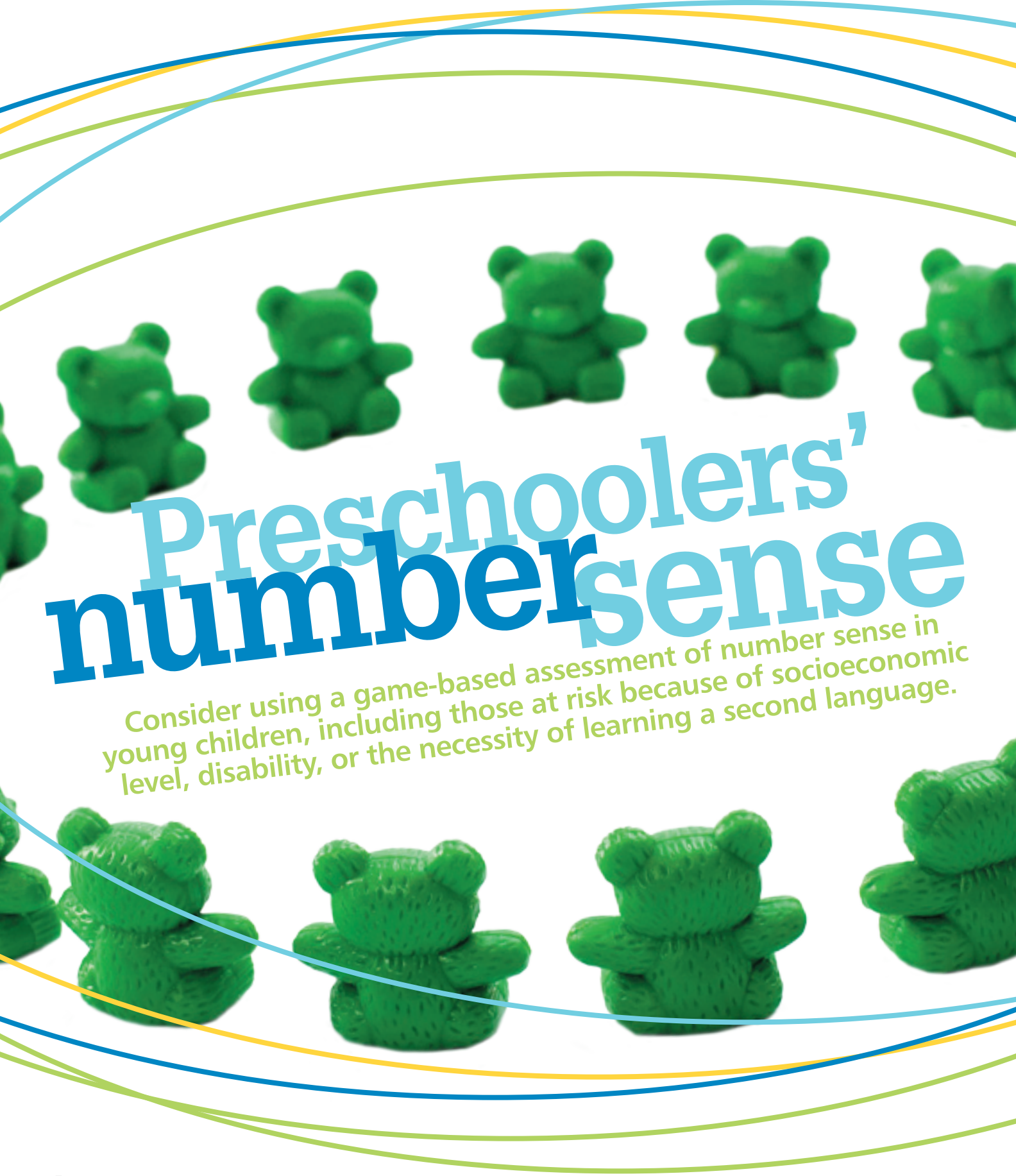
