CHAPTER

2

The base-ten structure of numbers

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In chapter 2, we continue to investigate students' understanding of how numbers are decomposed, but we take a step back for a different perspective. Rather than examine students who are working on computation, we consider students who are learning about how the components of a number relate to written numerals. It is easy for adults to see, for example, that the number 706 represents 7 hundreds, 0 tens, and 6 ones; or 6 hundreds, 10 tens, and 6 ones; or 70 tens and 6 ones, and so forth. However, when we study students who are in the process of learning, we discover that these ideas are not so simple. As you read chapter 2, take notes on the following issues:

- Some students in the cases offer incorrect answers. Look for elements of sound reasoning in their thinking. If the students are working from a logical position, where does their thinking go awry?
- In some cases, students discuss their own new insights. What are these insights, and how do they come about?

- How are the issues that sixth graders confront related to those of the primary-grade children? What new questions about place value arise for the older students?

After reading the chapter, reread this introduction.

case 6

Number of days in school

Dawn

KINDERGARTEN, DECEMBER

Each morning, as my kindergarten class gathers on the meeting rug, we run through a routine that helps set our day in motion. This set of rituals includes taking attendance, working with our classroom calendar and weather graph, and recording how many days we have been in school. It is amazing how much mathematics is involved in these activities, and I am often astounded by the thoughtful responses five- and six-year-olds give to such questions as "What number should we record on our days-in-school chart today?"

This morning, our sixtieth day of school, we turned to our days-in-school chart. Over the years, I have spent a lot of time thinking about the optimal way to record this data with kindergarten students. Ages ago I used a number line that spanned the top of the chalkboard, but found that this was too physically removed from the children, not to mention extremely cumbersome for me. Having seen the merits of using hundreds boards when working with older students in the past, I wondered if this type of grid might have a place in the kindergarten classroom as well. About ten years ago I made a switch to this type of recording system for tracking how many days we have been in school. Every day I record the number on our 10-by-18 grid, and the child who is the calendar helper adds one seashell to a cup that is kept nearby. This provides a set of concrete objects that corresponds to the number being logged on the chart. From time to time, the children count this set of shells, and we then compare our two types of recording systems to connect the quantity of shells with the number we count, read, and write. If necessary, we then adjust our data.

To begin our class discussion this morning, I asked, "What number should I write on our chart today?" Hands shot up and I began writing responses on the board next to our chart, hoping to allow as many children as possible to respond before recording the right answer on the grid. Often this type of discussion yields some interesting discoveries. Today was no exception.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	

Andrew: I think it's fifty-ten. [*I wrote 510 on the board.*]

Josef: Sixty. [*I wrote 60.*]

More hands shot up, and Bianca, Jared, Rhea, Terry, Toshi, Pat, John, Sione, Susan, and Brady all responded "Sixty." Some children added other comments, too.

Toshi: I know it's sixty. I just know it is.

Jared, John, and Sione were equally emphatic. Susan seemed a little less sure, but apparently wanted to go along with the general consensus. Still more responses kept coming as more hands went up.

Tamika: Forty. [*I wrote 40.*]

Jerrel: Eight. [*I wrote 8.*]

Nina: Seventy. [*I wrote 70.*]

"How can we find out which number we should write today?" I asked to move along our discussion.

Bianca: Counting, so we could know what comes next.

Josef: All the numbers are in front of it, 'cause 6 comes after 5, you know.

As Josef spoke, he walked up to the chart and pointed to the column of numbers on the right-hand side, stopping at the empty box under the 50.

Jared: Yeah, see, zero all the way down.

Andrew: Don't forget the ten.

Once again physical involvement seemed a necessity as Andrew moved up to the chart and dragged his finger across the row with the numbers 51, 52, 53, 54, 55, 56, 57, 58, and 59. Andrew seemed to be making use of the number (counting) sequence; thus his response of "fifty-ten" (510) made perfect sense.

Brady: But it's a 6. [*Again, moving up to point at the chart, Brady's finger slid down the right-hand column just as Toshi's and Jared's had as they spoke.*] See … 1, 2, 3, 4, 5, … and 6 goes here 'cause you're counting down.

Toshi: See, it's 60. I know it is.

Sitting toward the back of the group was Norman. Though he didn't speak out in front of the group, my aide was taking notes and later related to me that he was making comments under his breath.

Norman: It can't be 8. We already had that one. … Fifty-ten looks like five hundred and ten. That is too big to go there.

