

## Detailed Agenda

### Sharing exit-card comments and the main idea of the session (10 minutes)

#### *Whole group*

Use the first few minutes to share a few of the exit-card comments. You can use this as an opportunity to draw participants' attention to examples of practice 7 or to clarify any misconceptions.

Distribute the overview for the session, giving participants a minute or two to look it over. Then alert participants to the main idea of Session 2. Post this statement and read it aloud:

The value of a number is determined by multiplying the value of each digit by the value of the place that it occupies and then summing. For whole numbers, the value of the place farthest to the right is one; the value of every other place is ten times the value of the place to its right.

Let the group know that place value, while it might seem familiar and commonplace in their experience, is actually quite complex. Assure them that the activities of this session are designed to highlight features of the system. You might suggest that one way to become aware of aspects of the number system is to examine the kinds of errors students make as they work to learn how numbers are spoken and written.

Point out that the posted statement makes clear that multiplication is key to understanding how a multidigit numeral represents an amount. We multiply by 10 to find the next place value. And we multiply the value of a digit by the value of its place to find the value of the number. Suggest that participants keep this idea in mind as they work on the activities of the session. Point out that the statement will remain posted throughout the session.

### Discussion: Norms for learning (20 minutes)

#### *Whole group*

Now that participants have had a chance to work together for a session, this is a good time to collect ideas from the group about how to keep the focus on learning as they work together. Suggest that participants think about these two questions:

1. What do you do to make this a good learning experience for yourself?
2. What do you do to make this a good learning experience for others?

After a few minutes to think, ask the group to offer statements describing the way the seminar should operate to make it a good learning experience for all. Below are a few examples of what these might include:

- Allow time in small-group work for individuals to think before talking. Be prepared for sessions—everyone should have something to contribute. Listen carefully to take in another person's ideas.
- Find ways to disagree without being disagreeable. Start and end each session on time.
- Be open to a new idea or perspective.

Allow just 20 minutes for this discussion. The list does not need to be comprehensive or complete. The activity is designed to help participants know that reflecting on their own learning process and considering the dynamics of the group is a part of the seminar. Some facilitators post the list in the meeting room and encourage participants to add to it as the seminar continues. You should also alert participants that the “Norms for Learning” are intended to support practice 3. If necessary, clarify the difference between critique and criticize.

**Math activity: Multiplying by 10**

(45 minutes)

**Small groups**

(25 minutes)

**Whole group**

(20 minutes)

In this activity, participants will examine what happens when we multiply numbers by ten, one hundred, and other powers of ten, using various models for multiplication as tools. Begin the work by asking the group for examples of ways they can represent multiplication. Make a poster of what is offered. Suggestions might include making equal-size groups, drawing rectangles, drawing arrays, or devising story contexts that involve equal groups. If using such representations seems new to your group, you might want to demonstrate how the same multiplication statement can be shown with each representation (fig. 2.1).

