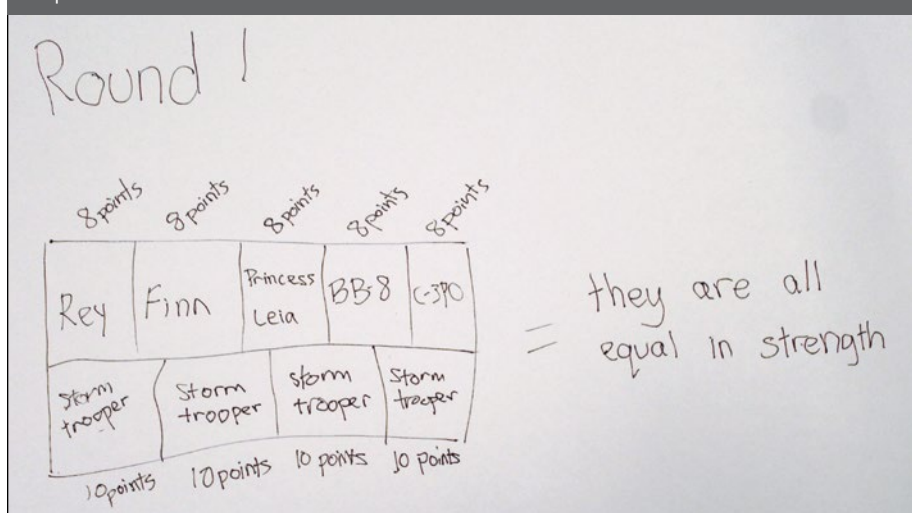
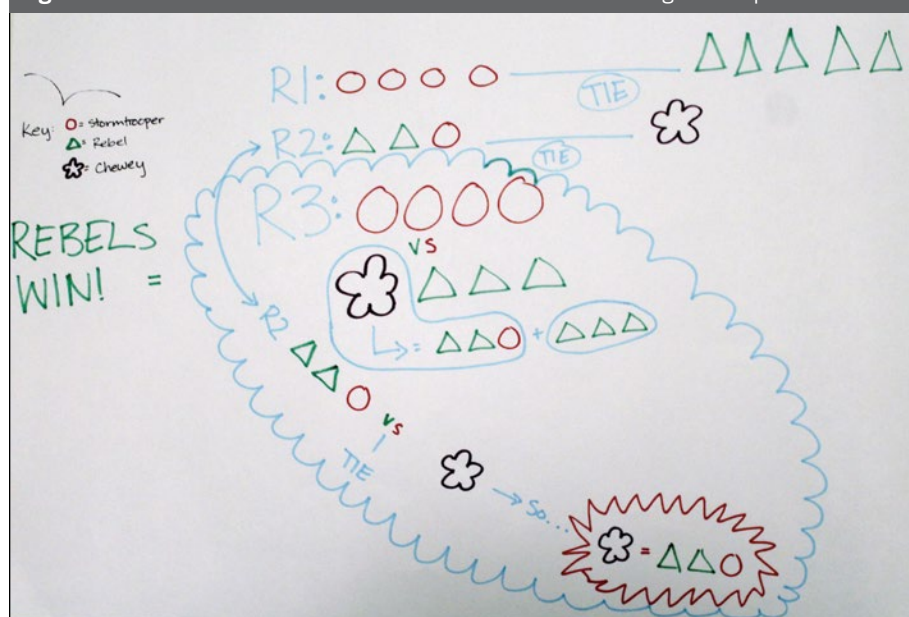


Fig. 6 Students’ tape diagram showed values assigned to rebel fighters and storm troopers.



two rebel fighters and one storm trooper—just below the purple cloud representing Chewbacca, the group has drawn two green triangles and one red circle + three more green triangles. This is evidence of this group's use of structure (SMP 7) as they “see complicated things . . . as single objects or as being composed of several objects.” Now the two teams in Round 3 are made up of five rebel fighters plus one storm trooper versus four storm troopers. Since Round 1 told us that five rebel fighters were equal in strength to four storm troopers, the side with the additional storm trooper wins.

Fig. 7 Some students used intuitive substitution to solve the tug-of-war problem.



SHARING STRATEGIES AND PROCESSING MATHEMATICS

After all groups had completed recording their solutions and strate-

gies, I had students move their chairs into a semicircle to present their work. I wanted the students to understand

that when their classmates were presenting, they were to pay close attention to the speakers; likewise,

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when they were the presenters, they were to speak clearly and loudly enough for their classmates to hear. This provided another opportunity for students to engage in SMP 3.

We began with groups that assigned values to the characters. As groups shared their solutions, I created a chart (similar to **table 1**) showing the values attributed to each and the resulting contests in Round 1, Round 2, and Round 3.

