Students work their way around four corners to reach mathematical consensus.

Unit Rates, and

Ratios, Rates, Debates!

Jessica Lynn Jensen

Watching a debate can help people understand multiple points of view. At the conclusion of a debate, a consensus might occur after a deep analysis and discussion of the issues. Most students will participate in a class debate at some point in their schooling, but chances are that debate will not take place in math class. What could there be to debate in a subject that is based on mathematical proof?

This article suggests using a debate format in math class to promote the use of critical thinking, academic language, and numerous Common Core Standards for Mathematical Practice (SMP) (CCSSI 2010). By debating classifications within various mathematical domains, students can use definitions and other resources to make arguments that represent their current understanding of a definition and then debate with classmates about opposing opinions. This use of academic language in a rigorous manner will allow students to build a deeper understanding of the interconnectedness of the content.

WHAT IS A FOUR-CORNER DEBATE?

The four-corner debate is an activity in which students move around the room as their ideas on classifications of elements change in response to their classmates' arguments. Four signs are posted, one in each corner of the room, and students use definitions and their own understanding to argue about the appropriate classification of an object, expression, and so forth. This article presents one example of a four-corner debate, which focuses on classifying comparison quantities; the possibilities are a ratio, a rate, a unit rate, or none of the above.

Ratios and rates are important parts of middle school mathematics: the Common Core deals with

Fig. 1 The first definition is from *Merriam-Webster's Dictionary*; the second is from *Holt Mathematics Course 2* (Bennett et al. 2007).

Ratio
M-W: The indicated quotient of two mathematical expressions
Holt: A comparison of two quantities by division

Rate
M-W: A fixed ratio between two things
Holt: A ratio that compares two quantities measured in different units

Unit Rate
M-W: A quantity, amount, or degree of something measured per unit of something else
Holt: A rate in which the second quantity in the comparison is one unit

ratio and proportion in both grades six and seven. One aspect of the concepts of ratio and proportion that is often troublesome for students is the hierarchical relationship among ratios, rates, and unit rates (Lobato, Ellis, and Zbiek 2010). When differentiating between classifications, understanding that an element can fit under multiple categories, even though we label it by its most specific name, can be challenging for students. Understanding the level of connectedness among ratios, rates, and unit rates allows students to increase their depth of understanding. This understanding can also be applied to other mathematical concepts, such as fractions, division, slopes, similar figures, and other areas of mathematics that can be strengthened by a solid development of proportional reasoning. In this particular debate, students (and teachers) can explore the concepts of ratio and rate as well as increase their understanding through active learning individually and as a group.

Students need to understand the connectedness among ratios, rates, and unit rates.

HOW IT WORKS

The teacher posts four signs in the corners of the room: Ratio, Rate, Unit Rate, and None. Students use definitions and their own understanding to argue about the appropriate classification of comparison quantities that are not all straightforward. The definitions given to students are not all necessarily mathematically precise or compatible. The purpose of this exercise is to promote discussion so that comparison quantities could be argued in multiple ways, thus encouraging students to use evidence for their claims and to think and reason beyond the definitions.

Students initially go to the corner that they believe the comparison is classified under, and then the debate starts as students try to persuade their classmates to join their corner. Students are encouraged to change corners if their understanding has shifted and they change their mind about the comparison's classification. The debate for a particular comparison quantity ends when the entire class is persuaded to move into one corner. At that point, a new debate starts for a different comparison.

Purpose

The purpose of this activity is for students to develop a deep understanding of the overlapping, hierarchical nature of ratios, rates, and unit rates while identifying their unique characteristics and considering their respective purposes in the real world. Moreover, students will have the opportunity to engage in numerous Standards for Mathematical Practice, namely SMP 3: "Construct viable arguments and critique the reasoning of others" and SMP 6: "Attend to precision" (CCSSI 2010, pp. 6–7). SMP 6 requires students to "communicate precisely to others" and "use clear definitions in discussion with others and in their own reasoning." Both of these practices are focal points in this debate activity.

Debate Participants

The open nature of this debate, which allows participants to make sense of the concepts in unique and varying ways, makes it appropriate for a wide range of students. The content is aimed at the grade 6 and 7 standards. Sixth-grade students would benefit from this activity after an introduction to ratios, rates, and unit rates; seventh-grade students would benefit before they are introduced to proportions.

Launch

Without definitions, this activity will not be as meaningful, so communicating about these definitions before the debate is important. I begin by having students write down what they think are the definitions of ratio, rate, and unit rate and also explain how they think the terms are related. After time for individual reflection, we discuss these ideas as a class. I then show students two different definitions for each term, one from a nonmathematical source and one from their textbook. In this case, I used *Merriam-Webster's Dictionary* (http://meriam-webster.com/dictionary/ratio) and *Holt Mathematics Course 2* (Bennett et al. 2007) (see **fig. 1**). It is important for students to understand that definitions are not always universal and that their argument might change on the basis of a given source. This understanding helps students form increasingly powerful arguments and allows for critical consumption of information. Once students have discussed their own definitions and read the definitions given by the published sources, they discuss differences between their original definitions and the published definitions. This process allows students to identify what they have understood and misunderstood in the past and deepen their own understanding of the definitions.

