Detailed Agenda

Sharing exit-card comments *(5 minutes)*

Whole group

Use the first five minutes to distribute the Overview and share comments from the exit cards. While it is a good idea to ask for names on the cards so you can understand the experience of your participants, do not identify the author when you share exit-card comments. First mention the math ideas that were commented on by several participants. One strategy for choosing comments about the learning process is to select those that illustrate the range of experiences; for instance, some participants mention they were comfortable because they teach this age group and had a lot to share while others state paying attention to young children's thinking was new to them. It is helpful for all participants to hear these remarks. Finally include comments about practice 2.

Sharing student thinking *(20 minutes)*

Pairs

Organize participants into pairs to discuss the written homework assignment in which they examined the thinking of their own students. While it is important that each participant have his or her own paper read and discussed, each pair can also be looking for common themes and ideas that arise in the student thinking in both classrooms.

Before the pairs start to share their student thinking assignment, remind them to include discussion of the mathematical practices they noted in their students' thinking. Announce that pairs should read each other's papers before beginning any discussion. Their conversation should reflect ideas from both papers. They might begin by sharing what surprised them or what they discovered about their own students' thinking. The discussion can also include what was the same and what was different in the ideas of the two sets of students.

At the end of the activity, let the group know that you will be reading and responding to all of the papers. Remind them of the procedure you established for collecting papers.

Discussion: Norms for learning *(20 minutes)*

Whole group

Now that participants have had a chance to work together for a session, this is a good time to collect ideas from the group about how to keep the focus on learning as they work together. Suggest that participants think about these two questions:

1. What do you do to make this a good learning experience for yourself?
2. What do you do to make this a good learning experience for others?

After a few minutes of "thinking time," ask the group to offer statements describing the way the seminar should operate to make it a good learning experience for all. Here are a few examples:

- Allow time in small-group work for individuals to think before talking.

- Be prepared for sessions—everyone should have something to contribute.
- Listen carefully to take in another person's ideas.
- Find ways to disagree without being disagreeable.
- Start and end each session on time.
- Be open to a new idea or perspective.

Allow just twenty minutes for this discussion.

The list does not need to be comprehensive or complete. The activity is designed to help participants know that reflecting on their own learning process and considering the dynamics of the group are part of the seminar. Point out how the norms they generated are related to practice 3. You might use this discussion to clarify the difference between critiquing and criticizing.

Some facilitators post the list in the meeting room and encourage participants to add to it as the seminar continues.

Viewing the video: Early multiplication and division *(10 minutes)*

Whole group

The video for Session 2 provides images of students using counting strategies to solve multiplication problems. The first two segments of a first-grade class and then a group of fourth graders illustrate connections between repeated addition, skip counting, and multiplication.

In the third segment, two third graders are explaining how they solved the problem of how many legs do three elephants have. The segment concludes with comments from their teacher. Embedded in their conversations are issues of equal-size groups, the connections between counting strategies and multiplication, and the need to keep track of both the number of groups and the number of things in a group.

After viewing the video, let the group know that there will be no whole-group discussion of the video. However, participants should look for the issues raised by the video as they come up in different contexts in the print cases.

Case discussion: Early multiplication and division *(50 minutes)*

Small groups *(25 minutes)*

Whole group *(25 minutes)*

The cases in chapter 2 present students working on multiplication and division. The chapter brings up the following main themes:

- In making sense of multiplication problems, children need to keep track of both the number in the group and the number of groups, and this is neither easy nor obvious.
- Children solve problems that adults might consider to involve division by using addition, subtraction, or multiplication.

Distribute the handout Focus Questions: Chapter 2. Let the group know that it is OK if they do not get to all of the focus questions. Remind them that the goal of the small-group time is to clarify their ideas and that they need not rush through the questions just to finish. Assure them that you will mention any specific questions you would like them to address before the whole-group discussion.

Small-group discussion *(25 minutes)*

Focus question 1, based on case 8, invites participants to analyze five different kindergarten students' representations of a multiplication problem. Question 2, also based on case 8, asks participants to examine the thinking of a student who is trying to work through a multiplication situation. Question 3, based on case 9, invites participants to analyze the thinking of second-grade students as they work with a situation involving multiplicative structures modeled with cubes.

