CHAPTER

2

Making meaning for multiplication and division

CASE 8
Bunnies and eggs
Bella
Kindergarten, April

CASE 9
Buildings, rooms, and floors
Annalisa
Grade 2, January

CASE 10
Candy canes in packages
Janine
Grade 4, December

CASE 11
How do kids think about division?
Georgia
Grades 3 and 4, October

CASE 12
Are these kids or seeds?
Melinda
Grade 2, November

As we move from addition and subtraction to multiplication and division, the questions that were posed in chapter 1 still apply, and we extend them to all four operations:

- When we see different number sentences that can model a single situation, what do we learn about relationships among operations?
- What does it mean to have a concept of addition, subtraction, multiplication, or division?
- What can we see about the operations through different representations (cubes, number lines, and so on)?

Furthermore, multiplication and division come with their own questions:

- What kinds of situations are modeled by multiplication and division?
- What issues must students work through in order to make sense of these operations?

- What ideas about addition and subtraction do students bring to their work with multiplication and division?

Ponder and take notes on these questions as you read through the following cases from kindergarten through grade 4. After reading the cases, review the questions again.

case 8

Bunnies and eggs

Bella
GRADE 1, SEPTEMBER

Because Easter was coming up, I decided to do some problems about bunny rabbits. After a discussion on the number of legs one bunny rabbit has, I drew a picture of a bunny on the board and asked the children to solve the following problem:

If one bunny has 4 legs, how many legs would 3 bunnies have?

I told them that they could use any manipulative they wished to solve the problem. I was quite interested in the various representations they came up with.

Jason built his rabbits from colored wooden blocks; each rabbit had a head, a body, and four legs. He came over to me and said, “Mrs. Waters, I got 12 legs.” I asked him how he got the answer, and he told me he counted them. I asked him to draw me a picture of what he had done. Figure 2.1 shows his drawing.

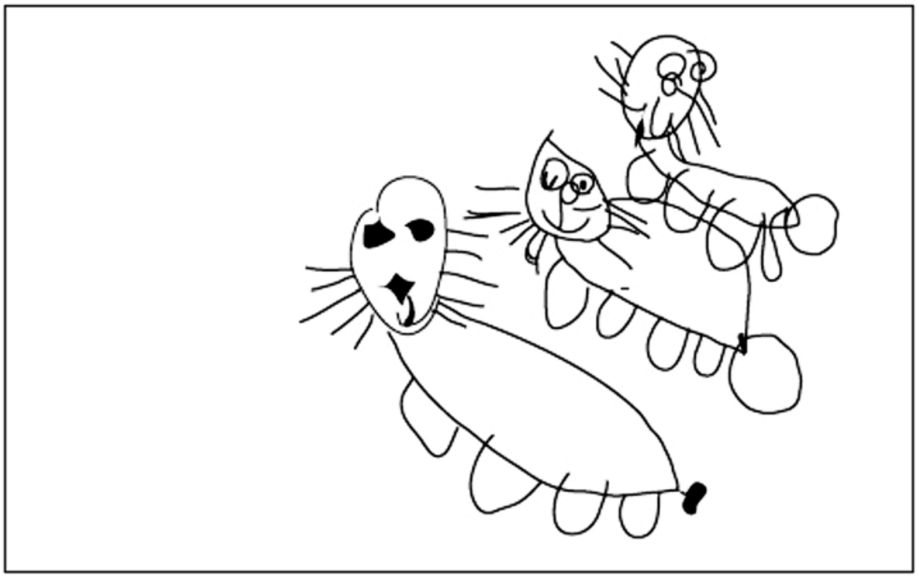


Fig. 2.1. Jason’s drawing of 3 rabbits, 4 legs each

Rashad used the large plastic keys we keep in a bucket in the room. He placed one key horizontally for each body, four keys perpendicular to it for the legs, and made three of these arrangements (fig. 2.2).

Fig. 2.2. Rashad’s arrangement of keys representing 3 rabbits

I asked, “How many legs did you get?” and Rashad told me 12. When I asked him to show me how he got 12, he counted, pointing to each “leg” in turn. I was surprised that he didn’t get confused about which keys were legs and which represented the body.

Carlita also used keys to represent the problem. First she made one row of four keys and told me that she had one rabbit. I asked her where the other rabbits were, and she said she would make them. I watched her build a second row of four keys, and then a third row (fig. 2.3).

