LEGOS:

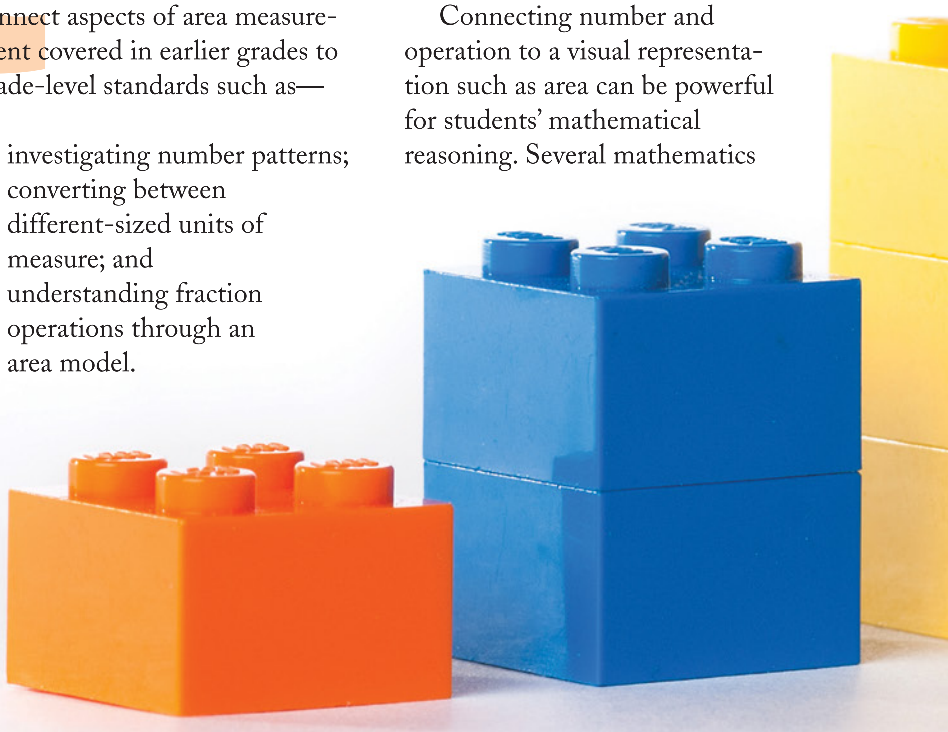
Use those multicolored linking bricks to help students connect measurement with an understanding of number and operations as well as fractions.

The fifth-grade curriculum presents many opportunities to use students' prior mathematical knowledge as a way to bridge new and more difficult mathematical ideas. In this article, we document an area tiling task given to fifth-grade students to connect aspects of area measurement covered in earlier grades to grade-level standards such as—

- investigating number patterns;
- converting between different-sized units of measure; and
- understanding fraction operations through an area model.

We chose a tiling task because the concept of measuring area is familiar to fifth-grade students. The area model is interwoven through many of the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010) and can act as a common thread for discussing and relating mathematical ideas that are more complex.

Connecting number and operation to a visual representation such as area can be powerful for students' mathematical reasoning. Several mathematics



Linking Units, Operations, and Area

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Fig. 1 Students were presented with a LEGO task in which they had to determine how many bricks would cover a square mat.

The LEGO Company is designing a jumbo kit that they would like to fill with enough of each type of brick piece to cover a green mat.

- How many of each type of brick will they need to include in the jumbo pack?
- What are different strategies or techniques you could use to find solutions? Why do they work?

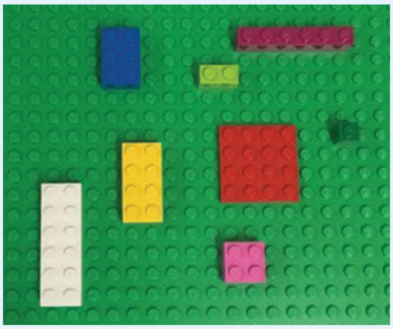


Fig. 2 Students recorded all their work.