Placing the different values in the same table allowed for a very productive conversation. Students noticed that strategies that initially looked different, such as assigning different values to the characters, were in fact the same as long as the relationships between the values were consistent. We did not address the equivalent ratios explicitly in this lesson, but the chart sets up the class nicely for this investigation in the future. The table also allowed the group that deemed the rebel fighters to each be worth $1/5$ of a point and the storm troopers to each

be worth $1/4$ of a point to “attend to precision” (SMP 6) and revise their values for Round 3. They agreed with the other groups regarding which team won Round 3, but they realized that one of their final values ($1\frac{1}{10}$) was incorrect.

REFLECTING ON THE TASK

As I wrote previously, this has always been a favorite task of mine, and this version did not disappoint! The students were instantly engaged, even though it can hardly be described as a “real-world” problem. Not only were they on task during the two periods we spent on the task, on the Friday following the introduction of the task, two students showed up wearing Star Wars shirts. Although this could have been a coincidence, I chose instead to believe that it was an indication of how much they enjoyed the challenge and the context.

The tug-of-war, whether in its original form or modified to reflect pop culture, has many benefits. First and foremost, it is a great example

of a task that lends itself to multiple strategies and multiple approaches within the same strategy. Students used two main strategies—assigning values and substitution. Within the groups that assigned values, three different approaches were shared. Students used equations, symbols, and tape diagrams, thus engaging in SMP 4—model with mathematics. In addition, the tug-of-war problem sets up the class for a natural segue into the area of ratios and proportional relationships—a focal, and often challenging, domain in the middle school standards. Second, the problem does not initially look like math. As presented, there are no numbers involved with the exception of the number of characters; thus, it does not have the effect of turning students off or causing them to shut down, as they might if the problem looked more “mathy.” The values and strategies that students used all came directly from them. They were never told, “This is an algebra problem that should be solved by assigning values or using substitution.” Finally, the problem encouraged students to use substitution intuitively. Once students understood that they had equivalent values to work with, it made complete sense to use substitution to solve the final round.

As teachers take on the challenge

Table 1 Student groups assigned values to the tug-of-war characters.

Rebel Fighter	Storm Trooper	Chewbacca	Round 1	Round 2	Round 3
1 point	$1\frac{1}{4}$ points	$3\frac{1}{4}$ points	$5 = 5$	$3\frac{1}{4} = 3\frac{1}{4}$	$6\frac{1}{4} > 5$
$1/5$ point	$1/4$ point	$13/20$ point	$1 = 1$	$13/20 = 13/20$	$1\frac{1}{10} > 1$
8 points	10 points	26 points	$40 = 40$	$26 = 26$	$50 > 40$

the results without the characters by formal instruction,

of providing their students with rich mathematical tasks that provide opportunities for engagement in the Standards for Mathematical Practice (CCSSI 2010), I encourage them to revisit some favorites from the past. Many tasks, like the Mathematical Tug-of-War, are ready to use as is, although it never hurts to call on Chewbacca and the rest of the rebel fighters to help solve problems, intergalactic or otherwise.

REFERENCES

- Burns, Marilyn. 2015. *About Teaching Mathematics: A K–8 Resource*. 4th ed. Sausalito, CA: Math Solutions.
- Common Core State Standards Initiative (CCSSI). 2010. *Common Core State Standards for Mathematics*. Washington, DC: National



Let's Chat about a Mathematical Tug-of-War

On Wednesday,
September 20, 2017,
at 9:00 p.m. ET,
we will expand on
“Return of the Tug-of-War”
(pp. 40–48),
by Julie McNamara.
Join us at #MTMSchat.

We will also Storify the conversation for
those who cannot join us live.
The *MTMS* monthly chats fall on the third
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K–12 classrooms any chance she gets.

Name _____

STAR WARS TUG OF WAR

Your task: Determine who will win the final tug-of-war (Round 3). The first two rounds will give you the information you need. You may want to make a sketch in the space below each round.

Round 1:

On one side are five rebel fighters: Rey, Finn, Princess Leia, BB-8, and R2-D2. Surprisingly, **they are all of equal strength.**

On the other side are four storm troopers. **They, too, are all equal in strength.**

In the contest between these two teams, the result is dead even. **Neither team can outpull the other.**

Round 2:

On one side is Chewbacca, who is known around the galaxy for being very strong.

On the other side is a team made up of Rey, Finn, and one storm trooper. **Again, it's a draw—an equal pull.**

Round 3:

It's the final tug that you must figure out.

It will be between these two teams: Chewy, Rey, Finn, and Princess Leia on one side and the four storm troopers on the other side.

Can you figure out who will win this tug-of-war?