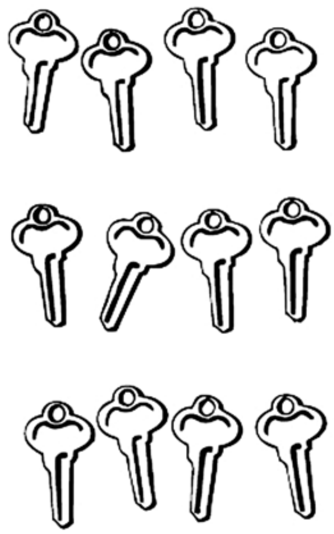


Fig. 2.3. Carlita’s arrangement of keys, showing 4 legs for each of 3 rabbits

“See, I have 3 rabbits.”

“How many legs are there in all?” I asked.

Carlita counted her keys: “12.”

Kenya used interlocking cubes, snapping together three groups of orange and white cubes in an a-b-a-b pattern. She counted them all together to get her answer and made a picture of what she had done (fig. 2.4).

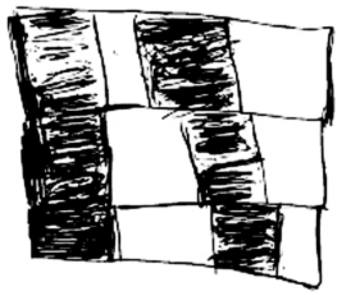


Fig. 2.4. Kenya's picture of cubes representing 3 four-legged rabbits

When I got to Flora, she had a row of 12 rocks in front of her (fig. 2.5).

Fig. 2.5. Flora's rocks, representing the 12 legs on 3 rabbits

"How many legs did you get?"

"I got 12."

"Can you tell me how you did it?"

"I counted one bunny first; then I counted the next bunny's; then I counted the next bunny."

"So how did you know how many legs there were in all?"

"I counted all of them."

Mark, Quincy, and Jerome were busy building their rabbits out of Legos. They built great looking rabbits with four legs, but time ran out before they could solve the problem.

The children were all so engaged in solving the problem that, even when I announced lunchtime, they didn't want to leave. There was no one in the group who was not working on the problem.

The children enjoyed being given the problem to solve, though I was aware that some of them still have difficulty counting even those small numbers. Besides the children whose representations I have described here, there were some who were not able to figure out the total.

Several days later, I decided to give a similar problem with Easter baskets; this time I wanted to see what was going on with the children who had difficulty with the bunny problem.

I made 3 Easter baskets for my friends. In each basket, I put 3 eggs. How many eggs were there in all?

Again, I told the children they could use whatever manipulatives they might choose.

Jenna used the interlocking cubes. First she made a unit of 3 cubes and brought them over to show me. I asked her what she had, and she said that she had 1 basket.

"Where are the other baskets?" I asked.

"I haven't made them yet," she said, heading back to the table. Soon she was back again, this time with 2 units of 3 cubes.

"How many baskets do you have now?"

"2."

"How many baskets did I say were needed?"

"3."

"Where are they?"

Jenna went off again and returned with the third unit of 3. "How many eggs do you have?" I asked.

"I don't know. I haven't counted them yet." She proceeded to count all the cubes and soon arrived at her answer of 9 eggs.

Jenna really surprised me with her understanding of the problem. She very rarely participates in an activity in a way that helps me understand what she really knows. Today I was impressed.

Junior also worked with interlocking cubes. He had 3 orange ones, 3 white, and 3 yellow.

"What do you have?" I asked.

"3 baskets."

"How many eggs are in each basket?"

"3."

"How many eggs do you have in all?"

"3."

Junior showed me how he counted: Each unit of 3 was "one" to him. I took one unit and broke it apart. He counted them, "1, 2, 3," and agreed that they represented 3 eggs. I put them back together with the other two units of 3 and asked him how many eggs were there. Again he said, "3." I tried several ways to help him see that I wanted him to count the individual cubes, but it didn't help. He was unable to see the difference or sameness between cubes once they became a single unit of 3.

Several of the children had no difficulty with this problem, so for them I posed the following extension:

> If I added 1 more egg to each basket, how many eggs would there be?

When I gave the second problem to Mark and Quincy, they both went away and soon returned with an answer. Mark explained, "I put 3 more on the 3 baskets. One more egg to 3 baskets, and that makes 12." Quincy had the same answer, but when I asked him how he got it, he said, "Mark told me."