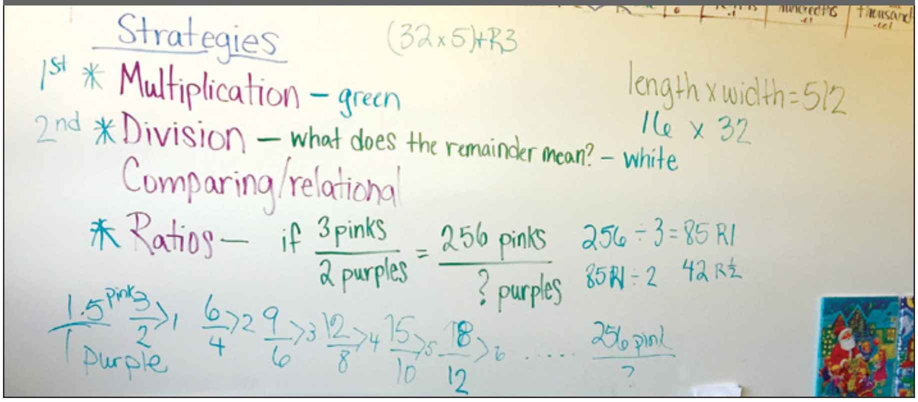
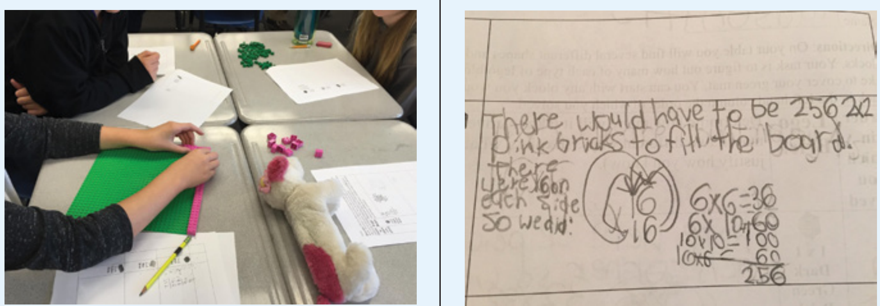


Fig. 3 Some students used a multiplication strategy and what they knew about finding the area of a rectangle.



education researchers, exploring this topic from different perspectives, have argued that making this connection is important, but it does not happen enough. From a *measurement perspective*, Battista (2003) argues that many students learn how to measure without making "connections between an enumeration strategy and the spatial structuring on which it is based" (p. 898). To be meaningful, students must understand how the calculations they are carrying out relate to the space and the units involved. From a *rational number perspective*, Lamon (2007) also suggests that measurement situations can act as a context for students to examine fractions in relation to the whole as well as how units relate to one another.

The LEGO® task stemmed from our research surrounding preservice elementary school teachers' (PSTs') conceptions of area tiling tasks (Wickstrom, Fulton, and Carlson 2017). In our research, we found that PSTs use five distinct, correct strategies when reasoning about tiling a space: counting, multiplication, addition of parts, compares unit, and division. From related research, we know that elementary school students use similar strategies (Barrett, Clements, and Sarama 2017). This task can be used with elementary school students to elicit different strategies through an area investigation and to foster connections between representations and numerical strategies. The task provides opportunities for mathematical discourse to occur among multiple viewpoints.

THE TASK

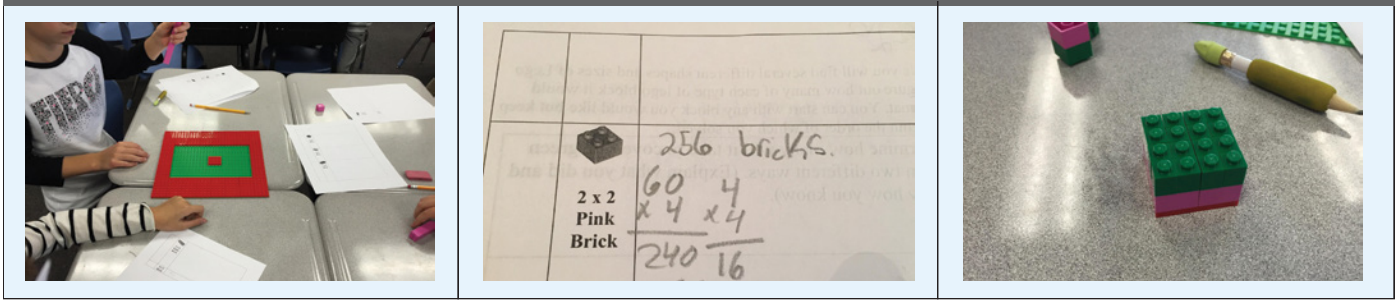
Nonstandard units, such as LEGO bricks, are interesting tools with which to explore area measurement because there is no inherent unit of measurement (e.g., an inch, a foot). Students must make decisions on defining the unit of measure, describing the pieces, and deciding whether units are related to one another. In addition, some bricks do not tile the space nicely and can be helpful in eliciting ideas about fractions and remainders. Although LEGO bricks are three dimensional, we have found that their tangibility helps students visualize the process of tiling a two-dimensional space.