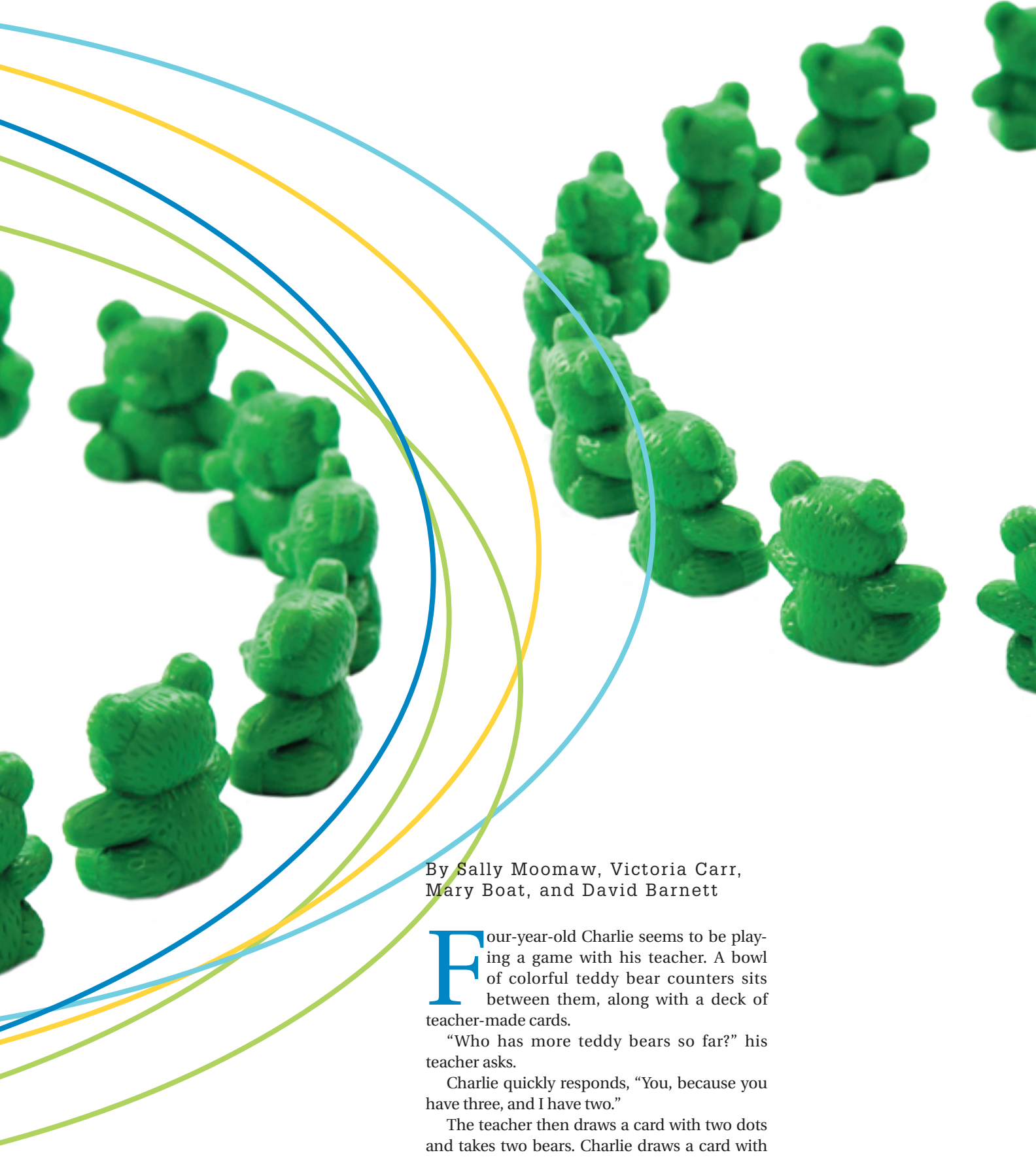