Jason presented me with 3 groups of 3 interlocking cubes to demonstrate his answer of 9 to the first problem. When I asked him to solve the second problem, he added only a single cube and said there were 10. I repeated the question at least twice to see if he could figure out what was wrong with his answer and how he could fix it, but he did not change his answer.

Rashad had no problem figuring out the first problem, using an arrangement of 3 rows of 3 keys each. When I gave him the second problem, he added 1 key to each group and said there were 12 eggs. Seeing how easily he arrived at the answer, I decided to challenge him with another extension:

> I have another friend, and I would also like to give this friend an Easter basket with the same number of eggs. How many eggs would there be in all?

"I added 1 more basket," he said. "4 baskets with 4 eggs: 16." Only after he gave an answer did he count the added keys. "See, I was right!" Rashad, it seems, has an ability to think about numbers at a level beyond his age.

As usual, I was unable to get around to all the children, but I learned more about what my class knows and how they handle problems. A couple of years ago, I wouldn't have attempted any of this kind of mathematics with children of this age. I would have thought that this is too difficult for them. How wrong I would have been!

case 9

Buildings, floors, and rooms

Annalisa

GRADE 2, JANUARY

I told the students that we would be making buildings out of cubes. (The activity comes from *Patterns, Teams, and Other Groups,* a second-grade unit from *Investigations in Number, Data, and Space,* 3rd edition [Pearson 2016].) Each cube was a room in a building, and we'd think about the number of rooms on each floor and how many rooms in the whole building. "There's one rule for our buildings," I said. "Each floor must have the same number of rooms and fit exactly over the room below it."

As the students watched, I built a building that had one floor and two rooms. Then I added cubes to make a three-floor building with two rooms on each floor. We imagined families of tiny people living on each floor and agreed that each floor had a bedroom and a kitchen.

Once the context and the words "building," "floor," and "room" had been introduced, I asked a series of questions about the building.

- How many rooms are on each floor of this building?
- How many floors does the building have?
- How many rooms are there altogether?

These three questions would be central to our discussions throughout this work.

I asked everyone to think about what would happen if I made the building higher. How many rooms would there be in the building if it had 5 floors?

Caroline: If there were 5 floors, we would double 5—5 kitchens and 5 bedrooms.

Roger: I know that there's a kitchen and a bedroom on each floor. If you do 5 floors, I would count by twos—2, 4, 6, 8, 10.

A show of hands indicated that everyone was on board with Caroline and Roger's thinking. I moved on to ask about 8 floors. I asked the class to turn to a partner and talk about how they would figure that out. When we came back together, Leighanne went first.

Leighanne: There would be 8 bedrooms and 8 kitchens—16 rooms.

About half the hands went up when I asked who shared Leighanne's idea.

Roger: I would count by twos again, starting from 5, because that was our last number.

I stopped Roger before he went any further to make sure that his classmates were focused on what numbers he was referring to in our building.

Teacher: Are you talking about what we already know: there are 10 rooms on 5 floors?

Roger: Yeah.

Teacher So as everyone listens to Roger's idea, you need to keep that in mind. He's remembering that when we had 5 floors, we had 10 rooms.

Roger: I just have to add 3 more.

Teacher: You have to add 3 more what?

Roger: 3 more twos.

Teacher: Why does Roger have to add on 3 more twos?

Luke: He has 5; 6, 7, 8 counting by twos.

I placed our cube building in the center of our circle. It now had 5 floors with 2 rooms on each floor. I asked Luke to continue.

Luke: What Roger means by 3 more twos, he starts with 5 floors. Then there would be 6, 7, and 8 floors—3 more stacks of 2.

Teacher: Three more floors of 2 rooms.

I added 3 more two-room floors to the building and looked around the class. Then I followed up with a new problem. "Suppose I built it with 10 floors. How many rooms would there be?"

Jack: You would just double it.

Teacher: Double what? What's the "it"?

Jack: Double the 10.

I wasn't sure if Jack was doubling 10—10 bedrooms or 10 kitchens—the way Leighanne had doubled the 8 floors, or if he was doubling the five-floor building—5 floors has 10 rooms so twice as many floors has twice as many rooms. As I asked Jack for more details, he became quiet. He may have lost track of what he meant, or perhaps, he had just lost track of the words. I checked in with the class.