Considering our students' backgrounds and interests, we began the first day of the lesson by posing the task shown in **figure 1**.

From our experience assigning this task, we began the lesson anticipating five different strategies.

1. Counting strategy: The student counts all the bricks needed.
2. Multiplication strategy: The student counts the bricks needed

Fig. 4 These students used a strategy of comparing units.



along the length and the width and multiplies to determine the total.

3. Addition-of-parts strategy: The student divides the tiling space into separate regions, finds the number needed for each region, and adds together the number needed for each region to determine the total.
4. Compares-units strategy: The student compares one brick to another to determine the number needed.
5. Division strategy: The student divides the number of studs on the space by the number of studs on the brick to determine the number needed.

Our goals were for students to make connections between calculations and visual representations and to work with classmates to understand multiple strategies throughout the lesson. In addition, we found that the transition from square bricks to rectangular bricks pushes students from mathematics they are comfortable with (e.g., finding the area of rectangular figures using square units) to grappling with new mathematics (e.g., patterning and using partial LEGO bricks).

DAY 1

Before enacting the lesson, we discussed how to structure the activity, questions to pose, and what we thought students might do. We decided to scaffold the lesson by supplying students with square bricks first (4 × 4, 2 × 2, and 1 × 1) because they tile the board completely. This elicits ideas about comparing units, multiplication, and division, and it is similar to tasks that students have done before. After students discussed their strategies for using square bricks, we intended to regroup and work with rectangular bricks.

After we launched the task, students began working immediately with the LEGO bricks, and we began observing and questioning them about their strategies. As groups reported interesting strategies, we often had them share their findings with the entire class to document what they had done (see **fig. 2**).

Multiplication Strategy

About half the groups used the formula for the area of a rectangle to help them determine how many units it would take to cover the mat. For example, one group noticed that it would take 16 pink (2 × 2) bricks to go across the length of the green mat and 16 pink (2 × 2) bricks to go across its width. Group members said they used what they knew about finding the area of a rectangle to multiply the side lengths together and concluded that it would take 256 pink bricks to cover the mat (see **fig. 3**). They used similar strategies for the other sizes of square bricks.

Division Strategy

We encountered two groups that divided 1024 by 4 to determine the number of pink (2 × 2) bricks. After being questioned further, students explained that they saw the studs on the mat and used multiplication to determine that 32 rows of 32 studs make 1024 studs. They also observed that each brick had 4 studs and conjectured that if they divided 1024 into groups of 4, then they could determine how many pink blocks they would need.

Compares-Units Strategy

We also noticed that members of one group found that they could look for relationships between bricks. They started tiling the space with the red (4 × 4) bricks and attached smaller bricks, like the pink, on top. For example (see **fig. 4**), one student stated, “First we covered the whole board in red squares and found out it took 64 red squares. Next we multiplied 64 by 4 because we saw it would take 4 pink bricks for 1 red brick.” Group members noticed patterning when they layered the bricks and used multiplication or division to determine the number they would need for each type (see **fig. 4**).

This group of students also pointed out that although it seemed to them that the red (4 × 4) brick should be twice as big as the pink (2 × 2) brick because 4 is two times bigger than 2, a red brick is really four times as big as a pink one (see **fig. 5**).

As we had anticipated, the first

Fig. 5 Students' comparison of a 2 × 2 pink brick and a 4 × 4 red brick shows that they realized an important point about scaling.