Because we needed to move on, I ended this discussion by reminding the children that another way to check was to use Bianca's suggested strategy of counting. Together, as I pointed to each number on our chart, we counted from 1 to 60 and agreed that 60 was the number to be written in the box for today. As we finished, there were two final comments:

Josef: Are we going to get to 100?

Brady: Yes, 'cause look, we can count by tens … 10, 20, 30, 40, 50, 60. I think we will get to more than 100.

As I reflect on these events, I am particularly taken by the ease with which some children are able to connect to the systematic way we use numbers. At the same time, I know for sure that not all children in my class are making sense of this experience at this time. Trying to provide opportunities where young children can investigate numbers in a meaningful way is a challenge.

case 7

One hundred and ninety-five

Danielle
GRADE 1, APRIL

In my classroom we do work on estimation, which gives the children experience in making reasonable guesses as well as the opportunity to count objects. Last week we were working with a bag of jelly beans. The students were trying to guess how many jelly beans they could hold in one hand, and then compare that to the actual amount they could hold. We also discussed whether the amount would be the same every time they took a handful. Next we estimated the amount of jelly beans in the whole bag. After we all made our predictions, I opened the bag and spread the jelly beans on a flat surface to give the children the chance to decide if they were satisfied with their estimate, to adjust it if needed, and to say if it was too high or too low.

The children in my class had not done any formal work with place value, although we had counted objects well beyond one hundred. Several times students had asked how to write a number greater than one hundred and I had showed them. But as yet, we hadn't discussed numbers containing three digits. As we finished up with the jelly-bean counting, we learned that the bag contained a total of 195 jelly beans. I asked if anyone knew how to write that number using numerals. Suggestions included the following:

1095
10095
195
1395
1295

The children who gave the first two responses were able to explain them somewhat. They heard "hundred" and remembered something about one hundred and zeros. They couldn't be more specific than that. Nathan, who gave the third response, said 195 looked like that and he just knew it. The children who gave the fourth and fifth responses said it was a big number and that's how you write big numbers.

I asked the children to look at these numbers, think about 195, and talk to each other about it. I also said that one of these was the right way to write it with numerals.

The children explained their answers to each other and arrived at the conclusion that the correct written form is 195. I don't recall the entire discussion, but I do remember that 1395 and 1295 were immediately discarded because they never heard anything about 3 or 2 in one hundred ninety-five.

Nathan, who was the first one to offer 195, was able to convince everyone that he was right, which, of course, he was. I was curious about how he knew this.

When I suggested to Nathan that we spend some one-on-one time discussing how he knew about 195, he was more than pleased to spend the time with me alone. As we sat down together, he was all smiles.

Teacher: Show me how to write one hundred ninety-five. [*Nathan writes 195.*] Why this way?

Nathan: I first thought it was [*writes*] 1095, but that would be ten hundred ninety-five.

Teacher: Why?

Nathan: Take away the 95 and the 1 and 0 is ten.

Teacher: What made you think of writing 195?

Nathan: When I see 100 and go up, you take away the zeros. When you go up 100, 101, 102, you keep going up. You take away the zeros.

Teacher: Why do you do that?

Nathan: If you don't take away the zeros it would be 10095. That would be 100 and 95 put on the end.

Teacher: How do you know about numbers?

Nathan: I can count up to 500. [*Starts to count, says 49, 30, self-corrects to 50, 51 ... Eventually, I ask him to stop.*]

Teacher: When you look at 195 [*I write 195*], what do the parts tell you?

Nathan: [*Makes circles around the numbers.*] Nineteen—if you take away the 5, just plain 5. If you take away the 19 [*he writes 195 again and circles parts of it*], 95 or 19. The middle can go either way. Plain 1, if you take away the 95, says "hundred."

Teacher: Can you write other numbers? [*I dictate the numbers 372, 249, 107, 950, and 401, and Nathan transcribes them as 372, 249, 1007, 950, 4011. Then he changes 4011 to 401, but doesn't comment on or change 1007.*]

I wish I understood better how to interpret what Nathan knew. It appeared that he had some initial thoughts about place value. For example, he saw that numbers have different values in different situations. How much did he understand about these values? A few weeks later, we were working with base-ten blocks. Nathan was trying to figure out how many days were in summer. He knew right away that 30 equaled three rods of base-ten blocks. He lay out three rods for June, three rods and one cube for July, and three rods and one cube for August. To find the total number of days, he didn't count by tens, but instead counted each individual space on the rods. Where do he and I go from here?

case 8

Who invented zero anyway?

