# Early Number Instruction

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Developing an understanding of number has historically been the focus of early childhood mathematics instruction and a foundation for subsequent instruction. When should mathematics "instruction" begin? On what topics should initial instruction efforts focus? How should such efforts be implemented? This article addresses these questions.

## When Should Mathematics Instruction Begin?

Opinions about when to begin number instruction vary greatly. Should such instruction be postponed until roughly 7 years of age, when children are intellectually more capable (see, e.g., Piaget [1965])? Should efforts to foster the number concept begin earlier—after children have mastered such skills as counting (that is, enumerating) a collection or at least after they have begun to say number words in the correct sequence (see, e.g., Baroody [1987]; Ginsburg [1977])? Or should it begin before children can count (see Huttenlocher, Jordan, and Levine [1994]) or even before they start to crawl (see Wynn [1998])?

Research over the last thirty years suggests that preschoolers have more competence than earlier scholars allowed (James 1890; Piaget 1965; Thorndike 1922). Indeed, some researchers have claimed, perhaps too optimistically, that pretoddlers can count without using number words and can even do simple arithmetic, such as recognizing that one item added to another makes two (Gelman and Gallistel 1978; Wynn 1998). Although the issue of whether pretoddlers really understand number is unclear (e.g., Mix, Levine, and Huttenlocher [2001]), we do know that children between 18 months and 3 1/2 years of age are learning much about the fundamental ideas of number.

## What Topics Should Initial Instruction Cover?

What essential number ideas are 1 1/2- to 3 1/2-year-olds attempting to understand? The following paragraphs discuss four of these ideas.

1. *A collection is composed of an exact number of discrete items.* Mix, Levine, and Huttenlocher (2001) argued that pretoddlers do not distinguish between discrete quantities (collections of distinct items) and continuous quantities (length or area) and that they use the same mechanism to arrive at inexact appraisals of both types of quantities. In other words, an infant may view a collection of three faces and the size of a stuffed animal as approximately how much space each occupies or roughly the total length of their contours. Fifteen-month-old Madeline, for example, chose the longer row of four widely spaced blocks and left the shorter row of five tightly bunched blocks for her

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*Edited by* **Julie Sarama,** *jsarama@buffalo.edu, and* **Douglas Clements,** *clements@buffalo.edu, State University of New York at Buffalo, Buffalo, NY 14260. This department addresses the early childhood teacher's need to support young children's emerging mathematics understandings and skills in a context that conforms with current knowledge about the way that children in prekindergarten and kindergarten learn mathematics. Readers are encouraged to send manuscripts for this section to "Early Childhood Corner," NCTM, 1906 Association Drive, Reston, VA 20191-9988.*

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FIGURE 1

**Ariana's tower-game strategy**

| Vignette | Comments |
| --- | --- |
| Ariana spontaneously announced, "I'm going to build my tower [of checkers] to ten." | Ariana apparently equated *ten* with a position or distance rather than with a collection of a particular number of items. Note that numbers can, in some other contexts, be used to designate position or distance instead of collection size. |
| A visitor remarked, "A tower to ten. OK, build your tower to ten." | |
| "Where's ten?" Ariana asked. | |
| Her mother responded, "Where's ten? Ten's a number . . . . You would have to count them to build ten, right? Remember how we count money, 'one, two?' " | The mother intuitively recognized her child's misinterpretation of number, provided feedback, and related the new situation of counting checkers to a familiar one of counting money. Note that children's number competences initially may be highly specific to the situation and that they may need help to relate new situations to activities that they can already perform. |
| Ariana counted "one" while placing one checker in her hand, "two" while placing a second checker, "three" without adding another checker, "four" while placing a third checker, and "five, six, seven, eight, nine, ten" while placing a fourth checker in her hand. | Ariana apparently recognized that enumerating a collection involved saying the number words in sequence while picking up, or pointing to, items. Clearly, though, she had not mastered the concept of assigning one counting word to one item. The adults did not intervene because enumerating ten items would probably be too difficult a task for the girl at the time. Note that Ariana used the number word *four* to represent a "large" collection. |

twin because the former appeared to be "more" than the latter. An important step toward distinguishing between discrete and continuous quantities and understanding number (identifying and representing exactly how many items are in a collection) may be constructing an elementary notion of one-to-one correspondence (Mix, Levine, and Huttenlocher 2001).

2. *A basic understanding of one-to-one correspondence involves realizing that the distinct items (objects or actions) in one collection can match those of another*. Research indicates that children can use a visible model of one collection to create a matching collection without counting (see, e.g., Wynn [1990]) and can point to items in a collection in one-to-one fashion before they learn to count collections (Beckwith and Restle 1966). Children may also use nonverbal production, the ability to create a matching collection for one that is not visible but is mentally represented, before they learn to count collections. For instance, Mix (2001) noted that her 21-month-old son retrieved two dog treats for two pet dogs in another room while saying, in effect, "This [one] is for [the first dog], and this [one] is for [the second dog]." Such intuitive, everyday one-to-one correspondences may help lay the groundwork for an informal understanding of numerical equivalence and number.