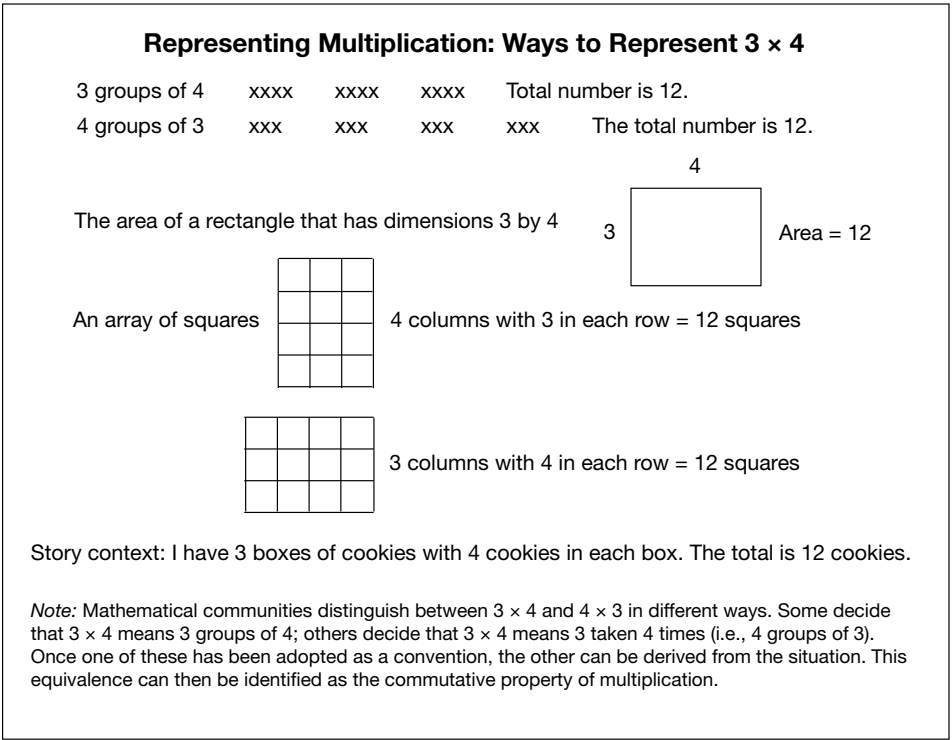


Fig. 2.1. Various representations of  $3 \times 4$

Another feature of this activity is making models to represent numbers beyond the ones, tens, hundreds, and thousands. One way for participants to create mental images for powers of ten is to replicate the geometric pattern used by ones, tens, hundreds, and thousands blocks in the base-ten models (fig. 2.2).

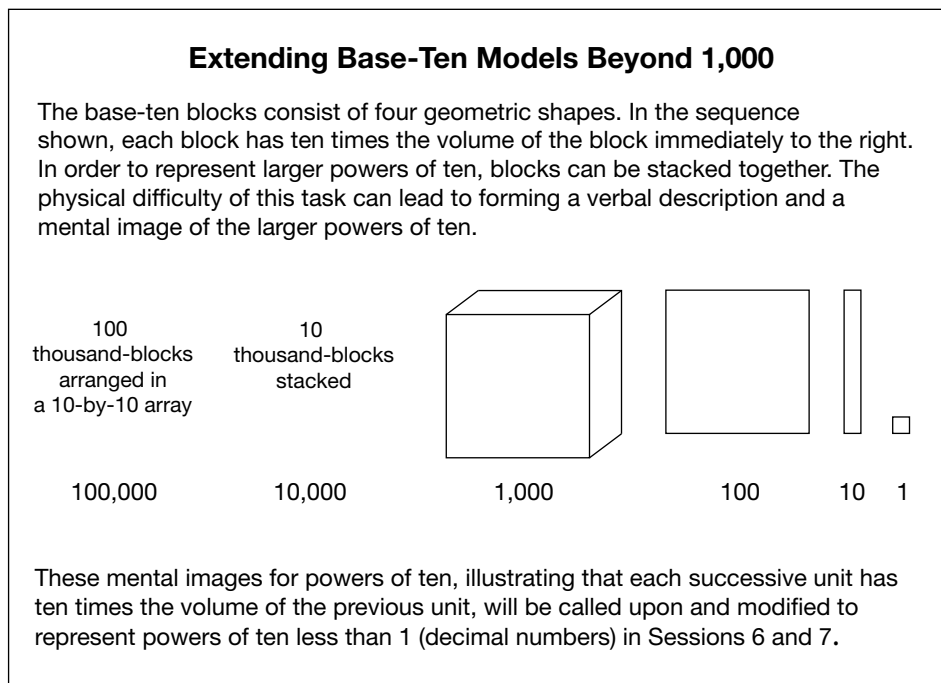


Fig. 2.2. Extending base-ten block models to create verbal descriptions of the larger powers of ten

### Small-group work: Representing multiplication

(25 minutes)

In “Multiplying by 10,” making models for problem 1 is particularly important. If a group is confused about extending the base-ten models beyond the thousand-block, hold up the thousand-cube and ask, “What would ten of these look like?” As you listen to small groups work on problems 2, 3, and 4, ask questions to help participants describe the connections between the quantity a number represents and the way it is written. Ask participants to articulate their own explanations for why multiplying by 10 does not change the sequence of digits. You might also ask small groups to write out numerical approaches, such as repeated addition or the multiplication algorithm, and examine the connections between those and the representations they have made.