Muriel
GRADE 2, APRIL

My second graders and I were looking at the hundreds chart set up in a 10-by-10 array. I had imagined leading a discussion toward the idea that moving down one space on a hundreds chart is actually adding ten. As we got into the discussion, I found that it is, of course, a complicated idea. Even if I tell my students that it just works this way, they don't get it in any kind of meaningful way. Besides that, several of my students raised a very different idea—an idea about their understanding of zero. Following is a portion of our discussion.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Beth is describing for me—and anyone who is listening—why it works to move down one and actually add ten on this particular hundreds chart.

Beth: See the nines? [*She points to the column of 9, 19, 29, 39 …*] This is 0 + 9, 10 + 9, 20 + 9, 30 + 9. The difference between 50 and 60 is 10, so the difference between 59 and 69 is 10.

Quite articulate, I think. Then Beth continues.

Beth: [*She points to 45.*] This has 4 tens and 1 five. These [*pointing to the columns*

with numbers ending in 6 or 7] all have sixes, sevens. [*Now she points to the 10, 20, 30 column.*] This one has zeros. But these aren't quite zero, because …

Her voice trails off, and my mind is racing, "What did she say? 'These aren't quite zero' "?

Teacher: Because why? [*I am trying desperately to figure out in that moment what she is saying and trying to sort out what I might ask next.*]

Beth: Because if zero isn't here [*she points to 60*], then this 6 is only 6. It depends on where it [the zero] is. See, 15. [*She writes 15 on the board.*] This [*points to the 1*] is a ten, not a one. Ten has everything in it up to 9. The ten section has from 10 to 99. The hundred section has—say it's one hundred and twenty-five. [*She writes 125 on the board.*] But you don't write it as 100 or else it would look like this. [*She writes 100205.*]

I'm in that place where I find myself spending a lot of time—hearing many rich ideas and wondering which ideas I should push on. I decide to push on Beth's ideas about the zero thing.

Teacher: You've written one hundred twenty-five separately as 100205. But tell me some more about zero.

Beth: It is sort of zero, but not exactly. [*In 30*] this zero makes it be 30. If this zero weren't there, it'd be 3.

Yessica: I have something to say about how these zeros aren't really zeros.

Attempting to include some other children in the discussion, I restate what Beth has said about the zero at the end of the number not being quite the same as zero by itself. Yessica comes up to the board.

Yessica: [*Writing 07 on the board*] That's 7. [*Now writes 0.7.*] That's 0 point 7. [*And then she writes 70.*] That's 70. Zero represents 7 tens.

I am completely intrigued by these ideas and love the term *represents*.

Teacher: So if this zero [*pointing to 0 in 70*] represents tens, in this number, 79, does the 9 represent tens?

Beth: Do you know what we really mean? Do you know the real thing?

Lately Beth has been responding to my questions as if I already know many of the answers I'm asking her to explain to me. I like to make my questioning as authentic as I can, but the fact is that I usually understand the mathematics that I'm asking the children to think through. Today my questions are framed to find out what the children are thinking, to hear their ideas. I know that the 9 in 79 represents 9 ones, but I'm not sure what Beth thinks.

I explain to her that in my math class for adults, we've talked a lot about zero and what it means to different people, and what it's worth, and whether it's odd or even, for example. And I express my honest interest in what second graders think about zero. This is heard by several other children in class, and they perk up a bit. The discussion continues.

Yessica: On the calculator there's a 07.

Teacher: And then is the 0 worth zero?

Yessica: Yes. [*Other children nod in agreement.*]

Teacher: But not in the 70?

Yessica: Right.

Brian: For just 7, the zero doesn't have to be there, just the 7.

Lamont: There are two ways to make zero. This is the 7 for the tens and it [the zero] makes 70.

Teacher: What's the other kind of zero?

Lamont: For the ones. [*He writes 08.*]

I'm thinking, "Two kinds of zeros? Wow."

Beth: It's like you sort of understand it, but nobody really understands it. Maybe someone will come around and figure it out. And who invented zero anyway?

I laugh and write the question on the board.

Wenona: Yeah, and who invented numbers anyway?

I write this down on the board also.

Teacher: I need to see if I can find any information for you to read.