3. *Number is an important way to categorize things.* Another important step toward understanding number is recognizing that it is an important attribute, like color, size, and weight, for categorizing things and, thus, for identifying and comparing them. Indeed, Mix, Levine, and Huttenlocher (2001) concluded that "an understanding of quantity [can]not emerge prior to the recognition of other categories" (p. 173). Although this process of categorizing probably starts before language develops, language provides labels that facilitate and accelerate the process.

Consider, for example, the development of 21-month-old Blake. He learned to use color words to categorize items just before he did the same thing with numbers. Consistent with Sandhofer and Smith's (1999) observations, he initially used color words indiscriminately. Next, he started to use one color word, then several, with increasing accuracy. His breakthrough may have been the result of realizing that color was useful for identifying objects. Similarly, Blake learned the number words *two*, *three*, and *four* but clearly did not attach any meaning to them. The boy then may have realized that numbers are also useful in categorizing objects in the environment and gradually used *two* and next *three* more and more discriminately to identify collections (even before he counts two items reliably).

**FIGURE 2**

**The "hiding game" (based on a task developed by Huttenlocher, Jordan, and Levine [1994])**

A caregiver places one to four chips on his or her mat, encourages a child to examine the collection, covers the collection, and says, "Make your mat just like mine." The child then tries to duplicate the now hidden collection. Note that, initially, young children may need and want to peek. Allow them to do so, but gradually encourage them to respond without peeking. The game can be played with a single child or a small group of children. In the latter situation, each child would have his or her own mat.

4. *What constitutes a collection does not depend on appearances.* Unlike physical properties that can be sensed directly, such as color, size, and weight, number is an abstract "property" because the collections that number is used to identify and represent can be arbitrarily defined. For example, a mother may ask a child who is playing with two blocks and two animals to put away "the blocks" or "the playthings." The former instruction refers to the collection of two blocks; the latter refers to all four items. In addition, a collection can be composed of physically dissimilar items, such as a blue button, a red button, a black checker, and a brownish penny, again, illustrating the abstract nature of number. We can also categorize as *the same number* two homogeneous collections of items that bear no physical resemblance, such as ○ ○ ○ and □ □ □. Indeed, one collection of dissimilar items (e.g., ✩ ● ■) and another collection of other dissimilar items (e.g., △ ✷ ◗) can be categorized as the same number. This abstract quality helps explain why children use number as a category later than they use physical properties, such as color, and may only gradually see the equality of collections that have increasingly different appearances (Mix, Levine, and Huttenlocher 2001).

**Number sense is not something that we can impose directly on children**

## How Should Initial Instruction Be Implemented?

Helping preschoolers understand the basic concepts underlying number may require a different type of teaching than that suggested by conventional wisdom. Many people view teaching as telling or showing children something that they need to know, then having them imitate and practice it. Such direct instruction undoubtedly plays a part in fostering number knowledge. For example, repeatedly reading children's books that involve counting to ten and having children imitate such readings can help children learn the arbitrary sequence of number words. Modeling may also be useful in underscoring the matching process, for instance retrieving two plates for two people and saying, "A plate for you and a plate for me."

Imagine, however, a caregiver explaining repeatedly to a child, "A collection is composed of an exact number of individual items" or "Number is an important way to categorize objects." Only someone who already understands the concepts could make sense of these statements, and the statements are unlikely to help young children learn the concepts. In other words, number sense is not something that we can impose directly on children.

In "teaching" preschoolers mathematics, caregivers should, for the most part, create opportunities or "problems" for children to explore, allow them to devise their own strategies, and interact with them as they think or work through situations (Anderson 1997). Consider, for example, 2-year-old Ariana's activity when a visitor gave her checkers to play a mathematics game. As the vignette in **figure 1** shows, Ariana used the checkers to devise her own game of building a tower, and the adults who were present wisely acceded.

### One-to-one correspondence

To help young children focus on distinct items, exact amounts, and one-to-one correspondence, caregivers can take advantage of everyday situations that involve matching. Initially, efforts should focus on one or two items, then on three, and later on four. In the following instructions, for example, note that the requests do not involve numbers: "Bring me a spoon." "Take an apple." "Bring a towel for mommy and a towel for the baby." "Give each dog a doggie biscuit." As a child becomes familiar with the number word *one* or to help familiarize a child with this term, incorporate it into matching requests, such as "Take just one cookie" or "Put one cup on each placemat."

To facilitate mental representation of exact amounts of distinct items, matching requests can be made for an unseen but previously viewed collection. For example, ask a child who is seated at a table with two friends to go into the kitchen and get a cupcake for everyone at the table. The "hiding game" can be an entertaining way to practice this nonverbal production skill (see **fig. 2**).

