Exploring Fraction Division

The Case of Kevin Richard

Mr. Richard's sixth-grade class is beginning work on division of fractions. He wants his students to understand that (1) dividing one number *a* by another number *b* means determining how many times *b* is contained in *a*; and (2) when dividing by a fraction, the remainder is expressed as a fraction of the divisor. In the long term, he wants students to understand the generalized procedures for dividing fractions (e.g., inverting the divisor and multiplying or finding a common denominator and dividing), but he wants students to first develop a conceptual understanding of fraction division. He decided to begin this work by engaging students in Max's Dog Food—a measurement division task. He selected this task because it aligned with his goals for the lesson, it could be solved in different ways, and he thought the context would help students to make sense of the situation.

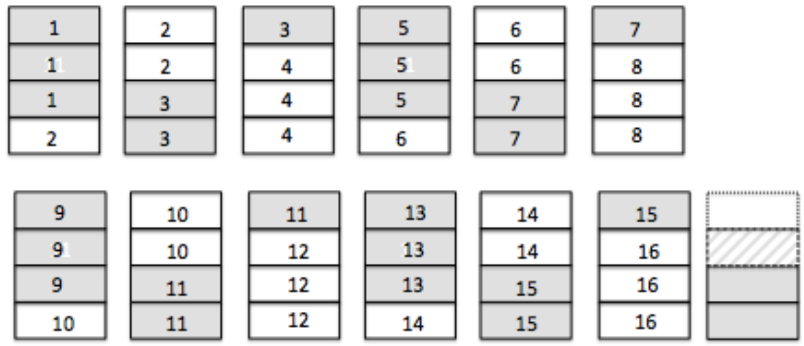
> Dog food is sold in a 12 1/2 pound bag. My dog, Max, eats a 3/4 pound serving every day. How many servings of dog food are in the bag? Draw a picture, construct a number line, or make a table to explain your solution (Institute for Learning at the University of Pittsburgh 2016).

As students began working with their groups on the task, Mr. Richard walked around the room, stopping at different groups to listen to their conversations and to ask questions as needed (e.g., How did you get your answer? What does your answer mean?). When students struggled, he asked them questions about the context (e.g., What does the 12 1/2 represent in the problem? What does the 3/4 represent in the problem? How many pounds would be in 1 serving? In 2 servings?) and suggested they try drawing or building a model to help them make sense of the situation. He noted that students used a variety of methods—group 1 represented 12 1/2 pounds using rectangles and marking off groups of 3/4; groups 4, 6, and 7 used repeated addition; groups 2 and 3 used a combination of repeated addition and scaling up; and group 5 used a number line that they divided into fourths. Although all groups eventually concluded that there would be 16 servings, many of the groups either ignored the 1/2 pound (in 12 1/2 pounds) or incorrectly labeled the answer as 16 1/2 servings.

Mr. Richard decided to begin the whole-group discussion by having group 1 (Jasmine, Marcus, Phoebe, and Kenyon) explain its model to the class. He felt that their model was accessible to all students in the class, could be related to other methods that students used, and would help students see the remainder as a fractional part of a 3/4 pound serving.

Teacher: Can you explain what you did?

Marcus: We started out by drawing 12 1/2 boxes (shown below). Each box represents a pound—the half box is 1/2 pound. Since Max gets 3/4 of a pound for a serving, we divided the 12 boxes into 4 equal pieces and the 1/2 box into 2 equal pieces. Then we found out how many 3/4s there would be in the 12 1/2.

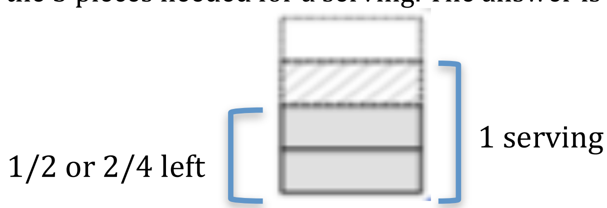


Teacher: Can someone else in the group be more specific about how you determined the number of times 3/4 would fit in the 12 1/2 pounds?

Phoebe: Well, every time we had 3 of the fourths we knew this would be one serving. So we started with the first group of 3 one-fourths and labeled them with 1s to show they were all part of the first serving. Then we just kept going. We found we had 16 groups of 3/4.

Teacher: So what did you do with the 1/2 pound?

Phoebe: So we made the 1/2 box a whole box and shaded in the 1/2 pound (see below). We knew that a serving was 3/4 of a pound and that would be 3 pieces of the box. So we had 2 of the 3 pieces needed for a serving. The answer is 16 2/3 servings.



Teacher: Does anyone from the group want to add to what Marcus and Phoebe have said?

Jasmine: The 1/2 pound that was left is 2/3 of a serving.

Teacher: Does anyone have any questions for the group? Duncan?