Several days later Lamont came with delight on his face to tell me that our librarian had seen the questions on the board and said she had a book called *Zero Is Not Nothing*. He eagerly went to the library to bring it back.

The next week, Henry came to me and said sincerely, "You see, Ms. Willis," holding his hands closed and then opening them palms up, "zero means there's nothing. See, there's nothing in my hands. That means zero."

I was thrilled that he had actually kept this issue in his head long enough to either talk to someone about it or come up with that explanation on his own. It was also somehow very touching to me that he seemed to be gently offering me an explanation about something I didn't yet understand.

I guess I've written up this episode for a couple of reasons. One is simply that I love the idea that even a few of my second graders can have this kind of discussion about number. This kind of chewing on ideas is exactly what I most hope and work for in my mathematics class (actually any class). I am genuinely intrigued to have this window into some second graders' thinking about what zero is. I also am thrilled that the assertion is in the air that someone invented this zero thing as well as the particular numbers that we use. It makes them much more accessible and "touchable."

I also wonder how making sense of zero affects children's understanding of place value. Actually, it's probably more to the point to wonder how *not* making sense of zero affects children's 230
understanding of place value.

I am sometimes just overwhelmed with the range of ideas that bombard me in a relatively short discussion. 233

case 9

Groups and leftovers

Donna

GRADE 2, MARCH

At the beginning of March, I was trying to decide how to introduce place value. My goal was to help the children develop an understanding of how our number system works so they would have a solid foundation when we began to work on adding and subtracting that involved regrouping. I adapted an activity I had seen in Van de Walle's *Elementary School Mathematics: Teaching Developmentally* (1990). My version asks the children to take a handful of kidney beans (less than one hundred—we had practiced eyeballing a hundred beans on our hundredth day of school) and count them, filling in the information in a table that started like the following one:

Number in group	Whole groups	Leftovers	How many all together
7			
3			
6			

The verbal instructions asked the children to put each new handful of beans into groups of whatever number was in the first column of the table. For example, if the number in the first column was 7, they had to put their beans into groups of seven and then fill in the rest of the information. I had chosen a variety of numbers to go in the first column, but I made sure that 10 appeared at least three times near the end of the sheet. The planned outcome was to have the children notice a pattern when they put the beans into full groups of ten—a pattern that didn't occur with any other number. I put the children into pairs and they set off to work.

As I circulated through the room, I noticed the children were using several different strategies to accomplish their task. Kathy counted all the beans by ones. Jelani put the beans into groups and then counted, while Ellen decided to "make sure" and check Jelani's work by counting their beans again. Marc and Karla seemed to enjoy the challenge of taking more than a hundred beans and were discussing their outcomes.

Amy and Tuan were the first group I heard remark about the pattern they saw in the tens rows. They were discussing the fact that the number of whole groups and the number of leftovers turned out to be the number of beans that they had all together—that the number in the last column was a combination of the numbers in the previous two columns. At this point, I also heard Kathy, who has a keen eye for patterns, make a similar observation.

Number in group	Whole groups	Leftovers	How many all together
3	7	2	23
7	5	5	40
10	6	7	67
10	3	8	38

The next day, the children finished their work and we came together to put our findings on a whole-class chart. The children offered examples of how to fill each line of the chart. Then I asked if they saw anything on the chart that they wanted to comment on. Many kids noticed what they came to call "the ten trick." I asked them why it worked.

Kimberly tried to explain what she understood: "The tens is what basically does it and the leftovers make it." Several other children tried to explain what seemed to be clear and yet confusing at the same time.

Eventually I asked, "Will it work with any other number of things in a group?" I expected their answer would be no, thinking only about groups of ten and fewer, but I heard something different as the discussion continued.

Sean stated that he thought this "trick" would work with anything that had a ten in it. I wasn't sure just what Sean meant, but several other children began to think about his comment and offered their ideas. Marc said that anything with a "one-zero" at the beginning would work. Sean responded that it worked with a hundred, a thousand, ten thousand, and a million.

Ellen said, "Fourteen groups of ten might not work." Most of the class agreed with this statement, but they weren't completely sure, so we all decided to test it. We took fourteen groups of beans with ten in each group, and I threw in a few leftovers. The children counted the groups by tens, and as we approached 140, the excitement mounted. I recorded our results on the chart and the class was abuzz.

Jelani said out loud, "Why does that work?"

