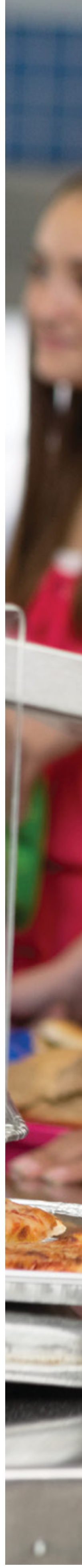
Real-life scenarios embedded in social, cultural, historical, and political contexts can help bridge a gap between critical mathematics education and mathematical modeling.

Empowering MATHEMATICIANS THROUGH Modeling

Lisa L. Poling, Nirmala Naresh, and Tracy Goodson-Espy

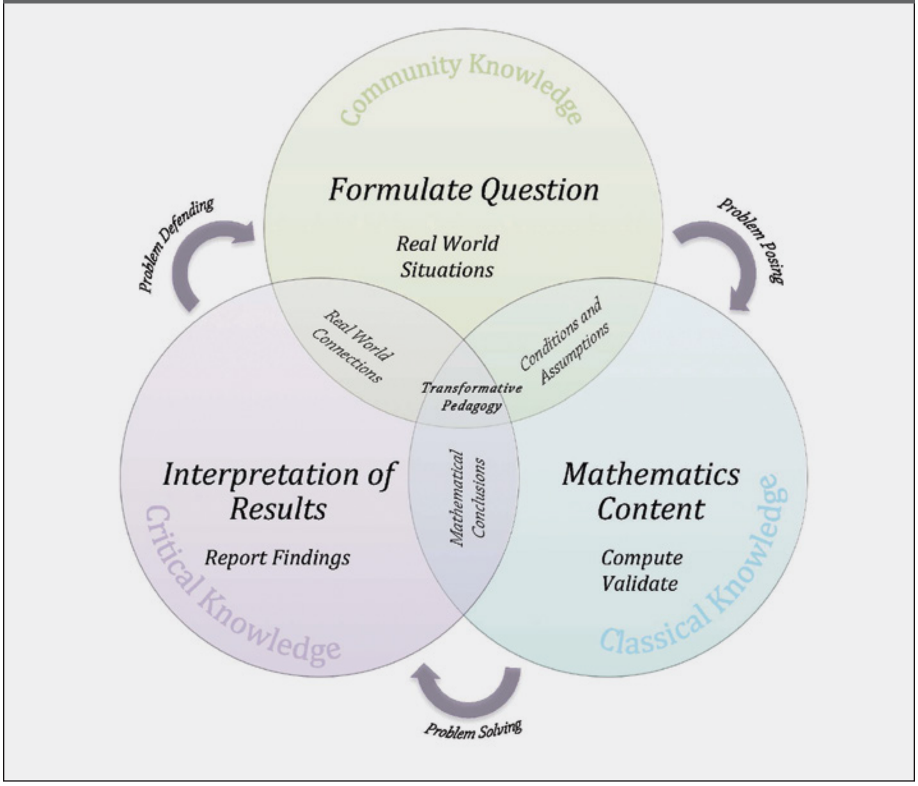


> Through my interactions with Ms. Pam, a lunch lady, I learned that the lunchroom is full of hidden mathematical concepts. She helped me realize the potential for using them to design math activities. I have grown confident in developing [models] and teaching modeling to elementary and middle school students as I no longer believe that these are abstract concepts for only a select group of students.
>
> (*Sam, a prospective middle school teacher*)

> By presenting an open-ended task, using an authentic context, where students had to take into account real-world constraints, I was able to make the task more challenging and interesting as we listened to the many different approaches and answers that [students] were able to generate. All of this happened thanks to Joe, who is a truck driver. His expertise and input helped me better understand the context and present a meaningful task to the students.
>
> (*Alan, a prospective middle school teacher*)

These excerpts capture two preservice teachers' (PST) reflections on mathematical modeling projects aimed at promoting connections between modeling and the principles of critical mathematics education (CME) (Frankenstein 1995). In this article, we show two examples to demonstrate how PSTs engaged in using

Fig. 1 The principles of critical mathematics education align with the CCSSM modeling framework.



mathematics in modeling real-world scenarios. The examples confirm how modeling projects deepened students' mathematical knowledge and how the use of a critical lens provided a way to acknowledge the influence of mathematics in occupations that are not traditionally associated with mathematics.