During the math activity, ask questions such as, “How does your representation capture or illustrate the base-ten structure of the number system?” After the discussion of question 4, when participants share their explanation of what happens when a number is multiplied by 10, invite participants to reflect on their experience and to note how practice 3 was enacted in their discussion. If participants refer to practice 4, clarify the distinction between modeling with mathematics and using blocks or materials to represent mathematical ideas, as discussed in the mathematical background note “Practice 4, Model with Mathematics: Highlighting a Common Misconception,” page 69.

As you interact with the small groups, take notes of the representations they use. Choose two or three that you want to include in the whole-group discussion about the rule for multiplying by 10. It will be useful to have base-ten models, story contexts, and numerical arguments to compare.

**Whole-group discussion: Sharing representations***(20 minutes)*

There are two main ideas for the whole-group discussion:

1. Sharing representations for multiplying by 10, 100, and other powers of ten.
2. Explaining why the rule “add a zero at the end of a number” works for multiplying by 10.

For the first point, ask participants to share how they mentally pictured  $23 \times 10$ ,  $23 \times 100$ ,  $23 \times 1000$ , and so forth. This discussion should help participants expand their images to include ways to represent numbers greater than 1,000. It also offers the opportunity to highlight the role of concrete materials as a starting point for forming mental images that can be extended beyond what is physically feasible.

Then turn to the second point by calling attention to problem 4. Ask participants to share some of the statements they generated to explain what happens when you multiply by 10.

Below are two possible explanations:

*A base-ten model for  $678 \times 10$  would begin with 678 as 6 flats, 7 longs, and 8 units. Multiplying by 10 produces 60 flats, 70 longs, and 80 units. However, 60 flats is equivalent to 6 thousand-blocks, 70 longs is equivalent to 7 flats, and 80 units is equivalent to 8 longs, which, when taken together, represent 6,780. In other words, the problem  $678 \times 10$  began with 6 flats, 7 longs, and 8 units; the result of multiplying by 10 produces 6 thousand-blocks, 7 flats, 8 longs, and no units. The digits are the same; that is, the number of blocks has not changed. However, the type of block, or the value of each block, has changed by a power of ten.*

Consider this *story context*: Sasha has 24 coins. If each coin is a penny, the total worth is  $24 \times 1$  or 24 cents. If each coin is a dime, the total worth is  $24 \times 10$  or 240 cents.

Consult Maxine’s Journal lines 203–285 for an example of such a discussion.

**Viewing the video: Interviews with three students***(25 minutes)***Whole group**

The video for Session 2 includes segments of three interviews: first-grade student Chris, fourth-grade student Cole, and sixth-grade student Jenna. The content of all three segments is the base-ten place-value structure of numbers. Let the group know that you will stop the video after each interview to ask them what the student understands and what is yet to be learned. Suggest that they take notes so they will have comments to offer in the discussion.

Some participants may want to discuss the nature of the interviewers’ questions and interactions with the students. It is important to set this discussion aside until after the analysis of student ideas. You might say, “First I want us to concentrate on what we are learning about each student’s thinking. After we have collected ideas about all three students, we will turn to the interviewers.”

Play the first clip; then stop and solicit comments on what Chris understands and what he still has to learn. You may find it useful to add a third category of comment by asking, “What might we ask Chris, and what would we hope to learn by asking that question?”

Play the second clip and ask similar questions regarding Cole. After discussing Cole, play the clip about Jenna and solicit ideas about her thinking. See Maxine’s Journal lines 286–398 for an example of this discussion.

Finally, turn to the nature of the interviewers’ questions and interactions with the students by asking, “What did you notice about the questions the interviewers asked?” Participants might observe that the interviewers did not tell the students whether they were right or wrong or did not provide feedback to the students. Take this opportunity to talk about the difference between *teaching* a student and *interviewing* a student to form a picture of what the student does or does not understand. Let participants know that for homework, they will be asked to conduct an interview similar to the ones they have just seen and write about what they learned. Assure the group that there will be another period in the session to talk about this assignment.