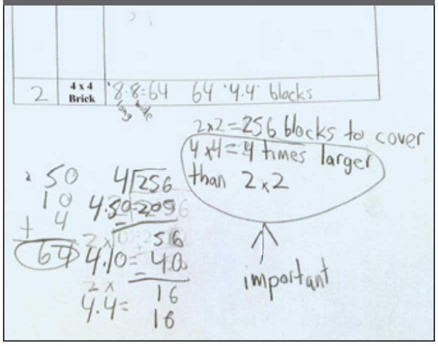
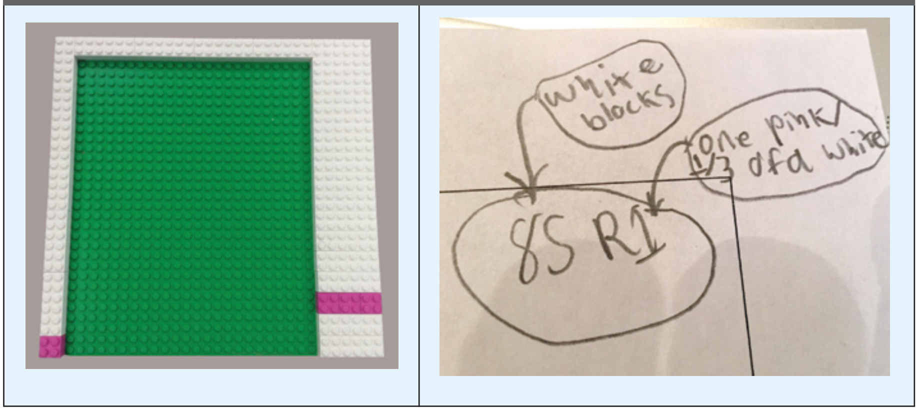


Fig. 6 This work is from a group that tiled a 2 × 6 LEGO brick and then discussed remainders.



day of the lesson was fairly straightforward. Recall that our purpose was to elicit strategies so students could see different, yet equally valid, ways of solving the problem and could make connections between these representations. To summarize their work, we selected groups to present the strategy they had employed. As they presented, we asked students to describe why their strategies worked and how their strategies connected to multiplication, division, or comparisons.

Students had two major realizations as they discussed. First, they realized that LEGO bricks could be described by the number of studs (as described in the division strategy). Some groups had not considered the small unit when describing the brick. One student stated, "The green mat and all our LEGOs covering the mat should share the same number of studs." They also realized that once they found the total number for one brick, they really did not have to consider the green mat again. They could compare the area of a new brick to a known brick (as described in the comparison strategy). The standards that students used to investigate the task on this first day were primarily fourth-grade standards of multiplying and dividing whole numbers. Through their conversations comparing the number of studs on a brick to the number of studs on a mat and comparing different sizes of bricks, students addressed fifth-grade measurement and data standards of converting among different-sized units of measure. We found it was important for students to spend the first day investigating the problem with the square bricks because students were able to understand the problem better; find patterns and relationships between bricks; and articulate the multiplication, division, and comparison strategies. On the second day, students used these findings and strategies to tile the mat with rectangular units.

DAY 2

On day 2 of the activity, we introduced the nonsquare bricks (1 × 6, 2 × 6, 1 × 2, 2 × 4, and 2 × 3) to help elicit concepts from CCSSM, such as multiplying fractional side lengths to find the areas of rectangles, observing number patterns, comparing units, and interpreting remainders. As students attempted the task, we noticed several important mathematical questions emerge as students used different strategies. For all these important questions, which are discussed below, we stopped the class and asked students to consider the problem together.

Dividing and Comparing Units

Several students, implementing the division strategy and the comparison strategy, had questions surrounding remainders. We asked one group to determine the number of white bricks (2 × 6) it would take to cover the green mat. Group members decided to compare the number of pink brinks to white bricks, and they discovered it would take 3 pink bricks (2 × 2) to cover 1 white brick (2 × 6). From there, they decided to divide 256 (the number of pink bricks it would take to cover the mat) by 3 to represent each white brick. When completing this division, they ended with 85, remainder 1. The group asked, "What does this 1 mean? Is it one LEGO brick? Is it one stud? What does it look like?"