Teacher: If we double the number of floors, what will happen to the 10 rooms? Jack has given us all something to think about. This isn't just his question. How do we figure out how many rooms? How does the idea of doubling the building help us? How many rooms are in the whole building when we have 5 floors?

Thomas: 10.

Teacher: When we double the building and we put on 5 more floors, how many more rooms will we be adding on to the building?

Thomas: 5.

Teacher: Do you mean 5 floors or 5 rooms?

Thomas: 5 rooms.

It seemed that Thomas was having a difficult time visualizing and keeping track of the rooms and floors. I thought it would help if he held the model.

Teacher: Could you take the cube model and show us where the 5 rooms would be?

Thomas pointed to the top of the building and said, "Right here."

Teacher: If we add 1 more floor, how many rooms will we add?

Thomas: 1.

Teacher: How many rooms are on the first floor?

Thomas: 2.

Teacher: How many on the second floor?

Thomas: 2.

Teacher: How many rooms are on the fifth floor?

Thomas: 5. I mean 2.

Teacher: Every time we add a floor, we had how many rooms?

Thomas: 2.

Teacher: So if we add 1 more floor and we have 6 floors, how many rooms will we add on to the building?

Thomas hesitated.

Teacher: There's a lot to keep track of here. You have to keep track of how many floors there are, how many rooms there are on each floor, and how many rooms there are in the whole building, altogether.

In fact, this is precisely what makes understanding multiplication challenging for many second graders—they have to keep track of different units and how they are related. It's a lot for a young mind to juggle.

I decided it was time for the students to move on to work on their own buildings with 3 rooms on each floor. There would be other opportunities to pursue the idea of doubling the whole building. We worked together to build the first two floors of 3 rooms each, and once I was sure that everyone knew how to continue building more floors, I had them continue the activity in pairs.

The sample of student work below shows three different approaches to solving the problem of the number of rooms in a building with 10 floors, 3 rooms on each floor.