Preschoolers' number sense

Consider using a game-based assessment of number sense in young children, including those at risk because of socioeconomic level, disability, or the necessity of learning a second language.

ISTOCKPHOTO.COM



By Sally Moomaw, Victoria Carr, Mary Boat, and David Barnett

Four-year-old Charlie seems to be playing a game with his teacher. A bowl of colorful teddy bear counters sits between them, along with a deck of teacher-made cards.

"Who has more teddy bears so far?" his teacher asks.

Charlie quickly responds, "You, because you have three, and I have two."

The teacher then draws a card with two dots and takes two bears. Charlie draws a card with three dots and takes three bears. He counts his bears and the teacher's bears. "We both got the same," he announces.

The teacher draws a card with six dots; Charlie's next card has five dots. After they take their bears, the teacher again asks who has

more or whether they have the same number of bears.

Charlie sounds a little disgusted. "You," he replies, "because you had a different number, and I got five again. I got two fives, and you have another number that's not five" (referring to the teacher's card with six dots).

The teacher is actually assessing Charlie's development of number sense, which refers to his ability to deeply understand numerical concepts and relationships (NCTM 2000). Charlie shows strong conceptual growth. He is able to construct equivalent sets of teddy bears for the cards he draws, and he can accurately compare the number of teddy bears he has with those of his teacher. He can even make logical inferences: If he and his teacher start with the same quantity and she draws a larger number on her turn, she then has more.

This demonstration of Charlie's conceptual understanding of number bodes well for his future success in school mathematics (Baroody and Ginsburg 1990; Ginsburg and Seo 1999; Klein, Starkey, and Wakeley 1999; Smith 1998; Starkey, Klein, and Wakeley 2004). In fact, the concepts embedded in number sense may be as important to early mathematics learning as concepts of phonemic awareness are to early reading (Gersten and Chard 1999).

Unfortunately—because of limited experiences with number, poverty and its associated risks to proper development, specific disabilities, emotional issues that affect learning and assessment, and difficulties related to learning English as a second language—many preschoolers already lag far behind their peers in understanding numerical relationships. Teachers must pay particular attention to monitoring mathematical development in children who are at risk for further developmental delays. Assessment should be used as a tool for enhancing students' learning (NCTM 2000), but traditional assessment methods may not supply the type of ongoing information that teachers need to make these instructional decisions.

The Arlitt Preschool Curriculum-Based Measure for Math (APCBM-Math), an interactive game with teddy bear counters and a deck of cards prearranged in a standardized order, can be used to monitor children with diagnosed disabilities and for program accountability.

SALLY MOOMAW

Challenges

Reliable assessment of young children's mathematical ability is difficult because of social and emotional factors related to their development. For example, preschool children may quickly become bored with the typical question-answer format of many assessments and refuse to respond. **Table 1** illustrates the challenges of assessing young children and how curriculum-based measures may address them. In contrast to traditional assessments, curriculum-based assessments align with classroom curriculum to offer teachers ongoing information on students' development and thereby guide teachers' planning and instructional decisions (Shinn 1998).

If assessing young children is challenging, it becomes even more so when the youngsters have developmental concerns, such as physical or mental disabilities. However, curriculum-based measures can be designed to be universally accessible to children both with and without disabilities. In the following example, Cara, a preschool-aged child designated as legally blind but with some residual vision, engages in the same curriculum-based measure as Charlie did in the initial example.

Cara draws a card with two large dots on it.

The assessor says, "You can take as many teddy bears as you have dots on your card."

Cara carefully sets one teddy bear on each of

TABLE 1

In contrast to traditional assessments, curriculum-based assessments align with classroom curriculum and offer teachers ongoing information on students' development.