"I learned that the lunchroom is full of hidden mathematical concepts."

A CRITICAL MATH PERSPECTIVE ON SOCIOCRITICAL MODELING

Principles and Standards for School Mathematics (NCTM 2000) and, more recently, the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010) emphasize modeling as an application process in which K–12 students engage with varying levels of sophistication. Students must understand that models can be developed, revised, and reapplied within a context. One may focus on modeling as content, emphasizing development of mathematical competencies that are necessary to model a phenomenon; or by considering modeling as a vehicle, emphasizing the pedagogy of modeling content (Julie 2002). Representative mathematical models are "fundamental to how people understand and use those ideas" and aid individuals to "acquire tools that significantly expand their capacity to model and interpret physical, social, and mathematical phenomena" (NCTM 2000, p. 4).

Although many people understand and emphasize the realistic-applied modeling approach, Kaiser and Sriraman posit that the sociocritical modeling approach provides an " 'emancipatory perspective' " [that] leads to a critical understanding of the surrounding world" (2006, p. 304). Critical mathematics education using sociocritical modeling (Kaiser 1995) empowers learners to use these elements: communal knowledge to frame problems that are central to their lived experiences, classical knowledge to develop mathematical competencies, and critical knowledge to gain a comprehensive understanding of the sociopolitical context for the problem (Gutstein 2007). **Figure 1** depicts the principles of critical mathematics education aligned with the CCSSM modeling framework (CCSSI 2010). The framework, used in tandem with community, critical, and classical knowledge (the three Cs) (Gutstein 2007), demonstrate the intricacies recognized as individuals construct and interpret mathematical models. Researchers have previously aligned modeling to the problem-solving process (Ciltas 2013), and we extend this to our framework. The problem-solving process requires that individuals formulate a question (community knowledge), collect data, analyze data (classical knowledge), and interpret data (critical knowledge). Modeling applications help to expand problem-solving performance, equipping students with the skills and knowledge to address and discern problems that they encounter in daily life (Niss 1989).

IMAGE ON PREVIOUS PAGE STEVE DEBENPORT/ISTOCK; LUNCH TRAY, KCLINE/THINKSTOCK

CONNECTING RESEARCH TO PRACTICE: MODELING IN TEACHER EDUCATION

Many teachers appear hesitant to teach modeling (Bautista et al. 2014). Teachers' views of modeling as applied primarily to high school may constrain their ability to perceive it as suitable for students in grades 5–9. Additionally, these grades 5–9 teachers may have limited access to professional development, curricular resources, or models of instruction to assist them in implementing modeling in their classrooms. Thus, attending to teachers' knowledge and beliefs about modeling is imperative. The work described below is part of a research study within a senior level course for prospective middle school teachers, aimed at identifying meaningful ways to promote connections between modeling and critical mathematics.

In this course, PSTs completed a modeling project focused on real-life scenarios embedded in a social context. The primary goals of the project were to empower PSTs by demonstrating how to blend modeling with critical math, empower middle-grades students through critical analysis, and empower individuals who sometimes may not be viewed as professionals who engage in doing mathematics. To promote the negotiation among community, critical, and classical knowledge, PSTs were first asked to identify members of a profession in real-life scenarios that are not traditionally associated with mathematics. Once a scenario was identified, PSTs created instructional activities focused on the use of mathematical modeling and critical mathematics that were suitable for grades 5–9. In so doing, many PSTs discerned connections between their modeling projects and recommendations involving modeling contained in the Common Core.

ENACTING A CRITICAL MATH APPROACH TO MODELING

Example 1: Social Setting—School Cafeteria

Sam developed her modeling task in collaboration with her mother, Pam, a school cafeteria worker. Here is the task, which she enacted with three sixth-grade students (X, Y, Z).

Modeling Mathematics: The Lunch Menu Task

Ms. Pam is a head cook in a school cafeteria. She must follow a set of guidelines to ensure that students receive the right amount of each food group per day. There is also a regulation that lunches must be within a certain calorie range.

- Create a weekly lunch plan that Ms. Pam would approve.
- On a given day, how much of each food type would Ms. Pam have to prepare?