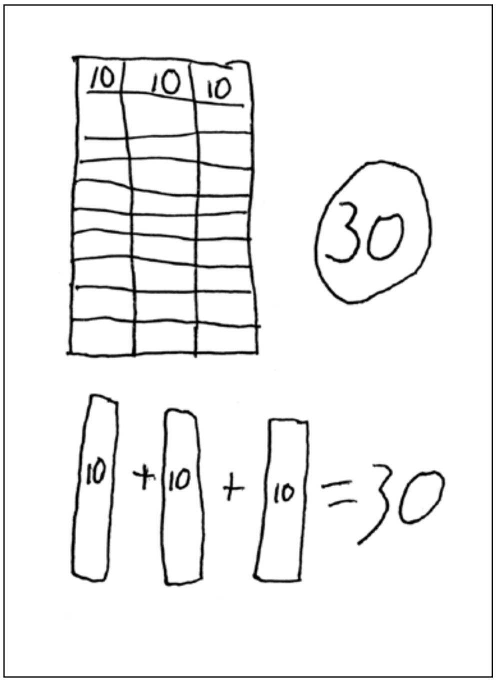


Fig. 2.6 Student A's work

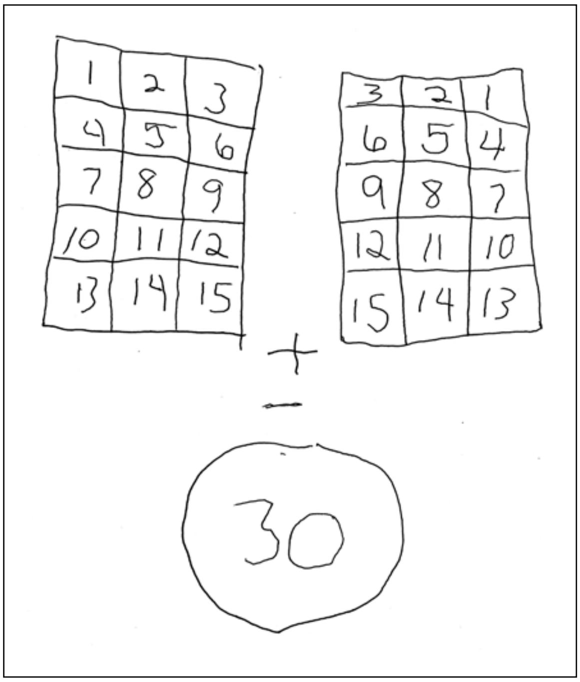


Fig. 2.7 Student B's work

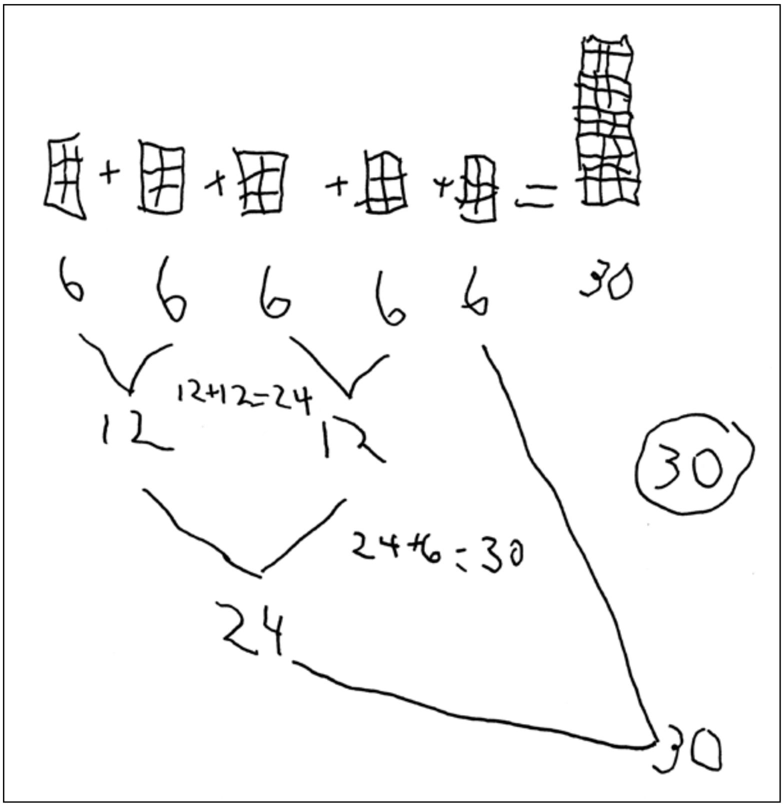


Fig. 2.8 Student C's work

case 10

Candy canes in packages

Janine

GRADE 4, DECEMBER

I sat with three girls today as they tried to tackle the following problem:

> There are 609 students at the school. How much would it cost to buy each one a candy cane? Candy canes are sold in packages of 6 for $ .29.

I have been trying to make the rounds and sit with a different group each day. It's interesting to see who has the concepts, who sits back, who leads the discussion, who is totally lost, which ones listen to each other, and which ones hear only their own voice.

The threesome that I sat with today is a new grouping. They have never worked together before, but they seemed to click on this problem.

Each started out working on her own for a few minutes; then they began talking. Almost immediately the three decided that the problem had something to do with the 6 times table—and that they had to go pretty far up the table to get to where they wanted to be. They started "6, 12, 18, 24, 30," and soon realized that there had to be a faster way.

Letitia, who grasps concepts pretty quickly, suggested that they should do 90×6 because she had figured out it would be 540. Wow! Good for her, I thought. We have been working on multiplying numbers by multiples of 10, so I guess it clicked for her that if $6 \times 9 = 54$, so 6×90 would equal 540. I was hoping she would take it one step further and see that 6×100 would be 600, but she stopped there and neither of the other two saw this either. Maybe this was because when studying the 6 times table in third grade, they only went up to 6×9.

From 540 they decided that they had to add up by 6s to get to 609. This is where keeping track of their numbers became very difficult. Letitia, after some thought, said that 540 + 10 more would equal 600. One of the others said that 540 + 10 is equal to 550 and not 600. Letitia struggled to explain. She had the idea that 10 groups of 6 would get her to 600, but she kept confusing 10 and 10 groups of 6. I could see her struggling to keep track of her thoughts; I could see her getting it and then having it fly out of her mind. She knew that 10 was important, but couldn't hold onto the idea that it wasn't 10 ones but 10 groups of 6 she was thinking about.

To help her out a little, I asked how many more she needed to get to 600 from 540. She answered 60. "Oh yeah—60 is 10 groups of 6!" She would grasp this idea for a while and then lose it again. She was really trying hard to hold onto the meaning of this 10!