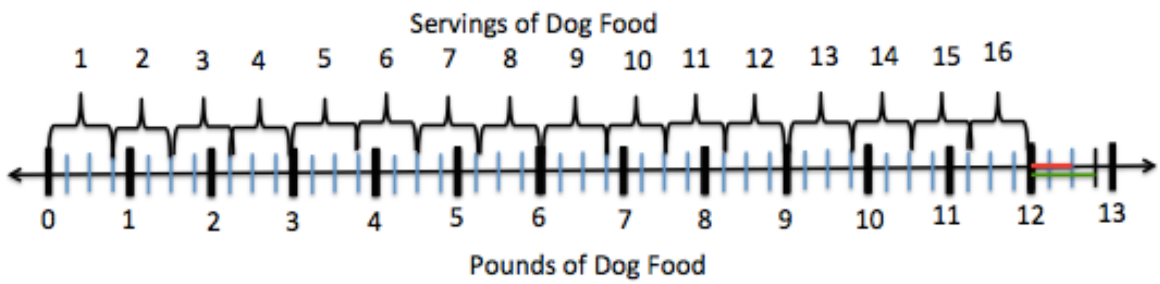
Duncan: I don't see how they got 16 2/3. We got 16 1/2.

Teacher: Can someone besides the members of group 1 explain why it is not 16 1/2? Kate?

Kate: Well there's one half in the 12 1/2 but we need to know what part of a serving the 1/2 pound is. We know that 1/2 pound is *not* a complete serving because the serving size is 3/4 pound, so the 1/2 pound is only going to be a portion of a serving. To make one serving size, which is three-fourths, you need to have another fourth, and that's why it is two-thirds because you need three of the fourths to get another serving and there's only two of those, so you have two-thirds of what you need for a serving.

Teacher: Tabitha, can you explain what your group did? This will give us another model to help us think about the problem.

Tabitha: We did a number line [shown on the next page]. We started by numbering 1 to 13 to show the number of pounds [bottom of the number line]. Then we divided it into fourths. We then marked off the number of three-fourths because that is the amount in a serving [top of the number line]. So we had 16 servings. But we still had the 1/2 pound [red on the number line]. So like Phoebe said, we need three-fourths for a serving [green line] and we have two-fourths left [red line] so we have two of the three pieces we need for another serving. So it has to be 2/3. So that gives us 16 2/3 servings.



Teacher: So how is the method Marcus's group used the same as or different from what Tabitha's group did? Take two minutes to turn and talk to a partner.

Teacher: [After 2 minutes.] Kayra, can you tell us what you and Michelle discussed?

Kayra: Well, each of the numbered servings in the picture that Marcus's group did was the same as counting the number of three-fourths like Tabitha's group did. So the ones in Marcus's picture are the same amount as the 1 on the top of the number line. Then they both compared the 1/2 left to the amount needed for a serving.

Sarah: They were both trying to figure out the number of 3/4s in 12 1/2. They just used a different model. It is really just 12 1/2 ÷ 3/4.

Teacher: [To the class.] Do you agree or disagree with Sarah?

Chris: I agree with her because when you do division you are trying to find how many 3/4s are in 12 1/2. They did it different ways but it is still the same. They both found there were 16 2/3 groups of 3/4 in 12 1/2.

Teacher: Do you agree with what Sarah and Chris are saying? Give me a "thumbs up," "thumbs down," or "thumbs sideways." [Most of the students give a "thumbs up."]

Teacher: Reilly, tell us why you disagree.

Reilly: My group didn't divide. We added.

Teacher: Tells us what you did.

Reilly: We just kept adding 3/4s until we got to 12. So we got 3/4 + 3/4 = 1 1/2, 1 1/2 + 3/4 = 2 1/4, and just kept going until we got to 12. Then we counted up the number of times we added 3/4, and it was 16. We didn't have enough for another serving, so we just ignored the 1/2 pound.

Teacher: Can someone use the number line to explain Reilly's method? Kevin?

Kevin: The numbers they got when they added three-fourths are on the number line. So 1 serving ends at 3/4s, two servings end at 1 1/2, three servings end at 2 1/4. So each of the numbers they got are names for points on the number line all the way to 12. It is like skip counting by 3/4s.

Teacher: So why did Reilly keep adding three-fourths? Why did they stop at 12?

Sade: He was trying to find out how many three-fourths it would take to use up the 12 pounds of dog food. He just started with 3/4 and built up to 12.

D'Angelo: He found how many three-fourths were in 12 just like Marcus and Tabitha.

Teacher: Let's go back to our discussion of division. Sarah said that we were really doing 12 1/2 ÷ 3/4. She is right. We were trying to figure out how many 3/4s there are in 12 1/2 and that is a division situation, but you didn't have to use a division rule. Most groups showed 12 ÷ 3/4 = 16 in some way. But Phoebe and Tabitha explained that we also need to do 1/2 ÷ 3/4 = 2/3.

With 5 minutes left in the class, the teacher told students that their exit ticket for the day would be to explain (1) Whether 12 1/2 ÷ 3/4 was the same as 12 ÷ 3/4 plus 1/2 ÷ 3/4 and why; and (2) What 1/2 ÷ 3/4 means. For homework he asked students to explain why the quotient (16 2/3) was larger than the dividend (12 1/2). He thought these prompts would give him valuable insights regarding whether students understood about division of fractions.

This case is based on a lesson taught by James Parson, a sixth-grade teacher at Meigs Middle School in the Metro Nashville Public Schools, who was working with Victoria Bill at the Institute for Learning (IFL) at the University of Pittsburgh. This case was written by Margaret Smith, Victoria Bill, and Mary Lynn Raith.