Challenge	Traditional Assessment	Curriculum-Based Assessment
Autonomy	Because preschool children are developing and asserting autonomy (Erikson 1950), issues of compliance often arise during assessment procedures.	The measures can be developed to align with young children's interests.
Attention	Preschool-aged children typically have much shorter attention spans than school-aged children; this characteristic affects the length of time that can be devoted to assessing them.	Curriculum-based measures are designed to be much shorter than other types of developmental assessments.
Trust	Because preschool children are also establishing trusting relationships (Erikson 1950) outside their homes, fear about working with an assessor is a concern.	The measures can be administered by classroom teachers who have training on procedures and scoring.
Development and Maturation	Maturation and learning occur quickly during the preschool years; therefore, ongoing assessment is critical for educational planning.	Curriculum-based measures are designed for making ongoing instructional decisions.
Alignment to Curriculum	Assessment information is often not related to standard preschool experiences and curricula.	Assessment is directly aligned to curriculum.

the two dots on her card. When the assessor asks how many bears Cara has, she replies, "One." When the assessor asks who has more teddy bears so far (Cara has two and the assessor has three), Cara quickly says, "Me," without looking at the assessor's bears.

Throughout the game, Cara consistently decides how many bears to take by aligning them in a one-to-one correspondence with the dots on her card. When asked how many bears she has, she always replies, "One." When asked who has more bears, she consistently answers, "Me."

Later in the game, when the cards have large numerals rather than dots, Cara is able to name two of the numerals but does not take the corresponding number of teddy bears.

Because the assessment measure uses cards with large dots or numerals and high contrast between the background and the symbols, children with visual disabilities can also participate. Also, because the game format offers objects to represent quantities, children who are not yet counters, such as Cara, can demonstrate their knowledge of number by matching the teddy bears to the dots on the card. This is an important stage of early quantification.

What teachers should know

Important research points out the emergence of key number sense concepts that teachers should look for as they interact with young children. First, building on the work of Piaget (1952), Kamii (1982; 2000) has examined young children's thinking as they attempt to quantify or create approximations of groups of objects. Children initially apply a *global strategy of quantification*, in which they rely on visual perception to estimate the number of objects in a group.

As children become more logical in their thinking, they begin to apply *one-to-one correspondence relationships* to quantification. Just as Cara did, they may align one object in the original group with each object in the set they are constructing.

Finally, children begin to use *counting strategies* to quantify. However, to apply counting to

the process of quantification, children must understand *cardinality*, the concept that when they count, the last object counted represents the total.

Additional research that should guide teachers comes from Gelman and Gallistel (1978), which clarified a series of counting principles that children must construct to successfully use counting to solve quantification problems. The first three principles are arguably most pertinent to preschool education (Baroody 1993):

1. *Stable order* is the understanding that when counting, specific number words must be said in a fixed order.
2. *One-to-one correspondence* refers to the realization that each object counted must receive one and only one number tag.
3. *Cardinality* is the knowledge that the last item counted represents the total.

A child's understanding of quantification and counting concepts can be quickly ascertained through a curriculum-based measure with a game format. A play-based measure format, in which the assessor takes turns with the child, provides a more natural context for young children than typical test situations. Furthermore, play provides an important means for children to understand and express relationships, because through play they can rearrange materials and observe the results (Piaget 1968). The following example shows how a teacher can use a curriculum-based measure with a play format to access a Head Start child's understanding of the quantification and counting concepts previously discussed, although the child is an English Language Learner.

Miguel watches as his teacher draws a card with three dots and then takes three teddy bears from the bowl. When it is Miguel's turn, he carefully looks at the dots on his card and then takes the correct number of teddy bears. When his teacher asks how many Miguel has, he holds up the corresponding number of fingers. He is unable to answer the question of who has more teddy bears. However, on a later turn, when Miguel draws a card with five dots, he spontaneously counts them in Spanish—"*uno*, *dos*, *tres*, *quatro*, *cinco*"—but takes six bears. Then, on his next turn, Miguel attempts to count the four dots on his card in English—"three, four, five, six"—but does not take any bears.

Although Miguel is just learning English, his teacher can gain important information from this interactive assessment. First, Miguel is attempting to apply counting as a means of quantifying. He is beginning to show stable order in counting but is inconsistent even in Spanish. On one occasion, Miguel correctly counts a set of five dots in Spanish but later mixes up the order of the counting words, on one occasion counting, "*uno*, *dos*, *quatro*, *ocho*."