So now they had 90 packages plus 10 packages for a total of 600 candy canes. They knew they needed 9 more candy canes. One more package would bring them to 606, but they still needed 3 more. Could they buy 3 separately? No, they came in boxes of 6. If they bought another box, they would have 3 extra. What would they do with them? Natalia said, “Let’s just eat them! We did the shopping. We deserve a prize!” All happily agreed that they would eat the 3 extras themselves.

Finally the group established that they had to buy 90 + 10 + 2 packages of candy canes. Because they didn’t know how to multiply 102 × $.29, they decided to do it with the calculator. Their answer came out 2958, which they reasoned would be $29.58.

case 11

How do kids think about division?

Georgia

GRADES 3 AND 4, OCTOBER

I decided to begin this year by focusing on the four operations. I wanted to investigate my students' ideas more closely.

As I discussed my plan with a colleague and said I was particularly interested in learning how my eight-, nine-, and ten-year-old students thought about division, she was horrified. She believes that children need to be taught the operations in a certain order. In her mind, first comes addition, then multiplication, then subtraction, and then division.

My initial idea was just to expose kids to division problems early in their experience with numbers so that the process would be familiar when they came to the numbers that themselves implied division—decimals and fractions. But through my classroom research, I'm learning much more about how kids think about division, what they call division, and how they define division. Much of this learning comes from my observations of their written work.

Each week I have given the students story problems that I considered to be division problems, problems I would solve by using division. Here are some examples and a few of the kids' responses to them.

> Jesse has 24 shirts. If he puts 8 of them in each drawer, how many drawers does he use?

Vanessa wrote 24 – 8 = 16, 16 – 8 = 8, 8 – 8 = 0, and then wrote 3 for the answer.

> If Jeremy needs to buy 36 cans of seltzer for his family and they come in packs of 6, how many packs should he buy?

This time Vanessa wrote 6 + 6 = 12, 12 + 12 = 24, 24 + 6 = 30, 30 + 6 = 36. I still need to ask her how many packs that gives her. But what made her add this time and subtract last time?

Other students use these same methods. Is it significant that sometimes they add and sometimes they subtract? What are their choices based on? I thought the problems about Jesse's shirts and the six-packs of seltzer water were the same kind of problem, and yet students treated them differently. On reflection, I wonder if the total number (24 or 36 in these cases) affects how kids approach a problem. If the total number is a familiar one, do they subtract (until they get zero), and if the number is less familiar, do they add, building up to the number, as Vanessa did for the second problem? Or do the contexts seem different to the children?

> Joni wants to build some bookshelves for her friends and family. If she bought 36 boards and she needs 4 boards for each bookshelf, how many bookshelves is she going to make?

Cory had clearly tried approaching this problem in more than one way. His paper was filled with tally marks, which seemed to be one approach he was thinking about it, while above the problem he had written $2 \div 36 = 28 \div 2 = 14$. I decided to ask him about his thinking. I assumed he meant $36 \div 2 = 28$, but I wasn't sure how he got the 28. I wanted to find out how he was really thinking about the problem instead of making assumptions.

Cory told me, "I thought it was times." Then he reread the problem aloud. "See, that's why I changed it to divided by. If it was 4 divided by, I would probably use 2 first so it would be easier. First I would do 2 divided by 36, and that equals 28, and 2 divided by 28 equals 14, so that's how I came up with 14. Because I knew divided by is half of whatever the number is, like 2 divided by 100 is 50."

I asked Cory, "Does that mean that 28 is half of 36?"

"Yes," he said, "because I know half of 30 is . . . wait a second . . . no, that isn't right. It would be 15 plus another 3, that's probably 18 and divided by that, which is 9."

Cory knew that 36 is made up of 30 and 6, and half of 30 is 15, and half of 6 is 3. Then he knew that 15 plus 3 is 18. He understands division in terms of halving, as he clearly states, "Divided by is half of whatever the number is." Halving is also implicit in his comment, "That's probably 18 and divided by that, which is 9"; he doesn't even have to say he halved it. And he knows that dividing by 4 is like halving twice. What about division that isn't based on half? Would that be division to him? What will happen when he has to divide by 3?

> You go into a pet store that sells mice. There are 48 mouse legs. How many mice are there?

Matthew organized his work beautifully. He wrote a key (m = mice, l = legs) and put his numbers in columns.