After students have a baseline understanding of these terms, we discuss the relationships among them. I set up an analogy that compares ratios, rates, and unit rates to fruit, apples, and Fuji apples. Students are familiar with the idea that all apples are fruits, but not all fruits are apples. This is an easy analogy for students to connect to the hierarchical relationship among ratios, rates, and unit rates. Just as all Fuji apples are apples and fruits, all unit rates are rates and ratios. Sometimes the relationship flows in the opposite direction, but you cannot assume this is always the case.

At this point, students are ready to start thinking more analytically about ratios. I then give them an activity sheet (see **fig. 2**) and have them independently place each of the comparison quantities in the most appropriate column. Making this task independent work is important because this assignment will fuel the debate.

Fig. 2 The ratio, rates, and unit rates activity contains multiple possibilities.

Rates, Ratios, Unit Rates, None

Place each of the following comparisons in the most appropriate column below.

$\frac{3}{4}$ 5 mi. : 2 mi. $\frac{\text{20 miles}}{\text{1 gallon}}$ miles per hour   7 to 1

\$2.68 per ounce $\frac{\text{12 boys}}{\text{13 girls}}$ $\frac{\text{45 words}}{\text{5 minutes}}$ 5 dogs : 3 hours

3 : 4 : 2   3 dogs and 4 cats   1 mi. to 1 mi.

3 tickets for 4 children   \$3.20 per 6-pack of soda

Ratios	Rates	Unit Rates	None

PREDEBATE DISCUSSION

A predebate discussion helps students focus on reasoning and learning, and distracts them from their worries about being wrong in front of the class. The discussion can focus on a few different ideas.

Fig. 3 The last two comments in this sampling of arguments illustrate student inquiries that occurred naturally.

Comparison	Student Arguments
3/4	"None, because 3/4 is one quantity, not two."
	"There are two kinds of ratios; part-to-part, and part-to-whole, and fractions are part-to-whole ratios."
	"If you're not given units, what do you assume then?"
	"Does a ratio have to have a unit (label)?"
5 mi. : 2 mi.	"A unit rate doesn't have to be compared to 1 because you could say that the unit is 2 miles. If every inch on the map represented 2 miles, 2 miles would be the unit."
Miles per hour	"I chose rate by using *Webster*'s definition because it says 'between two things,' but if you look at the textbook definition, it says between two quantities, and I don't think there are really quantities."
	"Does a fixed ratio have to mean a fixed numerical value, or can it be a fixed unit of measurement?"
	Teacher response: "What is the definition of a fixed ratio? If I say I am going to run however many miles I want per hour, is that a fixed ratio?"
\$3.20/six-pack of soda	"It's a unit rate because the price is compared to one six-pack of soda."
	"We need to find the price per can in order for it to be considered a unit rate."

1. **Mistakes are good.**
Everyone learns from shared mistakes, and they often lead to meaningful conversations that we would otherwise be unlikely to have. An additional bonus is that research shows that your brain grows when you make mistakes (Moser et al. 2011).

2. **Stick with your instinct.**
Unless someone has said something that has honestly changed your opinion, stay with your instinct. In my past experiences with this activity, if a student saw the majority of the class heading to a different corner, he or she would change an answer and go with the majority. I have seen a single student argue the whole class into his or her corner before, so be strong! State your reasoning and argument, and the worst-case scenario is that you are wrong and we have all learned from your ideas.

3. **Be clear and listen carefully.**
The best way to change someone's mind is to listen to their ideas and state clearly why their reasoning is not sound. It is the class's job to make sure everyone understands the mathematical concepts in this debate, so use your arguments to help others understand, not to prove them wrong.

This predebate discussion helps increase the productive nature of the debates. These are important messages for every mathematics class, but this is a perfect opportunity for students to live by these words.

DEBATE!

You will not have time to debate each of the comparison quantities in **figure 2** nor should you. It is your job, as the teacher, to choose which comparisons to debate and in what order.

I typically start with something easier, such as 3/4, to get students comfortable and then choose comparisons that will help them develop ideas in which the debate exposes inconsistency in their thinking. Other than this strategic selection, the teacher's job is to be as quiet as possible. I try not to guide students in any direction. I typically speak in only three instances. First, when I see students using excellent reasoning or arguing skills, I encourage them. Second, when the debate gets to a point where it is not progressing—which will happen—I step in to redirect the discussion. Students can argue some of these comparison quantities in multiple ways, so, depending on the situation and definition used, the time may come when the teacher needs to say it is acceptable to have two answers or point out an argument that students need to pay closer attention to. Third, sometimes I step in to play devil's advocate. If all students initially agree, I will try to throw in some cognitive dissonance. For example, all students originally thought that 12 girls/13 boys was a rate because "we have two different units of measure." So I jumped in the ratio corner and said that I thought it was a ratio because boys and girls are both people, so they might not be different units of measurement. Once you have students in your corner, back off and let them continue arguing.