In English, Miguel is beginning to learn the number words but does not start with *one*. Miguel does consistently apply the one-to-one correspondence principle when counting the dots on his cards. Whether counting in Spanish or English, he applies one number-word tag to each dot he counts. Because Miguel never uses a number word to indicate the total, he appears to lack use of the cardinality principle (although he does occasionally use his fingers).

The APCBM-Math assessment

Charlie, Cara, and Miguel were assessed using the Arlitt Preschool Curriculum-Based Measure for Math (APCBM-Math) (Carr and Boat 2003; Moomaw 2008). APCBM-Math has been validated, so it can be used for required monitoring of children with diagnosed disabilities and also for program accountability, unlike informal teacher assessments. APCBM-Math is an interactive game that is played between assessor and child. The game consists of a basket of teddy bear counters and a deck of cards, prearranged in a standardized order for consistency. The use of concrete objects is an important component of the assessment because it allows the assessor to document that children understand underlying number sense concepts, such as set representation and comparison (NCTM 2000). The cards contain either a set of three-quarter-inch dots, in both standard and nonstandard format, or a clearly written numeral from one to nine. The assessor's role is clearly scripted to ensure procedural reliability, but the regular exchange of turns between assessor and child frames the assessment as a game from the child's perspective and provides motivation to engage in the tasks.

The first section of the assessment requires children to perform several number tasks:

- **Quantify** sets of dots on a card and represent them with teddy bears.
- **Count** the bears.
- **Compare** a set of bears with the assessor's set.

The second section of the assessment involves numeral tasks:

- **Identify** a numeral on a card.
- **Take** an equivalent number of teddy bears.

The third section involves combining two sets of dots:

- **Draw** two cards.
- **Take** the number of teddy bears equivalent to the sum of the dots on the two cards.

Kali, Steve, and Maria

Using the first section of APCBM-Math, we looked at quantification strategies, application of counting principles, and ability to compare sets (more, less, or the same) for each of three children. We would, of course, play the game separately with each child so they would not influence one another

With each child, the assessor starts by drawing a card and taking a corresponding number of teddy bears—modeling how the game is played. Each time the student takes a turn, the assessor goes through a standard sequence of three directions or questions:

1. "You can take as many teddy bears as you have dots on your card." (This activity allows us to score the child's level of quantification as global, one-to-one correspondence, or counting.)
2. "How many bears do you have?" (This question allows us to assess the child's use of the counting principles, stable order, one-to-one correspondence, and cardinality.)
3. "Who has more bears, you or I, or do we have the same number?" (This question allows us to see which strategy the child uses to compare sets.)

When three-year-old Kali plays the game, she points to each dot on her card, one at a time, and takes one bear each time she points. We score her at the one-to-one correspondence level of quantification.

Four-year-old Steve counts the dots, and Maria, who is also four years old, can tell how many dots are on some of the cards just by looking (subitizing). For cards with five or more dots, Maria counts. We score both Steve and Maria at the counting stage of quantification (see **fig. 1**).

Next we ask each child how many bears he or she has. Kali holds up two fingers when she has two bears; however, as she continues to take more bears on each turn, she just says "more" when we ask her how many she has. She is not yet counting, so we cannot give her any points for counting principles.

Steve counts correctly until he has to quantify eight bears; then he counts faster than he points to the bears and says he has nine. For this item, we give him a point for stable order counting (he said the number words in the right order) and for cardinality (he told us his total by using the number of the last item he counted). We cannot give him a point for the one-to-one principle, though, because he did not say only one number word for each bear.

FIGURE 1

As youngsters' thinking becomes more logical, they apply one-to-one correspondence relationships to quantification.

0 1 2 3

Score 0	The child takes no bears
Score 1	Global quantification
Score 2	One-to-one correspondence
Score 3	Counting

FIGURE 2

Unlike Maria, who received a point for each counting principle, this child's assessment demonstrates stable order and cardinality but not one-to-one correspondence.