**FIGURE 3**

**The "same game"**

A caregiver puts out a card, or places two "choice" cards before a child (see example 1 below), and asks the child to match his or her card to the caregiver's: "Which of yours is the same number as mine?" The game can be made more challenging by adding a third choice or increasing the number of items shown (see example 2). Note in the examples below that if a child uses color, shape, area, density, length, or total perimeter (the total distance around the outer edges of all items in a collection) to make a choice, he or she will answer incorrectly or inconsistently.

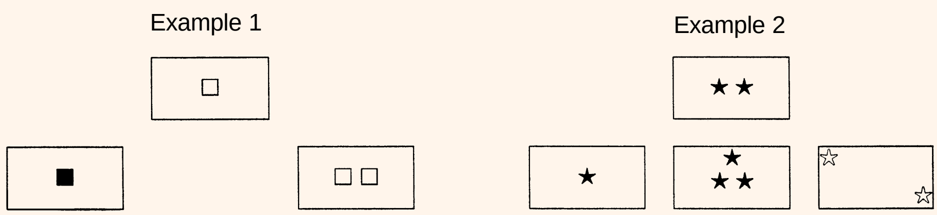
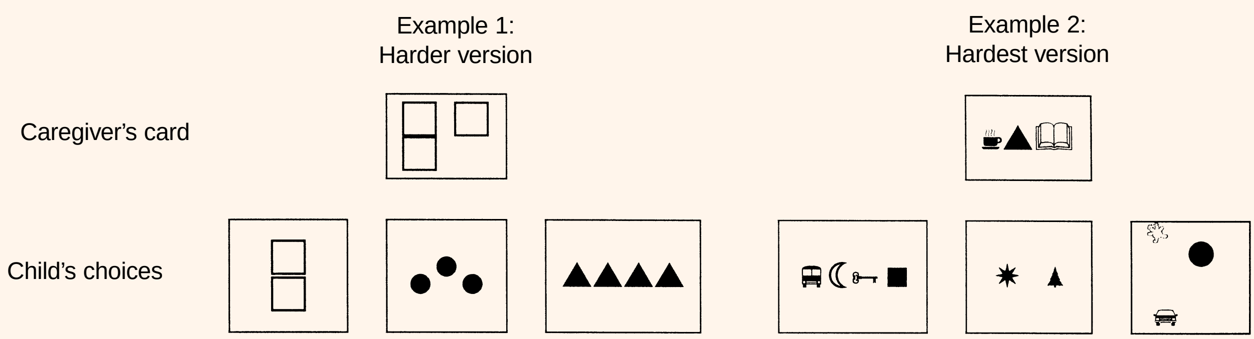



**FIGURE 4**

**Advanced versions of the "same game"**




## Number as a property

After children have had experience classifying objects by such obvious physical characteristics as size and color (e.g., "Get the red ball"), they can be helped to recognize that number can be valuable for categorizing things in a variety of ways. Caregivers can take advantage of everyday situations to foster this recognition. For example, children can be told, "The crayons are in the drawer with only one knob" or "I like the kitten with two black spots." The "same game" described in **figure 3**—like the game described in **figure 2**—illustrates how children's attention can be drawn to number even before they use number words reliably.

## Defining a collection

Even before children can accurately count small collections of things, they can begin the process of defining collections. For example, with two blue blocks and one red block available, a caregiver can request of a child, "Give me the blue blocks." The "hiding game" described in **figure 2** can be modified to practice categorization. For example, a caregiver might put two blue chips and a red chip on his or her mat, cover the mat, and request, "Make your mat just like mine." In time, the request can be revised to "How many blue chips am I hiding?" At first, the child can respond by putting blue chips on his or her mat; later, by specifying the number.

A more abstract understanding of collections can be fostered in everyday situations with simple requests, such as, "Put all your dirty clothes in the hamper" or "Put all the toys back on the shelf." (Note that even adolescents may have trouble comprehending or responding to such requests.) The game described in **figure 2** can be made somewhat more challenging by using different items to make up a collection and giving a choice of various items to the child to make his or her mat the same. As children learn number words or the skill of enumerating collections in a one-to-one fashion, they can first be asked to identify the number of items in small homogeneous collections, then in small collections of dissimilar items. Similarly, children can

be asked to compare two homogeneous and, later, two heterogeneous collections composed of different items (see examples 1 and 2, respectively, in **fig. 4**).

## Conclusion

Without question, children between 1 1/2 and 3 1/2 years of age can benefit from informal mathematics instruction, and the opportunities for initiating meaningful mathematical learning are limitless (see, e.g., Fromboluti and Rinck [1999]). By preparing a mathematically rich environment for children, caregivers and early childhood educators can lay a strong foundation for learning school mathematics.

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