Student Work

After beginning work, students realized that they needed additional information regarding nutritional and serving size regulations, which Sam provided. Consulting their monthly school lunch menu, they identified choices for each food category (see http://bit.ly/1AqISTY). **Figure 2** captures their responses to the questions.

Fig. 2 Grade 6 students responded to the first two questions.

I chose the same menu for all five days of the week.

Apple Sauce	1/2	50
Carrots	3/4	75
Bread Stick	1	100
Sausage Links	1	110
Regular milk	1	90

Total calorie count: 425

There are about 700 students in our school – not all of them eat lunch at school. So here is my answer for 600 students.

- Fruit: 600 × 0.5 = 300.0 cups
- Vegetables: 600 × 0.75 = 450.0 cups
- Grains: 600 × 1 = 600 servings
- Meat: 600 × 1 = 600 ounces
- Milk: 600 × 1 = 600 cups

Checking their work, Sam realized that this simplistic plan did not fulfill the nutritional requirements or the recommended serving size regulations. She challenged the students to experiment with different menu options, to interpret their solutions in the context of the problem, and to be explicit in communicating their thinking. In response, students created another model, this time using five different menu options, one for each day of the week (see **fig. 3**).

Sam asked if Ms. Pam would approve their lunch menu. Reflecting on their work, students realized that they still had not attended to the nutritional requirements and the serving size regulations. Students made three changes in the next revision: (1) they increased the serving size of the grain/meat item; (2) they made no changes to the milk and vegetable items because they conformed to the serving size regulation and (3) they increased the calorie count but did not exceed 700 calories. With these changes in place, students produced a third model (see http://bit.ly/1AqISTY), which they believed would get Ms. Pam's approval. Next, students revisited the question: How much of each food type will have to be prepared?

Student X: This seems a reasonable estimate, as I know that we have about

Fig. 3 In trial 2 of the Lunch Menu task, students created a model that uses five different menu options. The calorie count (CC) is per serving size (SS).

Food Type	Fruit			Veg			Grains			Meat			Fluid			Total Calorie Count
	Name	SS	CC	Name	SS	CC	Name	SS	CC	Name	SS	CC	Name	SS	CC	
Mon	Apple sauce	1/2	50	Carrot	3/4	75	Bread	1	100	Sausage	1	110	Reg. milk	1	90	425
Tue	Banana	1/2	75	Peas	3/4	60	Roll	1	65	Hamburger	1	185	Choc. milk	1	90	475
Wed	Grapes	1/2	60	Broccoli	3/4	80	Bun	1	150	Grilled Chicken	1	120	Str. milk	1	90	500
Thu	Orange	1/2	45	Grean beans	3/4	65	Hotdog bun	1	125	Breaded Chicken	1	160	Reg. milk	1	90	485
Fri	Apple juice	1/2	45	Green beans	3/4	65	Bread	1	70	Hamburger	1	185	Choc. milk	1	90	455
	Weekly serving		2½	Weekly serving		3¾	Weekly serving		5	Weekly serving		5	Weekly serving		5	

Fig. 4 Student X's work reflects that he knew that of 700 students in the school, about half ate the school lunch.

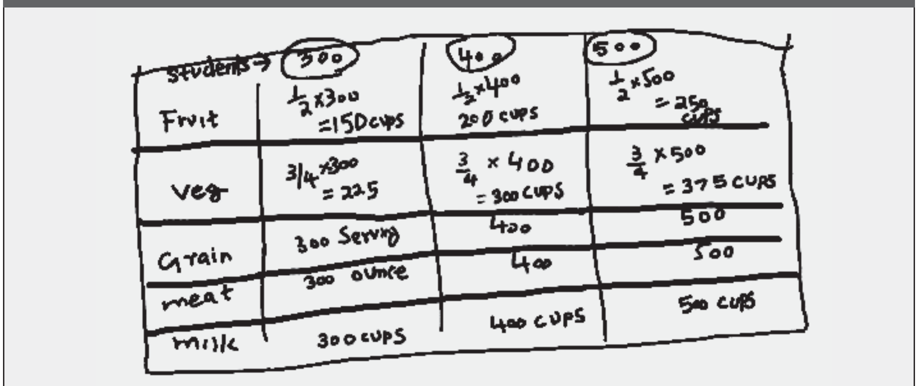
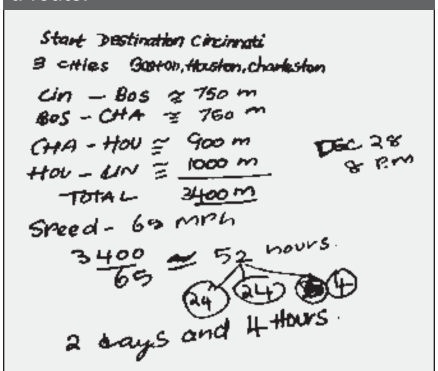


Fig. 5 Trial 1 illustrates the first attempt that sixth-grade students made to model a route.