0 1 (2) 3 4

Score 1 point for each (4 points possible)

(1) Stable order
1 One-to-one principle
(1) Cardinality
1 Correct answer

Maria's counting is accurate. We can give her points for all three counting principles (see **fig. 2**). When combining small sets, she sometimes does not even count. For example, when she already had two bears and got three more, Maria said, "Two and three—that makes five."

The third task is a little harder. We must ask the child, "Who has more teddy bears?" or whether we have the same amount. Scoring is based on whether the child gives a global response, attempts to align her bears to ours in a one-to-one relationship, or counts each set of bears to answer the question.

When we ask Kali who has more bears, she just glances at both groups and gives a response such as, "Maybe I have more," which we score as a global response based on visual perception.

At first Steve counts both sets of bears and makes such comments as, "We both have five; so, I would say we both have the same amount." We score this as counting. However, as the sets of bears get larger, Steve begins to rely on perceptual judgment, which we score as a global response for these items.

Maria is a solid counter, even when comparing sets of seventeen versus fifteen.

Using the information

We have already learned quite a bit about the numerical thinking of Kali, Steve, and Maria. On the basis of this information, we can design appropriate curricular materials and scaffold or model performance in the most effective way.

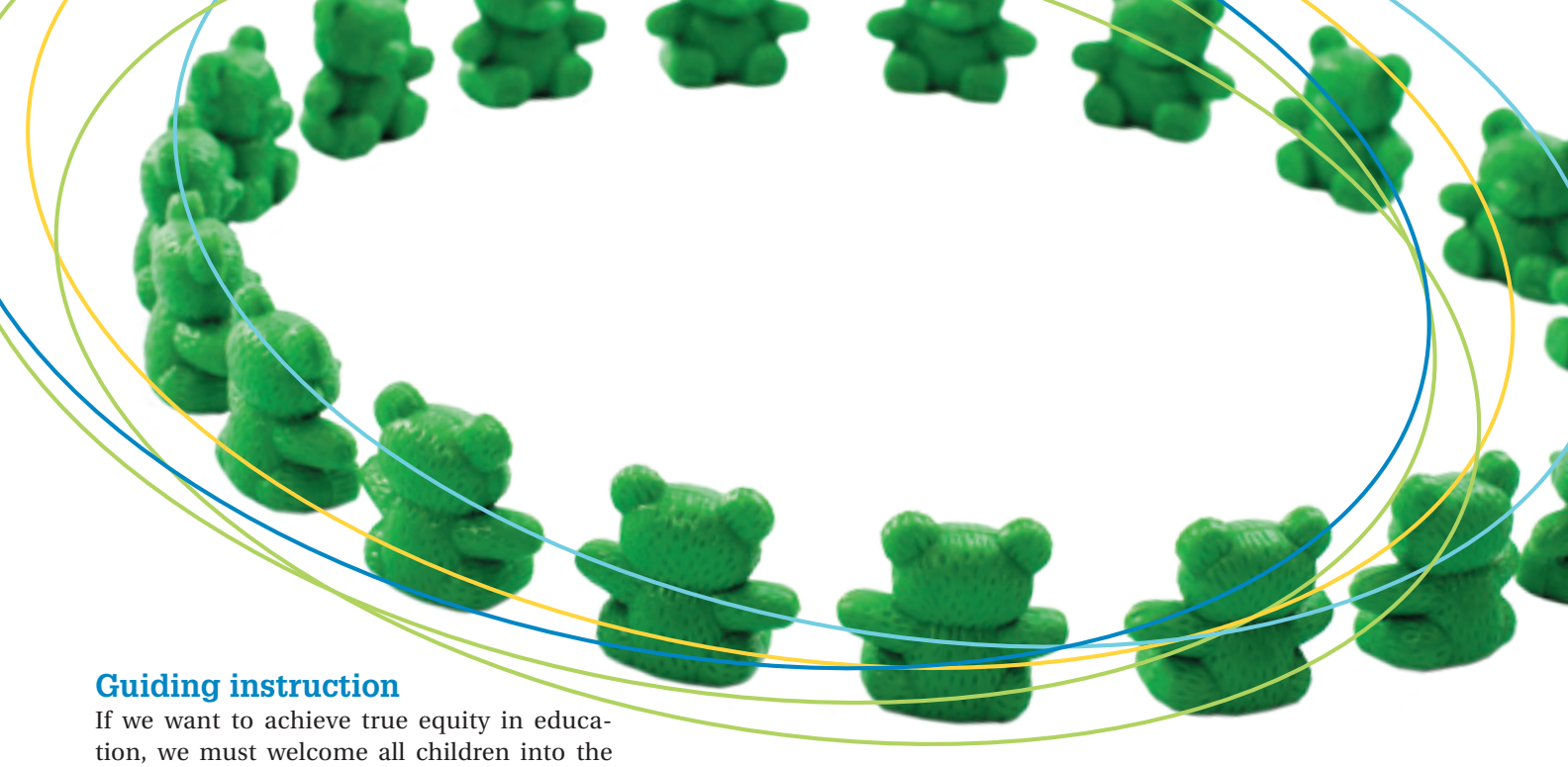
Recall that Kali used one-to-one correspondence to decide how many teddy bears to take each time she drew a card. Because counting is the next stage of quantification, we can model counting for Kali through play situations or daily activities. For example, at snack time, the teacher might say, "I am going to take one cheese cube for each cracker on my plate. Let's see, I have one, two, three crackers, so I will take one, two, three pieces of cheese."