Focus question 4, based on case 10, and question 5, based on case 12, ask participants to consider older students who are sorting out the same issue as the kindergarten student in question 2: in multiplication, you need to keep track of both the number in the group and the number of groups. Question 6 provides the opportunity for participants to illustrate practice 2 with examples from the cases. Question 7 focuses on the thinking of students in case 11 as they use a variety of operations to solve "division" problems.

As you work with the small groups, help them stay targeted on the specifics of the cases by asking for line-number references to support their ideas. After fifteen minutes, suggest they move on to questions 6 and 7 if they have not already done so.

Whole-group discussion *(25 minutes)*

The main points to come up in the whole-group discussion should include the following:

- In order to make sense of multiplication, students need to grapple with the idea of number in a group and number of groups; this is a multiyear process.
- Explicitly connecting representations to arithmetic sentences can help students note the relationship between repeated addition and multiplication.
- Examining the operations that students use to solve problems provides a window into how they are thinking about both the situations and the operations.

Remind participants that the small-group work is designed to help them use the thinking of the students in the cases to expand their own thinking about the operations. Indicate that many of the small groups had similar conversations (if this is true) and that you are not going to repeat those conversations in the whole group. Then pose these questions:

- What are you noticing about the operations of multiplication and division as you discuss the thinking of the children in the cases?
- What is involved for students as they work to make sense of these operations?

See "Maxine's Journal," lines 116–325, for an example of this discussion.

Break *(15 minutes)*

Math activity: Story problems for division *(55 minutes)*

Small groups *(25 minutes)*

Whole group *(25 minutes)*

Small groups *(5 minutes)*

This activity illustrates the importance of the problem context in making sense of division. While all the problems to be written will be represented by $32 \div 5$, the numerical answers will vary with the situations. In the same way that the math activity for Session 1 was designed to expand the participants' conceptions of subtraction, this activity helps them reconsider division.

What Does the Remainder Mean in a Division Problem?

Consider the division problem $32 \div 5$. Looking simply at the numbers, the mathematical answer is either $6\frac{2}{5}$ or 6.4, because $5 \times 6\frac{2}{5}$ and $5 \times 6.4 = 32$. Frequently, elementary textbooks suggest that the answer to $32 \div 5$ is 6 remainder 2. A more mathematically precise way to note this is to say $(5 \times 6) + 2 = 32$; that is, there are 6 groups of 5 in 32 with 2 items not in any of the 6 groups.

However, when this division expression is derived from a situation, the reality of the situation determines how to account for those 2 items that don't belong in any group. For instance, consider these three situations:

1. There are 32 children to place in my classroom. I have 5 tables in my room. I want to place the children at tables as evenly as possible. How many children will sit at each table?
2. There are 32 children to take on a field trip. A school van holds 5 children and 1 driver. How many vans will I need to transport the children?
3. I have 32 yards of material. It takes 5 yards of material to make a costume. How many costumes can I make?

In the Classroom Seating problem, because all the children must be seated and because children cannot be divided into parts, some tables will have 6 children and some tables will have 7. The 2 "leftover" children are distributed among the 5 tables. The answer in this case is "6 or 7"; that is, 6 at some tables and 7 at others.

In the Field Trip problem, because all the children need to be transported, 7 vans will be needed. Six vans will be fully occupied and the 2 children "left over" will need a seventh van.

In the Costume Sewing problem, I can make 6 costumes; $\frac{2}{5}$ of a costume or 0.4 of a costume is not sensible, because a costume cannot be broken into parts.

In each of the above situations, the objects (children and costumes) must remain whole. If the context allows for objects that can be divided up, different responses are possible. For instance, consider the following two situations:

> I plan to run for 32 miles over the next 5 days. How many miles will I run if I run the same distance every day?
>
> I have $32 to share with my 4 brothers. How much money will we each get?

Both of these situations illustrate cases in which the objects to be divided (miles and dollars) can be broken apart. The answer to the first of these is $6\frac{2}{5}$ miles or 6.4 miles. The answer to the second is $6.40. The difference between working with miles and dollars is not apparent in this example. If the problem were 32 ÷ 3, there would be a significant difference. Distance measured in miles is a continuous quantity; therefore there could be a reasonable answer ($10\frac{2}{3}$ miles per day). However, it isn't possible in our system of dollars and cents to express $10\frac{2}{3}$ dollars.)