700 students in our school and not all eat school lunch; about half do. In my grade, I know that at least half the students eat home lunch. So, I created a table [see **fig. 4**] to give a range: 300–500 students.

Student Y: But wait, this is not the lowest estimate. It will be "nothing" if no one ate lunch.

Student X: But that is very unlikely. We have a lot of students who eat lunch at school regularly.

Student Z: Not all students eat the same kinds of food. Even within a food group, there might be different choices. Taking that into account, it becomes a more complicated and difficult problem.

Sam's task gave students an opportunity to model a real-life practical problem, the creation of a school lunch menu, involving several bounding parameters (e.g., total number of calories, nutritional requirements, serving size requirements, minimizing the amount of wasted food, and so on). This task allowed students to use fractions and mixed numbers in a meaningful context, as well as apply the Common Core's Standards for Mathematical Practice (SMP), including SMP 4, Model with mathematics, and SMP 2, Reason abstractly and quantitatively (CCSSI 2010). After groups had the opportunity to create proposed solutions to the problem, Sam asked students to present their solutions to the Lunch Menu problem and answer the questions and challenges to their solution argued by classmates.

Example 2: Social Context—Truck Driving

Alan developed his modeling activity in collaboration with his Uncle Joe, a truck driver. Joe took pride in the fact that he delivered iPhones® to warehouses in different cities, on time, for a big launch. Alan used this context for his task, which he enacted with sixth-grade students.

Modeling Mathematics: The Delivering iPhones Task

Mr. Joe is a truck driver. As part of this job requirement, he has to deliver shipments to different warehouses across the country. He starts from [your location] and delivers shipments to three other cities in the United States [outside of your state].

- What is the shortest route that he can take? How long will it take him to complete it?
- The release date for the iPhone 6 is January 1 at midnight (12:00 a.m.).

When should Mr. Joe leave to make sure all of the stores get the iPhone 6s before the grand release?

Student Work

Three sixth-grade students (I, J, K) participated in this task (see **fig. 5**). The shortest distance identified by the students spanned 3400 miles. At an average driving speed of 65 miles per hour (mph), a driver can cover this distance in about 52 hours. Going back in time 52 hours from December 31, students reported that Mr. Joe should start his trip on December 28 at 8 p.m. Reviewing this work, Alan realized that the students had reduced this model-eliciting task to mere computation. Determined to engage the students in a purposeful discussion, Alan asked the following questions:

- How would Mr. Joe respond to your answer?
- What questions might he have?
- How would you convince him that this is the best possible answer?
- What evidence should you present to him?

To present their evidence, students had to document their problem-solving process.

During their second attempt, students outlined key steps that were instrumental in the model development, which are summarized below.

- Choose four destinations (Cincinnati, Boston, Charleston, and Houston as the four cities, with Cincinnati as the base destination).
- Find the distance between two cities (make a table).

Fig. 6 This grid models calculating distances.

Start	End	Distance		Google Map (Miles, Rounded to the Nearest 10)	Google Map Miles ÷ Map Miles (to the Nearest Tenth)
		Map Inches (Rounded to the Nearest 1/4 Mile)	Actual Miles (Map Inches × 300)		
Cincinnati	Boston	2.5	750	880	1.2
Boston	Charleston	2.5	750	970	1.3
Charleston	Houston	3	900	1100	1.2
Cincinnati	Houston	2.75	825	1000	1.2
Houston	Boston	5	1500	1800	1.2
Charleston	Cincinnati	1.5	450	620	1.4

Fig. 7 Several possible routes were visually analyzed.

Route 1 Route 2 Route 3

Students used a ruler to measure the distance between two cities and rounded to the nearest quarter of an inch. Using the map scale (1 inch = 300 miles), they found the actual distance. Since they did not know if this scale is accurate, they also used Google maps to check.