We can use a similar technique to help Kali begin to compare sets; however, because she used a global strategy for comparing sets, we should model the next stage, one-to-one correspondence. In dramatic play, the teacher might ask Kali who has more plastic eggs and then model aligning her eggs with Kali's in a one-to-one fashion. Kali will likely quickly observe who has some extra eggs and begin to use the alignment technique herself.

Steve is at the counting stage of quantification, but—like many young children—for larger sets, he loses the one-to-one relationship between the number words he says and the objects he counts. Rather than correct Steve, which may make him lose confidence and avoid larger quantities, we can model careful use of one-to-one correspondence and even use "self-talk" techniques. For example, the teacher might say, "Wow. I have a lot of toy dinosaurs on my game board. Steve, can you watch and make sure I do not skip any when I am counting them? Sometimes I count too fast and mess up."

Steve is good at comparing smaller sets of about three to five objects, but larger sets overwhelm him; he reverts to global quantification. His teacher can model using one-to-one correspondence with larger sets. For example, in the block area, she might say, "We both have lots of cars. Let's line them up and see who has more." She might even count each group to compare the amounts.

Maria is a secure counter who uses counting to quantify and to compare sets. Recall that she was even able to mentally add three and two. Maria needs many opportunities to practice adding. Games that use two dice will allow her to add two sets by counting all the dots. The teacher can model a counting-on technique. For example, if the teacher rolls a four and a five, she might say, "Well, I know that die has four, so five, six, seven, eight, nine" (while pointing to the dots on the second die).



Guiding instruction

If we want to achieve true equity in education, we must welcome all children into the classroom learning community. This requires developing assessment methods that accurately measure a wide range of children, including those at risk because of socio-economic level, disability, or the necessity of learning a second language. Research on APCBM–Math has shown that a curriculum-based measure, supported by acknowledged research about how children learn fundamental mathematical concepts, can supply teachers with necessary information to construct curriculum and scaffold learning for all children. The assessment itself provides an engaging learning opportunity. Such instruments can become a routine part of classroom activities and support all children in developing mathematical understanding.

BIBLIOGRAPHY

Baroody, Arthur J. "The Relationship between the Order-Irrelevance Principle and Counting Skill." *Journal for Research in Mathematics Education* 24, no. 5 (1993): 415–27.

