



Making Meaning for Operations

Session 2

Making meaning for multiplication and division

Overview

Agenda		
Sharing exit-card comments	Whole group	5 minutes
Sharing student thinking	Pairs	20 minutes
Discussion: Norms for learning	Whole group	20 minutes
Video: Early multiplication and division	Whole group	10 minutes
Case discussion: Chapter 2	Small groups Whole group	25 minutes 25 minutes
Break		15 minutes
Math activity: Story problems for division	Small groups Whole group Small groups	25 minutes 25 minutes 5 minutes
Homework and exit cards	Whole group	5 minutes

Mathematical themes

- When working with multiplicative situations, students frequently find it challenging to coordinate the different units, that is, the number of items in a group and the number of groups.
- The variety of students' methods for solving story problems involving multiplication and division illustrates relationships among operations.
- Different kinds of situations can be represented by the same division expression.

Connections to the Common Core State Standards for Mathematics

Standards for Mathematical Practice

Practice 2 Reason abstractly and quantitatively.

Practice 3 Construct viable arguments and critique the reasoning of others.

Standards for Mathematical Content

Grade 2: Operations and algebraic thinking

2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Grade 3: Operations and algebraic thinking

3.OA.1 Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5×7 .

3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$.

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = \square \div 3$, $6 \times 6 = ?$

3.OA.5 Apply properties of operations as strategies to multiply and divide. Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)

3.OA.6 Understand division as an unknown-factor problem. For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.

Focus Questions: Chapter 2

1. In case 8, children work to answer questions that adults would consider as multiplication problems. Think about the representations offered by Jason, Rashad, Carlita, Kenya, and Flora. How does each illustrate multiplication? How are they the same and how are they different?
2. Also in case 8, consider the thinking of Junior in lines 73–85. What ideas about multiplication is he grappling with? What does his confusion illuminate about the nature of multiplication?
3. Consider case 9:
 - (a) Explain the difference between the thinking of Caroline and Roger at the beginning of the case. How are their ideas related to a property of multiplication?
 - (b) What is illuminated as Luke explains Roger’s thinking in response to the teacher’s question, “Why does Roger have to add on 3 more twos?” How are their ideas related to a property of multiplication?
 - (c) What ideas related to multiplication are evident in the student work samples?
 - (d) What new ideas or questions about developing concepts for multiplication does this case highlight for you?
4. In case 10, we see children working on a multiplication problem, making mistakes, and sorting out their misunderstandings. Where did these children get confused? How did they sort it out? How are the ideas in this fourth-grade class similar to Junior’s confusion in case 8’s kindergarten class?
5. Consider case 12. Explain the thinking of Su-Yin, Derrick, and William. What ideas about multiplication and division does the thinking of these children highlight?
6. Look over your work for questions 1–4. Locate one or two examples of student work that exemplify practice 2; then be specific as you explain how your examples illustrate the practice.
7. In case 11, we see students using addition, subtraction, and multiplication to solve problems that most of us would consider division problems.
 - (a) In particular, consider Vanessa’s work on the first two problems (lines 261–274). Vanessa subtracts to solve one problem and adds to solve the other. Why do you think she uses different operations for these two problems?
 - (b) Consider the work of Cory (lines 278–290) and Matthew (lines 299–306). What does each student’s approach indicate about his thinking about division?

Math Activity: Story Problems for Division

1. Write story problems for $32 \div 5$ so that the question you pose would be answered by each of the following:
 - (a) $6\frac{2}{5}$
 - (b) 6.4
 - (c) 6 or 7
 - (d) 6
 - (e) 7
 - (f) 6 remainder 2
2. Write story problems for $5 \div 8$ so that the question you pose would be solved with each of the answers below. Consider how to modify your responses from problem 1 so they fit the $5 \div 8$ situations.
 - (a) $\frac{5}{8}$
 - (b) 0.625
 - (c) 0 or 1
 - (d) 0
 - (e) 1

Third Homework

Reading assignment: Casebook chapter 3

In the casebook, read chapter 3, “When Dividing Doesn’t Come Out Evenly.” Consider the questions posed in the chapter introduction as you read the cases.

Writing assignment: Story problems for $5 \div 8$

You may have worked on this problem briefly in the seminar. This assignment provides the opportunity for you to revisit your thinking by writing your own answers. Your work can also include questions you have about any parts of the problem that are still confusing or unclear. The seminar facilitator will read what you have written and respond to your ideas and questions.

1. Write story problems for $5 \div 8$ so that the question you pose would be answered by each of the following:

(a) $\frac{5}{8}$ (b) 0.625 (c) 0 or 1 (d) 0 (e) 1

One way to approach this task is to examine the stories your group wrote for $32 \div 5$ and determine how to modify them to align with $5 \div 8$.

2. For each problem, explain what in the story context makes the answer appropriate for the situation.
3. Describe how your math thinking offers an example of practice 2.