Catalyzing Change in High School Mathematics: Initiating Critical Conversations

Matt Larson
NCTM President

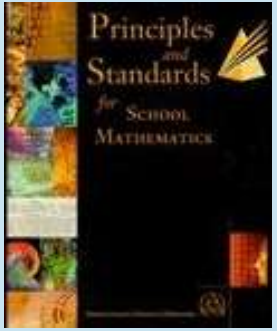
Thank You to the Writing Team!

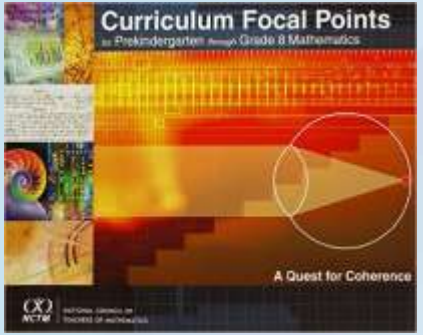
Karen J. Graham, Chair **University of New Hampshire** **Durham, New Hampshire**	
Gail Burrill **Michigan State University** **East Lansing Michigan**	**Jennifer Curtis** **NC Math Alliance Supporting Teachers,** **Durham, North Carolina**
Ed Dickey **University of South Carolina** **Columbia, South Carolina**	**Kanita DuCloux** **Western Kentucky University** **Bowling Green, Kentucky**
Christine Franklin **American Statistical Association** **University of Georgia** **Athens, Georgia**	**Damarrio C. Holloway** **Discovery High School** **Lawrenceville, GA**

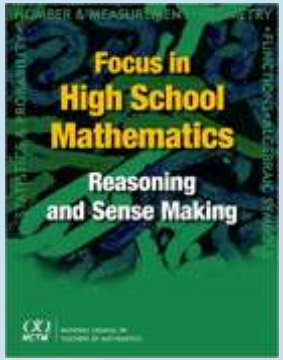
Thank You to the Writing Team!

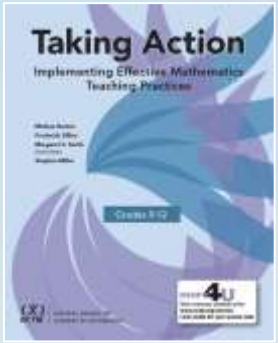
Paul Kelley Anoka High School Anoka, Minnesota	Yvonne Lai University of Nebraska-Lincoln Lincoln, Nebraska
Max Ray-Riek NCTM Reston, Virginia	John W. Staley Baltimore County Public Schools Towson, Maryland
Francis Su Harvey Mudd College Claremont, CA	Daniel J. Teague North Carolina School of Science & Math Durham, North Carolina
Gwendolyn Zimmermann Adlai E. Stevenson High School Lincolnshire, Illinois	David Barnes, Staff Liaison NCTM Reston, Virginia
Matt Larson, Contributing Author Lincoln Public Schools University of Nebraska-Lincoln Lincoln, Nebraska	Robert Q. Berry III, Contributing Author University of Virginia Charlottesville, Virginia

The Last Three Decades Have Seen Significant Progress in the Teaching and Learning of Mathematics