Baroody, Arthur J., and Herbert P. Ginsburg. "Children's Mathematical Learning: A Cognitive View." In *Constructionist Views on the Teaching and Learning of Mathematics*, Journal for Research in Mathematical Education: Monograph No. 4, edited by Robert B. Davis, Carolyn A. Maher, and Nel Noddings, pp. 51–64. Reston, VA: National Council of Teachers of Mathematics, 1990.

Carr, Victoria W., and Mary B. Boat. "Improving Outcomes for Head Start Children: Active Curriculum Training-Accelerated Child Trajectories (ACT-2)." Annual report, Head Start Child Outcomes Research Support Consortium, Washington, DC, July 2003.

Erikson, Erik, H. *Childhood and Society.* New York: W. W. Norton and Company, 1950.

Gelman, Rochel, and C. R. Gallistel. *The Child's Understanding of Number.* Cambridge, MA: Harvard University Press, 1978.

Gersten, Russell, and David Chard. "Number Sense: Rethinking Arithmetic Instruction for Students with Mathematical Disabilities." *Journal of Special Education* 33, no. 1 (Spring 1999): 18–28.

Ginsburg, Herbert P., and Kyoung-Hye Seo. "Mathematics in Children's Thinking." *Mathematical Thinking and Learning* 1, no. 2 (June 1999): 113–29.

Kamii, Constance. *Number in Preschool and Kindergarten: Educational Implications of Piaget's Theory.* Washington, DC: National Association for the Education of Young Children, 1982.

———. *Young Children Reinvent Arithmetic: Implications of Piaget's Theory.* 2nd ed. New York: Teachers College Press, 2000.

Klein, Alice, Prentice Starkey, and Ann Wakeley. "Enhancing Prekindergarten Children's Readiness for School Mathematics." Paper presented at the Annual Meeting of the American Educational Research Association, Montreal, Quebec, 1999. ERIC Document Reproduction Service No. ED431556.

Moomaw, Sally. "Measuring Number Sense in Young Children." EdD diss., University of Cincinnati, 2008.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, VA: NCTM, 2000.

Piaget, Jean. *The Child's Conception of Number.* New York: W. W. Norton and Company, 1952.

———. "Quantification, Conservation, and Nativism." *Science* 162 (1968): 976–79.

Rogers, Cosby S., and Janet K. Sawyers. *Play in the Lives of Children.* Washington, DC: National Association for the Education of Young Children, 1988.

Shinn, Mark R. *Advanced Applications of Curriculum-Based Measurement.* New York: Guilford Press, 1998.

Smith, Susan Sperry. "Early Childhood Mathematics." Paper presented at the Forum on Early Childhood Science, Mathematics, and Technology Education, Washington, DC, 1998. ERIC Document Reproduction Service No. ED416994.

Smock, Charles D. "A Constructivist Model for Instruction." In *Models for Learning Mathematics,* edited by Alan R. Osborne. Columbus, OH: ERIC Center for Science, Mathematics, and Environmental Education, 1976.

Starkey, Prentice, Alice Klein, and Ann Wakeley. "Enhancing Young Children's Mathematical Knowledge through a Prekindergarten Mathematics Intervention." *Early Childhood Research Quarterly* 19, no. 1 (1st quarter 2004): 99–120.




The authors are colleagues at the University of Cincinnati. **Sally Moomaw**, **sally.moomaw@uc.edu**, taught preschool and kindergarten children for twenty-five years. She is now an assistant professor, teaching and conducting research in early childhood mathematical development. **Victoria Carr**, **victoria.carr@uc.edu**, is an assistant professor and directs the Arlitt Child and Family Research and Education Center. She is interested in evidence-based practices in preschool settings. **Mary Boat**, **mary.boat@uc.edu**, is an associate professor of early childhood education who coordinates early childhood licensure, distance learning, and graduate programs. **David Barnett**, **barnetdw@uc.edu**, is a professor of school psychology. His teaching and research interests include early intervention, measurement, and decision making.