Small-group work *(25 minutes)*

Organize the teachers into small groups and ask each group to write a story problem for 32 ÷ 5.

Collect two or three examples. After each one is shared, ask, "Does anyone have a problem situation that is different from those that have already been offered?" Distribute the math activity handout and display the six posters for collecting different types of story problems. Point out that as participants come up with new problems for the different solutions, they should write them on the appropriate posters.

If participants are having trouble varying their word problems, you might refer them to the problems in case 11. As you circulate among the small groups, keep an eye on the collection of answers. If you see one or two posters with limited or no responses, call attention to this and ask the groups to concentrate on those answers for the remaining time. By collecting responses in this way, participants will be able to see a variety of problem situations for each possible answer.

Note: Participants should understand that although answers to particular story problems involving 32 ÷ 5 may vary, it is incorrect to write 32 ÷ 5 = 6 or 32 ÷ 5 = 7. Only the notation 32 ÷ 5 = 6.4 is used to express the numerical answer. The variation among the answers in this activity is related to how we interpret the numerical answer differently, depending on the specific story situation.

Whole-group discussion *(25 minutes)*

Begin the whole-group discussion by asking if anyone has questions about any particular problem situations that have been posted. Once any difficulties have been resolved, focus the whole-group discussion on these questions:

- What is it about the story context that determines whole number answers such as 6, 7, and "6 or 7"?
- What is it about the story context that discriminates among the three whole number answers?

- What is it about the story context that provides the possibility of answers expressed with decimals or fractions, such as $6\frac{2}{5}$ or 6.4?
- What do you notice about the answer 6 remainder 2?

This discussion should bring up the following factors:

- In some contexts, the objects or the groups can be broken apart, and in others they cannot.
- Some contexts require that the entire amount be accounted for, and some allow for leftovers.
- Some contexts are more likely to produce fractions as answers and others, decimals, even though $\frac{2}{5}$ and 0.4 are equivalent.

Small-group discussion: Story contexts for 5 ÷ 8 *(5 minutes)*

Once these points have been clarified, ask participants to move back into their small groups to think about question 2 of the math activity. Suggest that they examine their responses to 32 ÷ 5, and then think explicitly about the story contexts from that discussion as they work with the 5 ÷ 8 problems. "How can you adapt the problems for 32 ÷ 5 so they would result in the given answers for this new task?"

After five minutes, let the group know there will not be a whole-group discussion on this question, but that they will continue to think about it for homework.

Possible Answers to Story Problems Represented by 5 ÷ 8

Answer: 0

It takes 8 yards of material to make a wedding dress. I have 5 yards of material. How many dresses can I make?

Answer: 1

I have 5 children to transport and a van holds 8 children. How many vans do I need?

Answer: 0 or 1

I have 5 children to place in my room. I have 8 tables. I want to place the children at the tables as evenly as possible. How many children will be at each table?

Answer: 0 remainder 5

I have $5. Tickets for a ride cost $8. How many rides can I buy? (I can purchase 0 rides and have $5 left.)

Answer: $\frac{5}{8}$ or 0.625

I want to run 5 miles over the next 8 days. I plan to run the same distance every day. How many miles should I run each day?

I have 5 gallons of liquid to put into 8 containers evenly. How much liquid will be in each container?

Homework and exit cards *(5 minutes)*

Whole group

Distribute the Third Homework sheet. Point out that the writing assignment is based on problem 2 from the math activity.

If you are planning to assign the reading of chapter 8, "Highlights of Related Research," in sections throughout the seminar rather than all at once at the end of Session 7, it is appropriate to assign sections 1 and 2 at this point. Let participants know they won't be discussing this reading at the next meeting, but that these sections are pertinent to the work of the past two sessions and thus make useful reading at this stage of the seminar.

Distribute index cards and pose these exit-card questions:

- What was important about the math discussed in this session?
- How is the seminar working for you as a learner?
- Practice 2 and practice 3 have been highlighted in this session. Explain something you figured out or are wondering about regarding one of these practices.

Before the next session...

In preparation for the next session, read what participants wrote in their student-thinking assignment and write a response to each participant. For more information, see the section in "Maxine's Journal," lines 452–545, on responding to the second homework. Make copies of the papers and your response for your files before returning the work.