We encouraged the class to place the white bricks and then use what they observed to make sense of the remainder. After laying out the LEGO bricks (see **fig. 6**) and splitting the board into different parts, students saw they could get 5 groups of 16 (5 bricks across and 16 down), or 80 bricks. They still had a space on the side of the mat to fill, which they discovered could hold another 5 bricks. When they saw a leftover 2 × 2 space, they filled it with a pink brick.

Now that the students could see where the bricks would go, we asked

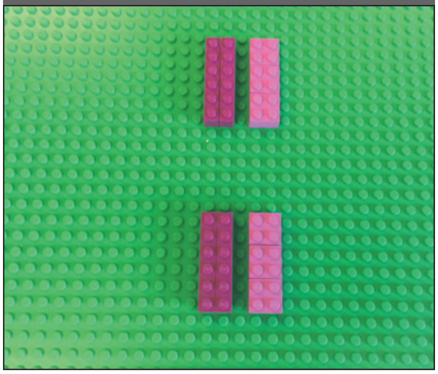


Fig. 7 One group of students struggled with describing the relationship of purple bricks to pink bricks.

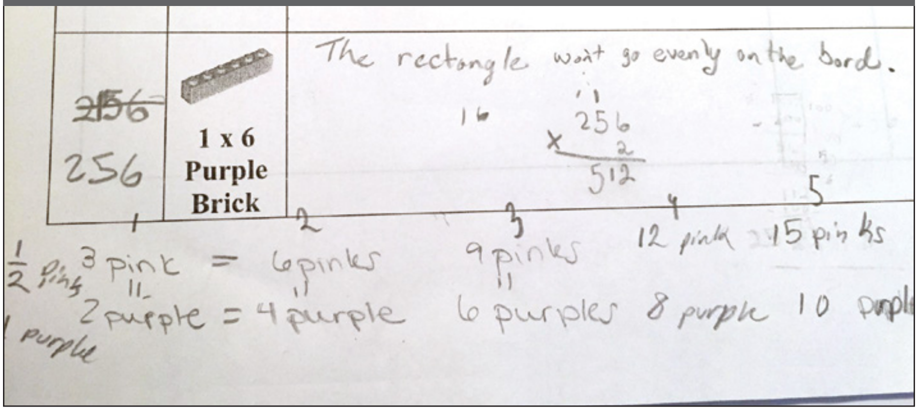


Fig. 8 Trying to keep track of two patterns at the same time prompted this student to create a double number line.

them what they thought the remainder represented. One student thought it could be interpreted as 1 pink brick. Another student saw that it took 3 pink bricks to make 1 white brick, so the remainder could also be thought of as 1/3 of a white brick. At the end of the conversations, students agreed that we could interpret the number needed as 85 and 1/3 white bricks. Moreover, another student stated that we could think about the relationship as 5 1/3 × 16 because it takes 5 and 1/3 groups of bricks to go across and 16 bricks down.

To challenge students and address standards surrounding fraction multiplication, we envision teachers asking students to consider the multiplication strategy explicitly as a second method for all bricks. Students would have to wrestle with questions about partial groups of bricks and determine what an appropriate denominator would be, similar to the previous conversation about why 3 is the denominator with the pink and white bricks.

Comparing Units Strategy and Connecting to Ratio and Proportion

Many students found success in comparing the pink (2 × 2) unit to the yellow (2 × 4) unit and the lime-green (2 × 1) unit by using multiplication or division. However, one group discovered a pattern that was a little harder to articulate. They stated, "We notice that 2 purple (1 × 6) bricks are the same as 3 pink (2 × 2 bricks)." (See **fig. 7**.)

Even though students do not learn about ratio and proportion formally until sixth grade, we took time to have a discussion and see what students could infer about the relationship. Students began by creating their own version of a double number line (see **fig. 8**) while trying to keep track of both patterns at the same time. They saw that as the pink bricks increased by 3 each time, the purples increased by 2.

We asked students what other patterns they saw. A student stated that he saw that the gap between the numbers changed by 1 each time. He explained that 3 pink bricks are 1 away from 2 purple bricks, 6 pink bricks are 2 away from 4 purple bricks, 9 pink bricks are 3 away from 6 purple bricks, and so on.