1 *m*	4 *l*
2 *m*	8 *l*
3 *m*	12 *l*
4 *m*	16 *l*
5 *m*	20 *l*
6 *m*	24 *l*
7 *m*	28 *l*
8 *m*	32 *l*
9 *m*	36 *l*
10 *m*	40 *l*
11 *m*	44 *l*
12 *m*	48 *l*

Then in a neat box he wrote, "12 $m \times 4\ l = 48\ l$." Above the box he wrote the number 12. What does this say about Matthew's understanding of division? He knows that 12 is the answer, but he feels satisfied with a multiplication number sentence in which the answer is part of the problem rather than the answer to the problem. He knows how to find the answer, but instead of the number sentence I had expected, $48 \div 4 = 12$, he wrote a multiplication number sentence.

During a conversation with classmates about a similar problem, Matthew said, "This is another division problem. It's 63 divided by 9. What number times 9 equals 63?—7." When I asked him to explain what there was about the problem that made it a division problem, he said, "I don't know, but it is. But my thinking is multiplication."

What does this say about kids' understanding of division if they use all the operations except division? As I look at how kids think about division, and how I had initially hoped that the work would help kids with fractions, I wonder in what way the two topics are connected. Do kids bump up against the same ideas in both division and fraction work? What does this say about how kids think about wholes and parts?

case 12

Are these kids or seeds?

Melinda

GRADE 2, NOVEMBER

On Halloween I had my second graders work on this word problem:

> There are 6 children sharing 45 roasted pumpkin seeds. They want to share them as evenly as possible. How many will they each get?

First I have to say that I was really amazed and impressed with the industry and confidence with which the children approached the problem. Almost everyone got right to work and seemed to be engaged. Here are some things that seemed interesting to me.

Maria, Nikita, and Su-Yin worked together. They each got 45 interlocking cubes and snapped them together in a row. Su-Yin's initial strategy involved taking one cube off her collection of 45 so that she would have an even number left. When I asked her why she wanted an even number, she said so she could divide it in half. I asked her why she wanted to divide it in half, and what she planned to do next. She said she didn't know. I wondered whether she had some idea that she might be able to split her two even groups into three groups each somehow. On the other hand, maybe dividing something fairly made her think of halves and even numbers, without having any specific thought about needing six groups.

I never did find out because Nikita and Maria had a different strategy, which Su-Yin adopted. Maria started making 6 piles of cubes, first breaking off a stick of 5 cubes for each. She then gave each pile 2 more, and had 3 cubes left.

I left these three girls for a while and worked with other children. When I returned to them, Maria was working by herself, drawing her 6 piles of 7 cubes, and Nikita and Su-Yin were arguing. Nikita was saying that each child got 7 seeds, and Su-Yin was saying each got 6 and Nikita was counting a kid as a seed. Su-Yin indicated her 6 piles of cubes and asked me, "Are these kids or seeds?" I asked her what she meant, and she said that Nikita thought that they were all seeds, but that she had made each of her groups by putting a cube to be a kid in the center and then putting seeds around it. Therefore, her groups were 6 seeds and 1 kid each.

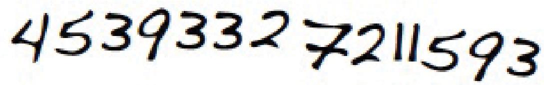
I was interested in Su-Yin's plan for representing and solving the problem, and I wondered what she had thought about in the process. I asked her to explain and show me exactly what she had done and thought from the beginning. She said she got 45 cubes, then put the 45 cubes in a big pile and made sure there were 45. I asked her why she had to have 45, and she said, "That's how many seeds there are." She explained then that she wanted to make groups by putting a kid in the middle and putting seeds around it, so she started to take cubes from the 45, and then said, "Oh! I need to get 6 more cubes!" I asked her why, and she said, "Because those are seeds!"

My question: “Is this about keeping track of units again?” Su-Yin had to keep track of the numbers 45 and 6 from the start of this problem, and eventually there were some 7s and a 3. She had to think not just of the numbers and how they relate to each other but also what they represent in this problem. I think that her plan to represent a child and then give the child a fair share of cubes representing seeds was a good one.