Throughout the discussion, I was thinking about some of the ideas that had come up the week before while we were doing some mental math (arithmetic in our heads). The children had stated the following theory: "Ten plus any number less than 10 and more than 2 is a teen that has the same number at the end." Now I reminded them of their theory and asked if there was a connection between what we were seeing on this chart and that statement.

Marc's answer seemed to sum up my thoughts about teaching place value. He said, "There is a big connection, but it's hard to explain."

case 10

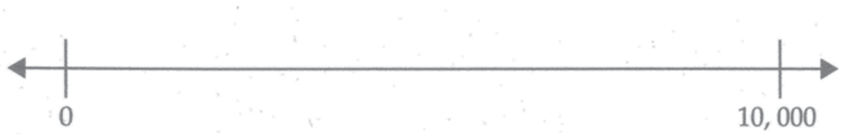
Thinking with number lines

Leslie
GRADE 6, SEPTEMBER

Each year, I begin with a similar goal: to pose mathematical tasks that require my sixth-grade students to use different models to represent and explain their problem-solving strategies. Some students come to our middle school from classrooms with similar expectations and experiences, but for many, explaining and representing their ideas this way is a new challenge.

Our class begins with a daily warm-up, and I thought this might be a good way to get students thinking about the relative size of numbers on a number line. Both confident and struggling students had limited experience with number lines, so I expected to both probe understandings and uncover misconceptions. I posed the following question:

> Where would you place 375 on this number line? Explain how you decide where it is located.



During warm-ups, students are given time to write about their ideas in their notebooks, and then they share with their table partners. What follows is what I saw and heard as I moved around the room to view their work, listen to their conversations, and decide what ideas to bring to the whole group for discussion.

Lucas and Jake both started by marking 5,000 in the middle (figs. 2.1 and 2.2). I listened to their conversation.

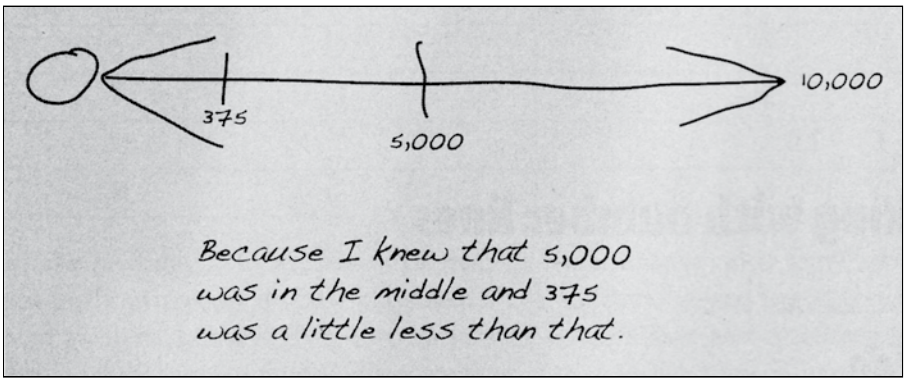


Fig. 2.1. Lucas's diagram and explanation

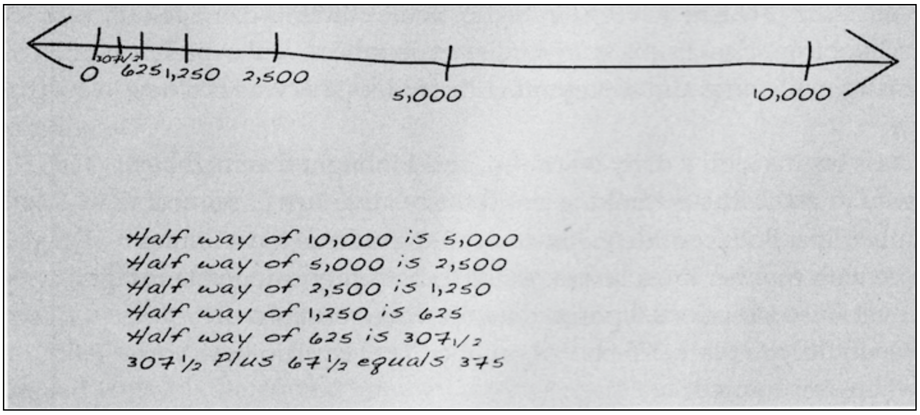


Fig. 2.2. Jake's diagram and explanation

Lucas: Well, I knew 5,000 is half of 10,000, so that's why it is in the middle; then I put 375 between 0 and 5,000 because I knew that 375 is less than 5,000 and more than zero.