Another student exclaimed, "The purple is one-and-a-half times as big as the pink." He described the pink brick as having 4 studs and the purple brick, 6 studs. Students continued the pattern until they found that 255 pink bricks are the same as 170 purple bricks, but we still had 1 extra pink brick of space to fill. Similar to the previous problem, students reasoned that the 4 studs on the pink brick resembled 2/3 of a purple brick.

This strategy can be revisited with students in the sixth and seventh grades as an extension of the task to discuss ratio and proportion more formally. To investigate the bricks, we could ask students to compare bricks that differ in only one dimension (e.g., a 2 × 3 and a 1 × 3 brick) to make a conjecture about the relationship between the size of the bricks and the number needed. Then we could ask students to conjecture about the relationship between bricks that differ in both dimensions (e.g., a 1 × 4 and a 2 × 3 brick). Investigating relationships between different sizes of blocks through ratio or proportion will allow students to explore the sixth-grade ratio standards or seventh-grade proportion standards defined in the Common Core.

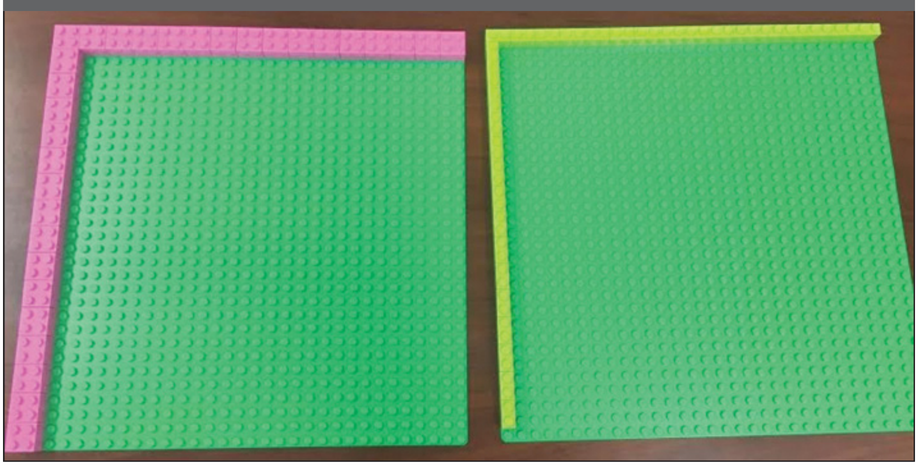
The Disappearance of Multiplication

Midway through the lesson, we noticed that the multiplication strategy had disappeared. Students found difficulty using partial side lengths in terms of multiplication and relied on strategies that were easier to grapple with, such as comparing units. We decided to force students to examine multiplication to build connections.

Table 1 Students used various strategies for the LEGO problem.			
Size of LEGO Brick (in units)	**Number Needed**	**Strategy**	**Alternative Strategy**
1 × 1	1024	**Multiplication:** I can multiply 32 × 32 because I see 32 rows with 32 bricks.	**Counting:** We can count the studs.
1 × 2	512	**Comparison:** It will take half as many compared to the 1 × 1 brick because it is twice as big.	**Division:** There are two studs on each brick, so we could take the number of total studs, 1024, and divide by 2.
1 × 6	170 2/3	**Addition of parts:** We can get 5 across and 32 down, which is 160 bricks, but we still need to count and cover more of the board. That portion is another 10 and 2/3 pieces.	**Comparison:** The 1 × 6 is 3 times as big as the 1 × 2. I can take the number of 1 × 2 and divide by 3.
2 × 2	256	**Multiplication:** I can multiply 16 × 16 because I see 16 rows of 16 2 × 2 bricks.	**Comparison:** The 2 × 2 bricks are 1/4 of the size of the 4 × 4. We can take the 4 × 4 and divide by 4.
2 × 3	170 2/3	**Comparison:** The 2 × 3 brick is the same number of studs as the 1 × 6 brick, so it should take the same amount.	**Division:** The 2 × 3 brick has 6 studs, so I can take the total number of studs, 1024, and divide by 6.
2 × 4	128	**Multiplication:** I can see that it will take 16 bricks to go across and 8 bricks to go down, so I can multiply 16 × 8.	**Comparison:** The 2 × 4 bricks are twice the size of the 2 × 2 bricks. We can take the number of 2 × 2 bricks and divide by 2.
2 × 6	85 1/3	**Addition of parts:** We can get 5 across and 16 down, which is 80 bricks, but we still need to count and cover more of the board. That portion is another 5 and 1/3 pieces.	**Division:** The 2 × 6 brick has 12 studs, so I can take the total number of studs, 1024, and divide by 12.
4 × 4	64	**Multiplication:** I see 8 rows of 8 bricks, so I can multiply 8 × 8.	**Counting:** We can lay the bricks out on the board and count how many we need.