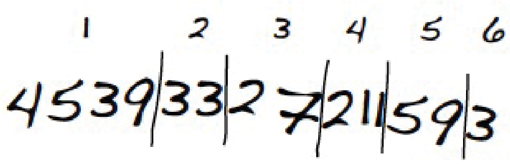
But was it too much for her to hold onto at once? First she didn’t realize that she couldn’t take any of the 45 cubes to make children because those 45 represented the whole collection of seeds. Then, in her representation, she thought each group of 7 cubes showed a child and his or her seeds, still not realizing that “children” couldn’t be taken from the “seeds” and that all the cubes had to be counted as seeds in the end. It was a lot to hold onto!

I am also intrigued by the fact that Nikita and Su-Yin were so clear about what they were disagreeing about. Su-Yin’s statement that Nikita was turning a kid into a seed showed some clarity about what was going on and where the confusion was, even though it was in fact she who had turned a seed into a kid. I also wonder why Nikita was so sure she was right about the problem, the solution, and her representation of them, but could not convince Su-Yin.

Meanwhile, Derrick and William were having an interesting time with the problem. William, who is fairly able but not very confident, said immediately, “I have an idea!” His idea, which he shared with Derrick, was to start with 45 and keep taking off 6s. They did not want to use manipulatives or draw pictures. They did use William’s plan, and subtracted 6 repeatedly from 45 “in their heads.” They ended up with a list of numbers, run together and hard to read:

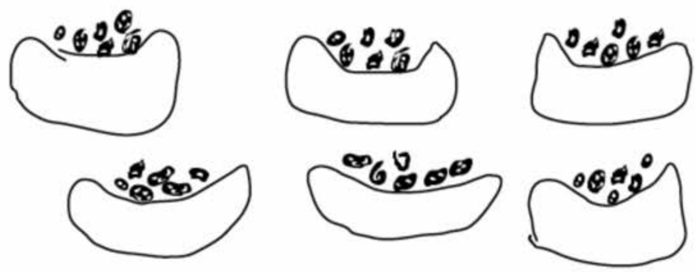


William did say that he took out 6s because that was 1 seed for each child. I wasn’t sure whether their list of numbers told him how many seeds each child ended up with, but my colleague Lydia was in the room, and she had a conversation with him about it. She later explained to me that he started to divide their list of numbers in a way to show that each time he subtracted 6, he was giving each child 1 seed. However, he made mistakes about where to divide the run-together numerals, and ended up with 6 groups. It didn’t bother him that the numbers he created by dividing up his string did not make sense in the context of the problem.



When I joined William and Derrick, they had their list of numbers and had drawn 6 little bowls with 6 little seeds in each. I asked them what the bowls were and how many seeds they had in their picture. William replied that the bowls were the seeds for each kid. Derrick figured out how many seeds they had drawn by adding 6s, and said there were 36. William agreed.

Neither boy seemed bothered by the fact that they had accounted for only 36 seeds when there were 45 in the problem.



I asked them, “How many seeds would be left then?” They figured out that there would be 9 by counting on from 36 up to 45. When I asked what they could *do* with the extra 9 seeds, they disagreed. William suggested that each child could have one more seed, but Derrick said, “No, there’s only enough for one kid. You would need another kid.” I asked why, and he said because there was only “one more 6.” He decided that maybe one of the existing kids could get 6 more seeds, or 12 total, and that 3 could be thrown out. For some reason, he was unwilling to divide up a group of 6 seeds in order to give each child one more.

Apparently Derrick convinced William to give one person 12 and everyone else 6 (perhaps because I joked that maybe one of the kids was bossy or bigger or hungrier). When William explained his thinking to the class (I had him put out cubes to show his bowls of seeds), he had a group of 12 and 5 groups of 6. He went back and forth between labeling his groups of 6 as one seed for each kid and as a single kid’s share of seeds. I think he was confused because he initially took out groups of 6 and planned to give one seed in the group to each kid and do this as many times as possible, but then he began to see those original piles of 6 as the seeds for each kid. I wondered why he didn’t distribute 6 of the remaining 9 seeds in the end. Why did he stop distributing seeds when each of the 6 kids had 6?

Again, I have questions about how children can keep the “whole” group of seeds in mind while they think of particular ways to divide it into parts. There are many issues for children to sort out:

As they start dividing up the whole, what do their groups mean?

Are they collections for each child?

One seed for each child?

Are the children themselves represented in their diagram or model of the problem?

Although there was much confusion while these second graders worked on the problem, I have to say that they were remarkably able to make sense of a very complex question.