The last column of **figure 6** gives the ratio of the two distances.

- Find possible routes (using the map). Once they created the table, the next step was to identify possible routes on the map (see **fig. 7**). A careful analysis of the data led them to identify several possible routes and calculate the distance for each route (see **fig. 8**).
- Find the shortest route (use map and table). Using the table, students determined the shortest route (circled in the table) and that it spans 3950 miles. Student J commented, "We should exclude the distance to travel from the third city to the base destination Cincinnati, since our goal is to suggest a suitable start time so that he can deliver the iPhones by the deadline; the problem does not say that he has to be back home by then." Thus, the total distance was recalculated to be 3950 – 880 = 3070 miles.
- Find the duration of the trip (use the rate formula). To determine the duration of the trip, students needed to know Mr. Joe's driving speed. They looked up the speed limit guidelines on the website http://www.motorists.org/speed-limits/state-chart and

Fig. 8 The data decided the shortest route.

Start	City 1	City 2	City 3	End	Total Distance	
					Map	Google Maps
Cincinnati	Boston	Charleston	Houston	Cincinnati	3400	3950
Cincinnati	Houston	Charleston	Boston	Cincinnati	3225	3950
Cincinnati	Charleston	Boston	Houston	Cincinnati	3505	4390
Cincinnati	Houston	Boston	Charleston	Cincinnati	3525	4390
Cincinnati	Boston	Houston	Charleston	Cincinnati	3600	4400
Cincinnati	Charleston	Houston	Boston	Cincinnati	3690	4400

discovered that the speed limit (for trucks) ranged between 55 and 65 mph. Assuming 60 mph as Mr. Joe's average driving speed, they calculated the total time needed as around 51 hours (distance/rate = 3070 mi./60 mph ≈ 51 hours).

The next question that students answered was this: When should Mr. Joe start so that he can deliver the merchandise on time?

Student J: If the stores close at, say, 5:00 p.m. on New Year's Eve, then we have to deliver the phones before that time. Let's aim for 4:00 p.m. on December 31. Minus 24 hours [is the] 30th at 4:00 p.m.; minus 24 hours [is the] 29th at 4:00 p.m.; minus 2 hours leaves us at 1:00 p.m. on December 29.

Student K: But wait, he cannot drive nonstop. He has to take some breaks for eating and going to the bathroom, and sleeping.

Student J: He may also stop to fill the gas tank.

Student I: There may be accidents on the way, too. So, we have to factor in time for such things. Let's say, bathroom breaks take 3 hours, eating and filling gas take 6 hours, and two nights of sleep is 16 hours. So, the total is 25 extra hours.

Student K: Let's add 5 more hours for other things, like delays and accidents. So, the total is 30 hours to add to 51 hours, which is 81 hours.

Student J: Then we go back 1 day (24 hours) to 1 p.m. on December 28 and go back 6 more hours that day: 12, 11, 10, 9, 8, 7. So he should start at 7:00 a.m.

Student K: No, it will be 8 a.m. since we are going back from 1:00 p.m, and we have to stop at 8.

The students were positive that Mr. Joe would approve of this answer and their findings.

These examples show evidence that students engaged in the steps of the CCSSM modeling framework while completing the tasks: making sense of a situation; determining given and needed information; making assumptions; developing mathematical representations, such as tables, equations, formulas, and charts, and using them to find a solution; interpreting the results in the context; and validating their findings. Students realized the importance of engaging in iterative cycles of modeling to sufficiently address a modeling task, and this realization holds a key to their successful engagement.

IMPLICATIONS FOR PRACTICE

The two examples presented here illustrate that interactions of community, classical knowledge, and critical knowledge are instrumental in developing an authentic modeling task and in fostering a fruitful modeling experience for students. We use PSTs' work samples, discussions, and reflections to argue for promoting and explicitly connecting modeling and critical mathematics education. Several tangible outcomes resulted for the preservice teachers:

1. They developed an understanding that although relatively few preexisting modeling tasks are available for middle school students and although developing such tasks can be challenging, they are up to this challenge and that such knowledge is empowering.
2. They learned that supporting students through their engagement in a modeling task is critical. Giving too little (or too much) information may result in a procedural or a simplistic answer that completely ignores the contextual parameters. Asking the right questions, maintaining a sustained focus on the context, allowing students to gather additional information as they see fit, and being willing to accept a reasonable line of thought are all keys to a successful implementation of a modeling task.
3. Their perceptions of who is viewed as being a mathematician changed after witnessing the transformative properties of critical mathematics education. In the

Students realized the importance of engaging in iterative cycles of modeling to sufficiently address a modeling tasks.