We asked students to examine the lime-green bricks (1 × 2) and write a multiplication statement that matched the work they had done comparing units. We found that students initially wanted to measure the lime-green bricks the same way they had measured the pink bricks, calculating that it would require 16 × 16, or 256, lime-green bricks (see **fig. 9**).

Fig. 9 Ignoring the width of the blocks led to an error.



When students realized that their strategy did not yield the same answer as the division strategy, we asked them to think about why they were not getting the same answer. After 5–10 minutes of brainstorming, students realized it was not 16 groups of 16 for the lime-green bricks, but rather 16 groups of 32. When we asked why, one student explained, “We have to think about both our length and our width. Before, when we laid it out as 16 × 16, we were really doing length times length and ignoring the width.” His group further explained that the lime-green bricks (1 × 2) were relatives of the pink bricks (2 × 2) and the dark-green bricks (1 × 1). The width of the lime-green brick is the same width as the pink brick, so it will take 16 units across. The length of the lime-green brick is the same as the dark-green brick, so it will take 32 units down. With square units, we do not have to consider the orientation of the length and the width because they are the same.

Students went back to the definition of multiplication to relate the number of groups and number in a group to other bricks similar to the white (2 × 6) block discussed previously.

Let's Chat about Linking Units, Operations, and Area

On Wednesday, April 17, 2019, at 9:00 p.m. ET, we will expand on "Linking Units, Operations, and Area" (pp. 338–46), by Megan H. Wickstrom, Elizabeth Fulton, and Dacia Lackey.

Join us at #MTMSchat.

CONCLUSION AND REFLECTION ON THE LESSON

At the end of both days of instruction, we concluded by having students share solutions as well as thoughts about different strategies. **Table 1** shows common responses from students, but there are many other possible ways to arrive at these solutions. For this particular task, we had students share the strategy they thought most efficient for a specific brick and explain their choice. For example, the multiplication strategy is usually easier to use with square bricks rather than rectangular bricks because they tile the board using a whole number of groups. Then, we often challenge students to consider and apply a secondary strategy.

In our discussion, students reflected on two ideas more broadly: efficiency and attributes. In discussing efficiency, one student stated, "The division strategy is easier to use if you have a LEGO that is not square," but another student commented that although the division strategy may be easier, it does not show us where the bricks will go, which the multiplication strategy does. Students commented that different strategies illustrate different aspects of multiplication, division, and visualization. They can use different strategies to check their work or understand the task from a different perspective.

A second realization was that the strategy that students used was tied to different attributes of the LEGO. The multiplication strategy allowed them to think about the problem in terms of the side length of the space in comparison to the side length of the brick. The division and comparing-units strategies allowed them to think about the problem in terms of area of the brick related to area of the space. For example, some students noticed it would take the same number of 1 × 6 bricks as 2 × 3 bricks because they have the same area. Students were able to link their understanding of operations to the physical attributes of the brick and the space.

Physical tools can be powerful in helping students create mathematics and understand more deeply the strategies they employ. The LEGO bricks elicited and extended students' thinking about number and operations in relation to area measurement. The task gave students an opportunity to use known mathematics to connect

The LEGO bricks elicited and extended students' thinking.

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to grade-level standards and beyond as well as to visualize the mathematics in their strategies through physical representations.

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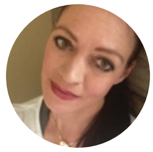

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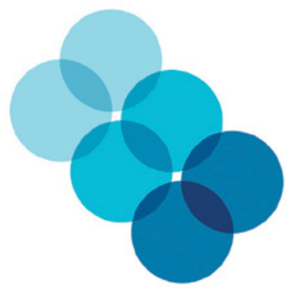

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