Jake: How did you know where to put it? I mean, I know it's less than 5,000, but how much less?

Lucas: I didn't really think about that.

Jake: Well, if you look at my number line, you can see that I started with 5,000 like you did. Then I only had half of the number line to worry about.

Lucas: What do you mean? You have the whole number line like I do.

Jake: Yeah, but once I knew where 5,000 was, I didn't think about this half of the number line again [*pointing to the side greater than 5,000*]. I only looked at this half [*indicates the side less than 5,000*] because I knew 375 would be in this half somewhere, I just didn't know where yet. So I kept cutting it in half until I got really close to 375, and that's how I decided where it went.

I moved on to Tyrese and Anthony.

Tyrese: Well, I knew that 5,000 would be in the middle, so I put it there. Then I thought about how I could divide up the part between 0 and 5,000 so it would be easier to see where 375 would go, because 375 isn't easy. So I decided I would put 1,000; 2,000; 3,000; and 4,000 on the number line. And then I knew where 375 would go.

Anthony: OK, I get what you did. I did it a different way. I started at the 0 and counted by fifties until I got to 400. I knew 375 would be halfway between 350 and 400. Then I thought that my number line would have to be a lot longer because the way it was 400 was too close to 10,000, so I added all of these lines to show that it would be far away.

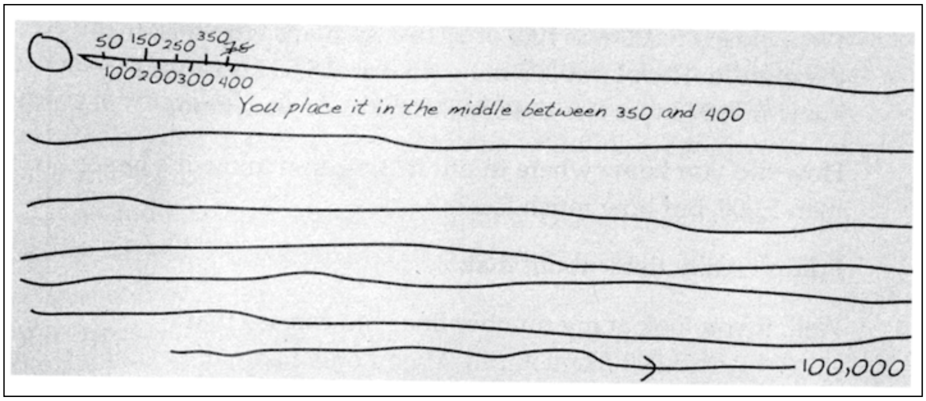


Fig. 2.3. Anthony’s extended number line

Tyrese: Are you allowed to move 10,000?

Anthony: Well, I had to move it, or it would be wrong.

Shaquille and Chris were talking about how their number lines were similar.

Shaquille: Ours look the same at the beginning. Your numbers are by hundreds, and so are mine, if you look at the numbers on the bottom. After I finished with hundreds, I went back and added fifties in between and you didn’t. I stopped when I got to 1,000, but if I had kept going, ours would have been almost the same.

Chris: I kept going to 10,000 because it’s supposed to have numbers all the way across. It has to be all divided up all the way across.

Shaquille: I think if we put yours and mine next to each other, where we have 375 would match.