PSTs' view, Pam and Joe became credible sources of knowledge who were able to demonstrate the use of increasingly sophisticated mathematics.

4. They moved from thinking of modeling as something that only secondary students can do to recognizing that elementary and middle school students can also engage in this activity. PSTs' view of modeling was transformed.
5. These teachers contributed to the knowledge base by developing modeling tasks and discrediting the myth that meaningful math cannot coexist when we teach using a critical math perspective (Gutstein 2007).

Let's Chat about Empowering Mathematicians!

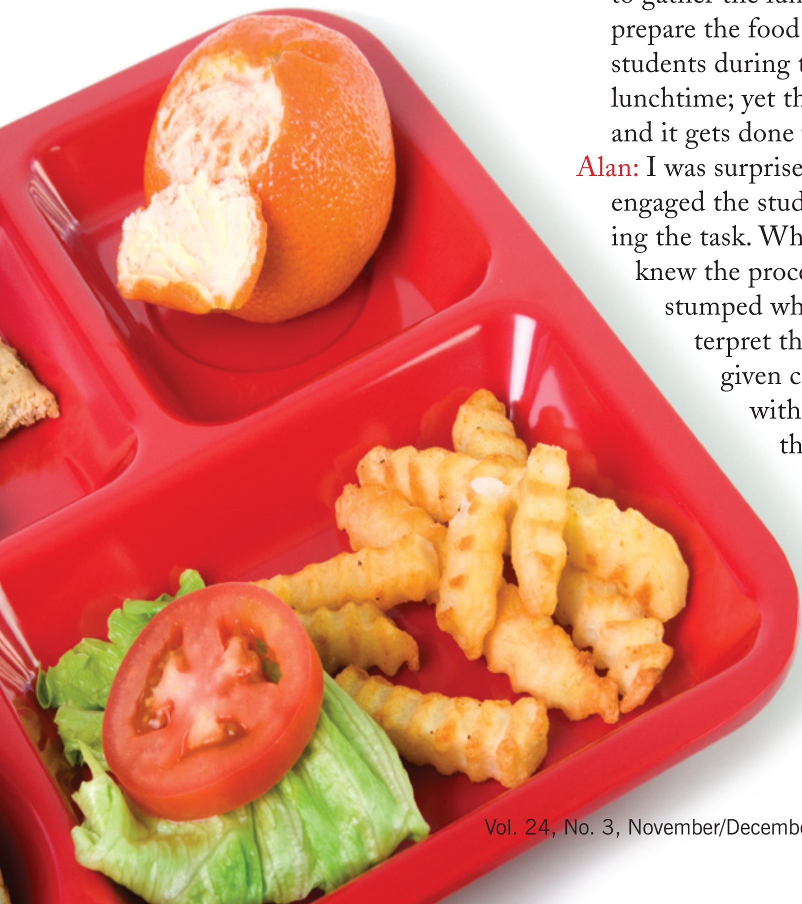
On Wednesday, November 21, 2018, at 9:00 p.m. ET, we will discuss "Empowering Mathematicians through Modeling" (pp. 138–46), by Lisa L. Poling, Nirmala Naresh, and Tracy Goodson-Espy.

Join us at #MTMSchat.

Engagement and interactions with Pam and Joe resulted in authentic modeling tasks. This process was empowering for the PSTs, their collaborators, and students. This gave PSTs much-needed confidence in their competency to develop modeling activities and thereby enhance content and pedagogy.

Sam: From speaking with my mom and from reviewing the documents, I can tell that the workplace is hectic, and that the workers do not have much time to gather the lunch count and prepare the food for each group of students during their designated lunchtime; yet the work gets done, and it gets done well!

Alan: I was surprised to see how engaged the students were during the task. While the students knew the procedures, they were stumped when asked to interpret their answer in the given context. However, with a little guidance, they were able to do it. They were thrilled to know that I developed this task in collaboration with Mr. Joe. I too have begun to question my own perceptions on mathematics and my view of who can and cannot have ownership of mathematics.



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