Comparison quantities without referents (e.g., 3/4, 7 to 1, or 3: 4: 2) may be difficult because they do not have a context to argue from. Involving these comparisons helps students realize the importance of context and teaches them how to define the lines between when context is important and when further context is unnecessary.

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STUDENT ARGUMENTS

It would be impossible to list all the arguments made over the numerous debates I have seen students engage in, so I have chosen a few comparisons to share a sampling of statements. Student arguments for the comparisons are found in **figure 3**, and comments about these arguments are found below.

Students were surprisingly divided for the comparison quantity 3/4. Their arguments about whether the comparison represented one or two quantities originated from their understanding of fractions. This was an important connection for students to make. Even though a fraction represents a single value, it is also a comparison between the 3 parts that you have and the 4 parts that it takes to make a whole.

Students also became owners of their own learning and immediately started coming up with their own questions to ask the class (see **fig. 3**). One interesting misconception, exposed through the comparison 5 mi. : 2 mi., was centered around the understanding of what comprises a "unit." After this student's comments, the discussion turned to how we would be able to discern the difference between a rate and unit rate if the second quantity for unit rates was not 1.

Miles per hour was always a highly debated "comparison." Students were usually in three corners: rate, unit rate, and none. Students tended to use the definitions very heavily for this debate. Many students were uncertain about what the word *fixed* meant in the rate definition. This discussion moved students to a point at which they figured out miles per hour was a unit of measure of an intensive quantity and not a unit rate. Students realized that we needed numbers to connect that unit of measure to a unit rate. Even after I confirmed that this "comparison" fell under the none category, students continued to argue. This was a major teacher win for me because I knew I had removed myself from the position of "answer book." Students cared more about thoroughly understanding the ideas than getting an answer from the teacher.

A debate about $3.20/six-pack of soda helped students understand that the purpose of a unit rate was to be able to compare quantities measured with the same unit. To push students' thinking, I offered a question. I asked, "Behind curtain A is a six-pack of soda for $4.50, and behind curtain B is a six-pack of soda for $5.50. Which one is the better buy?" We then discussed how the purpose of a unit rate was to be able to compare amounts easily. In this case, the unit of a six-pack was not a standardized unit of measurement, and therefore, not useful. We could not compare prices for the six-packs because the amount of soda in a six-pack varied, and therefore, it was not technically a fixed rate.

SUMMARIZE

By this point, students are exhausted—a true testament to the active, cognitive engagement that this activity encourages. However, because of the nature of this activity, some students might still be confused about a few ideas, which is OK. I tell my students that confusion always comes before clarity. Because the debate itself easily takes up an entire class period, I give students an exit slip that asks them to reflect on the debate and list three new things that they learned. The responses to this prompt usually vary widely, so I use these responses as a point of discussion during the beginning of the next class to help students solidify their understanding. Some of the students' reflections are correct, and some are not; so I have students complete a 3-2-1. They must find three responses that they know are true, two responses that they know are false, and one response that they are unsure of (see **fig. 4**). Green indicates that the statement is true; blue statements need more context or clarification to determine the validity; and red statements are false.

Student debates are valuable tools for building deep understanding of mathematical concepts. The student-centered and discussion-based

Fig. 4 Student exit slip statements were used as a summary activity.

Find three statements that you know are true, two statements that you know are false, and one statement that you are unsure about:

1. A ratio does not have to have a label.
2. If something does not have units of measurement, then it cannot be a rate or unit rate.
3. 7 to 1 would be a ratio, not a unit rate, because there are not any labels.
4. Unit rates are comparing amounts, not numbers.
5. The word per means 1.
6. Per means division.
7. Rates and unit rates must have units of measure.
8. If something has a 1 under it, it is always a unit rate.
9. Ratios always have to compare the same types of things, for example, miles to miles.
10. Unit rates help you find the price of one thing in a pack.
11. All ratios are rates.
12. Rates need different units.
13. 75 miles per hour is a unit rate because the word hour implies that it is 1 hour.

nature of this activity exposes student misconceptions, encourages connections, and helps students become owners of their own learning. In a traditional lesson on these concepts, definitions would not be analyzed as thoroughly and students would not have the opportunity to realize that they did not fully understand the meaning of a simple term like *fixed*. Similarly, without teacher provocation, students may not have made as strong a connection/differentiation between fractions and ratios. This activity provides students with the time needed to sort through misunderstandings as a class and participate in true collaborative inquiry.

Four-corner debates can easily be adapted to engage students in analytical thinking, argument, and justification for numerous concepts. For example, provide students with situations that represent various mathematical properties and have them argue into corners of associative, commutative, distributive, and identity properties. Help students identify operations that are necessary for various situations by giving them scenarios and arguing among corners of addition, subtraction, multiplication, and division. The opportunities for four-corner debates are endless, so get creative. You will be amazed at what you can learn both from and about your students' thinking through debates.

Let's Chat about Ratios!

On Wednesday, October 17, 2018, at 9:00 p.m. ET, we will discuss "Ratios, Rates, Unit Rates, and Debates!" (pp. 74–80), by Jessica Lynn Jensen. Join us at #MTMSchat.

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