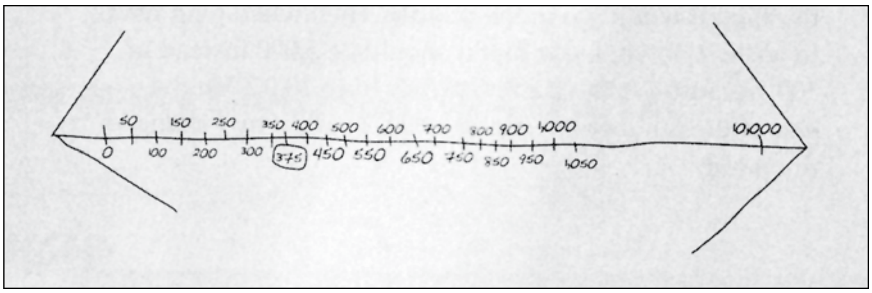


Fig. 2.4. Shaquille’s number line

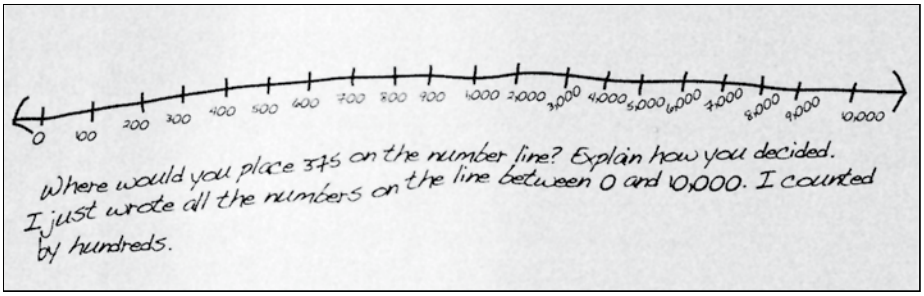


Fig. 2.5. Chris's number line

Olivia explained what she was thinking about as she got started on the task. She constructed several number lines (fig. 2.6).

Olivia: Well, at first I thought that 375 was hard to start with, so I asked myself what would be an easier number to start with, and I decided 500. Then I thought some more, and I realized it would have to be 5,000, not 500, because it's 10,000 instead of 1,000.

Teacher: I'm not sure what you mean. Can you tell us why you were thinking about 500 and 5,000?

Olivia: Well, I was confused at first, because I thought 500 would be an easy number to put on the number line. I was thinking it would go in the middle. Then when I got ready to write it down, I saw that it should be 5,000 instead of 500, because 5,000 is half of 10,000. It's a 10,000 on the end of the number line, not a 1,000. That's why I changed my mind.

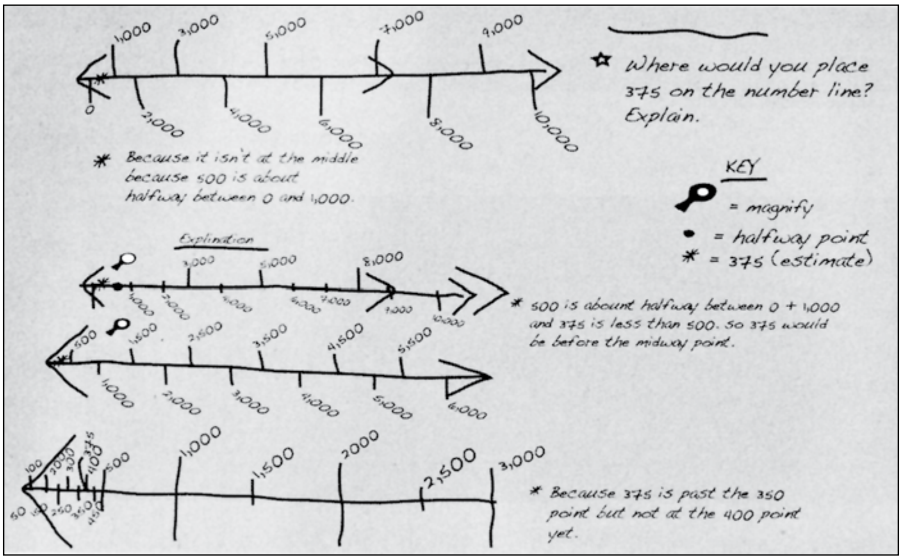


Fig. 2.6. Olivia's number lines

Now that I see the work students did individually and in pairs, I need to think through what has been revealed about what my students do and do not understand. Once I think that through, I can decide which ideas I want the class to consider in whole group. Which students should I ask to present their number lines and what questions should I pose about them?

case 11

How many thousands in 437,812?

Susie

GRADE 6, NOVEMBER

As a math coach, I have been working with teachers and students in lower grades, and I know how difficult it can be for students to acquire an understanding of place value. Because of this work, I have been wondering about older students. Do they struggle with place value issues as well? I recently had an opportunity to explore these questions when a sixth-grade teacher invited me into her classroom. We agreed that I would teach the lesson while she observed, and then we would discuss what we learned.

In order to find out more about student understanding of place value, I wrote the following numbers, each on a separate card:

437,812
6,437,812
36,437,812
536,437,812
1,536,437,812

I started the lesson by holding up the card for the first number. I asked the students if they could tell me the number. Students responded by raising their hands, appearing confident and anxious to share. I asked Rachel if she would read the number, which she was able to do without error.

Teacher: How did you know this was four hundred thirty-seven thousand, eight hundred twelve?

Rachel: Because of the commas!

Teacher: What do you mean, "because of the commas"?