## **Break**

(15 minutes)

## **Case discussion: Chapter 2**

(55 minutes)

### **Grade-level small groups**

(30 minutes)

### **Whole group**

(25 minutes)

The cases in chapter 2 have three main themes:

1. Identifying the conceptual issues that students face in making sense of the way numbers are written and spoken.
2. Examining the mathematical ideas involved in representing both small and large numbers on a single number line.
3. Exploring place value ideas that arise for older students as they work with large numbers.

Distribute the handout Focus Questions: Chapter 2. Place participants in grade-level groups and suggest that primary teachers spend most of the small-group time examining questions 1 through 5. Teachers of older students should concentrate on questions 3 through 7.

## **Small-group discussion: Ideas about the number system**

(30 minutes)

Through their discussion of questions 1 and 2, based on Dawn’s and Danielle’s cases, participants work to understand both what is correct in a student’s thinking and where the thinking went awry. They explore why someone might think “fifty-ten” is the number after 59 or why someone would write 1005 for one hundred five. It may be new for participants to look for elements of correct thinking in student errors. Ask questions to keep the focus on “What is *right* in this thinking?”

Question 3, based on Donna's case, highlights the relationship between an amount and the way we write two-digit numbers. Donna helps her students focus on the special pattern that emerges when they divide objects (in this case, beans) into groups of ten and then list the number of groups and the ones remaining.

Question 4 prompts participants to review their thinking on the first three questions with a focus on practice 7.

The next three questions, based on Leslie's case 10, examine the number lines. Question 5 invites participants to create their own number line and to place a set of numbers on it. Question 6 engages participants in the examination of two different number lines to see how they are the same and how they are different. Question 7 introduces the idea of multiple number lines, each with different scales.

Question 8 focuses attention on the thinking of students who are working on the question of how many tens are in 437,812.

As you interact with the small groups, take note of the ways participants use the number line in question 5 and decide which of those representations would be productive for the whole group. For instance, if a group approached the problem one way and then discarded that method for another, it might be useful to have them explain what they had originally thought and why they decided to change their approach. Capturing a range of numbers (some small, some large) on a single number line requires thinking about how to represent quantity consistently or how to use multiple scales. The whole-group discussion should illustrate the variety of methods your participants used.

### **Whole-group discussion: Number lines**

*(25 minutes)*

While the main focus of the whole-group discussion should be on the number-line work, use the first few minutes to gather participants' comments about the elements of correct logic in either "fifty-ten" following 59 or writing 10095 for one hundred ninety-five.

Then turn to question 5 and ask, "What did you need to take into account as you placed the given numbers on the number line, and how did you do that?" After a variety of methods have been illustrated, ask how the methods are the same and how they are different.

Ask participants to reflect on the thinking of Shaquille, Chris, and Olivia in questions 6 and 7. Ask, "What ideas about the number system are present in their approaches?"

If your small groups worked on question 8, use the last few minutes of this discussion to invite participants to compare the thinking they did in the math activity "Multiplying by 10" with the thinking of the students about dividing by 10 in this case. Ask, "What ideas about the number system are highlighted by examining what happens when you multiply or divide by 10?"

## Homework and exit cards

(10 minutes)

### *Whole group*

Distribute the Third Homework handout and have participants read the interview assignment. Point out that this is an opportunity for them to find out what ideas about numbers, number lines, place value, and the number system their own students have. Remind participants that the intent of this interview is to find out as much as they can about the way the student is thinking. Provide ten minutes for pairs to brainstorm some of the questions they would like to ask in an interview.

As the session ends, distribute index cards and pose these exit-card questions:

- How is the seminar working for you as a learner?
- What ideas about the meaning of practice 7 has the session illuminated for you?
- What questions about practice 7 are you pondering?

## Before the next session...

In preparation for the next session, read participants' written homework on analyzing student methods. Write a response to each participant. For more information, see the section in Maxine's Journal on responding to the second homework, lines 563–634. Make copies of both the papers and your responses for your files before returning the work.