Rachel: The commas break the numbers up, and each part has a different name.

When asked, Rachel explained that "each part" referred to thousands, millions, billions, and so forth. In her explanation, Rachel traced the comma with her finger while saying thousand. She described picturing the number in her mind like the following figure:

437 , 812
Thousand

Most students were able to identify the numbers correctly. During our discussion, I asked several questions to confirm that the students could read and write numerals to one billion, as required by our state core curriculum. When the numbers became increasingly more difficult, I wondered what processes the students were using to correctly identify the number.

Teacher: What were you thinking when you read the number [536,437,812]?

Gabriella: I memorized the columns for place values and I know that 5 is in the hundred-millions column. I was following a pattern.

Teacher: [*Holding up the card for 1,536,437,812*] What about this number?

Trinia: I know that number; it's one trillion, five hundred thirty-six million, four hundred thirty-seven thousand, eight hundred twelve. Wait, I forgot the billions.

Teacher: When you said trillion, what were you thinking?

Trinia: I just got mixed up which place the trillion goes into.

Trinia's comments were typical. I continued to question how the students determined "which place" they were working with in each number. Proceeding with the lesson, I asked the class to look at 437,812 and find how many thousands are in this number. I told the students to think about it for a few minutes and then work with their partners. As they began this task, I walked around the room, listening to their ideas.

Mike: I think there are three.

Ali: Maybe four.

Mike: I think it might be three because we have 4 hundred 37 thousand; I think three might be the right answer.

Anna: I think it's seven thousand, because this [*pointing*] is the hundred-thousand place, and this is the ten-thousand place, and this is the one-thousand place. I think it's seven.

Trent: What makes this hard is that there is a 4 in the hundred-thousand place and a 3 in the ten-thousand place and a 7 in the thousand place. All three numbers are in the space for thousands.

Ali: I think it's 4 or 3 or 7. I don't know for sure, but if you turn all of these numbers to zero [*referring to 812*], this is what you have left.

Mike: I know why the 3 is the right answer! I looked at the number and I saw these are in the thousands place because I knew this is the ones, tens, and hundreds. I knew that these were the thousands, so I looked at it and saw that there were three numbers in the thousands place, so there are three thousands in that number.

The students were struggling more than I had anticipated. They seemed to have a limited understanding of place value. They had learned the name of the place of each digit, but did not understand the value or the relationship of the digits within a number. Why is place value such a difficult concept to grasp? As I pondered this question, I moved to the next group. They stopped their conversation as I approached.

Teacher: What do you think the answer is?

Trinia: We think it's 250.

Teacher: Tell me about your thinking.

Chase: Each of these [*referring to each digit in 437,812*] have ten, so we just times'd each number by ten. Then added the numbers together.

Teacher: I think I understand why you chose to multiply each number by ten, but I'm not sure. Will you explain it to me?

Maggie: Each number is ten times the other number.

Teacher: Does that mean there are 10 tens in one hundred?

Maggie: Yes!

Teacher: Then how many tens are in one thousand?

Chase: Uh-oh.

Teacher: What do you mean, Chase?

Chase: We forgot that each number has to have all of the other numbers times'd.

Teacher: How will this help you find how many thousands are in our number?

Trinia: I'm not sure, but we need to keep working on it.

Although these students still have quite a lot to sort out, I was encouraged to see that they started to make connections between the values of the digits. Now that they seemed to have a more productive direction to pursue, I moved on to the next group.

Sadie: I think it's just 37 thousand.

Teacher: Why do you think that?

Sadie: Because the 4 means hundred thousand, but the 37 just sticks together and makes 37 thousands.

Teacher: Do you mean 37 groups of one thousand, or do you mean 37 thousand?

Sadie: I think 37 thousand, but I'm not sure.

Meagan: I think all we have to do is divide.

Juan: What do you mean?

Meagan: If we want to know how many things are in a number, you just divide. It's like if you have a number, let's say 100, and you want to know how many fives are in the number, you have to divide to figure it out.

Juan: OK, you divide; I'm not going to divide that number!

The students in this group retrieved their calculators and determined there were 437.812 thousands in 437,812.

I found this teaching experience to be quite revealing. Although the students can read and write numbers to one billion, they seem to be missing many ideas about the base-ten structure of numbers. Now I must prepare a meeting with the teacher to discuss what her students understand, what her students still need to learn, and what would be fruitful next steps for her class.