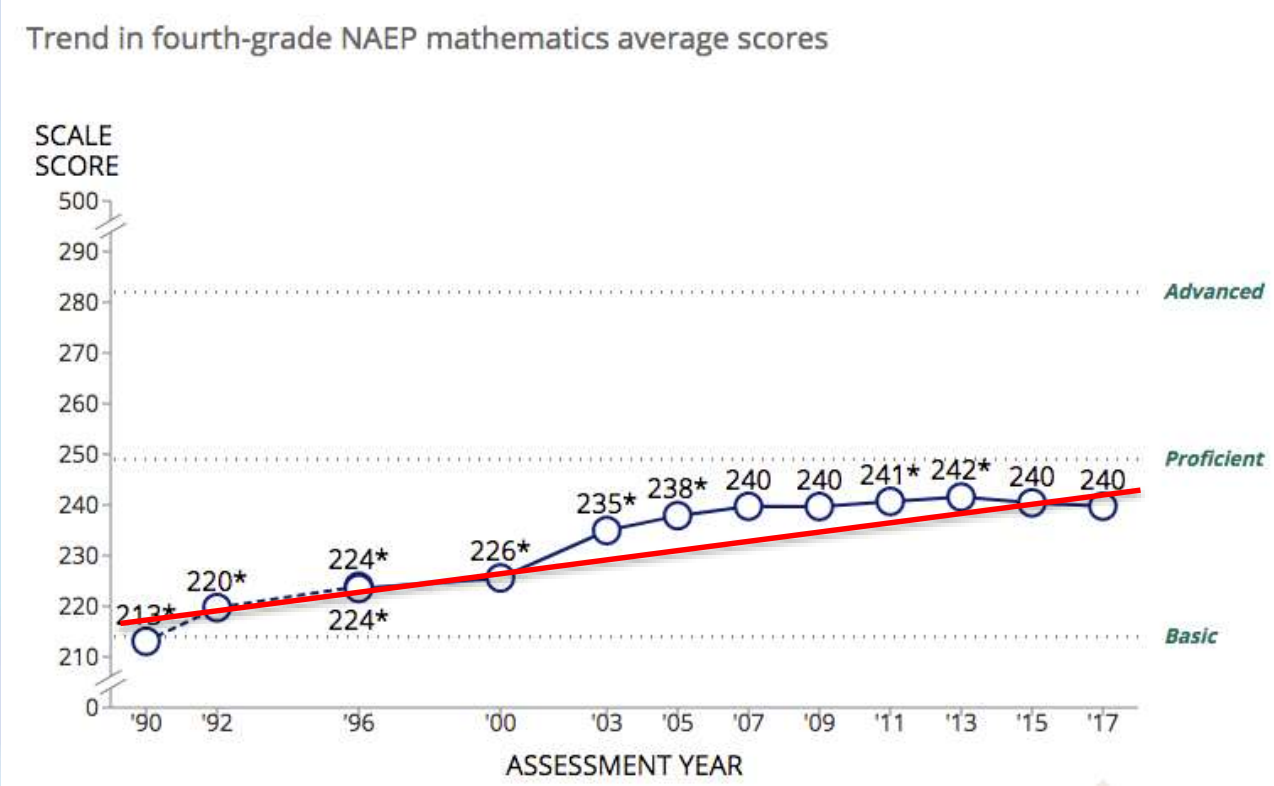






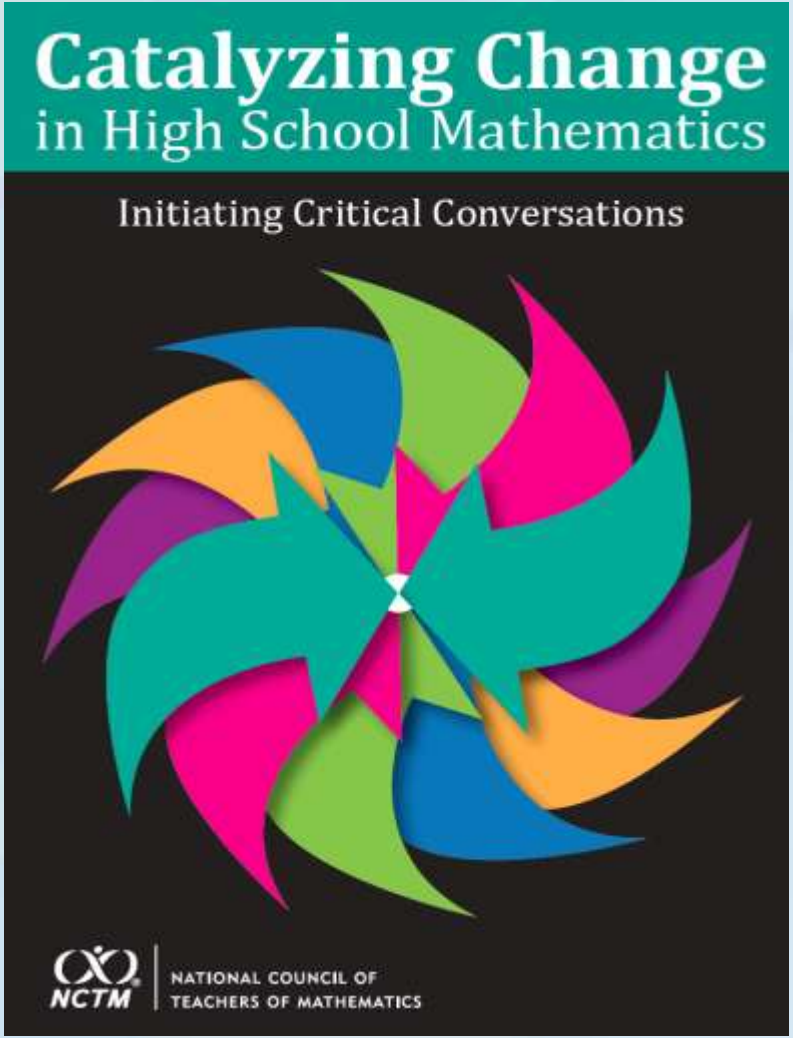


We Have Seen Improved Mathematics Learning at the Elementary and Middle Levels



Why *Catalyzing Change in High School Mathematics*?

Why now?

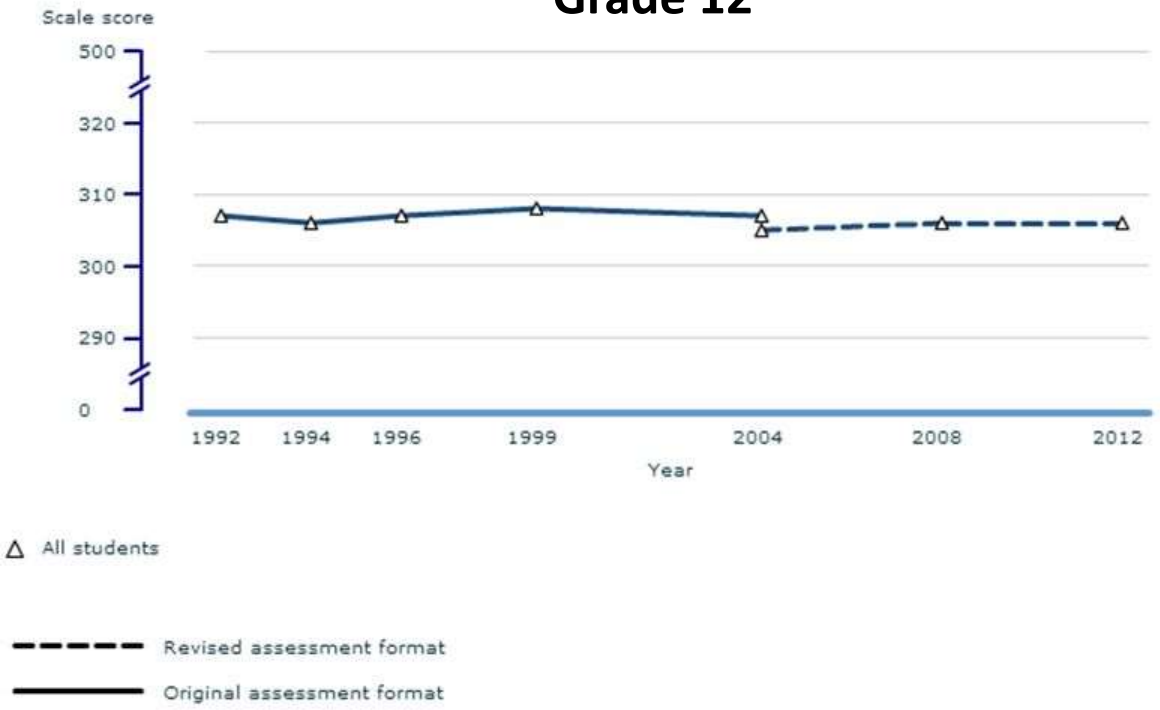


The steady improvement in mathematics learning at the elementary and middle levels has not been shared to the same degree at the high school level.

NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

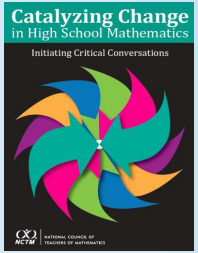
NAEP 10 Year Trend Line (Math)

Grade 12



Source: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1992, 1994, 1996, 1999, 2004, 2008 and 2012 Long-Term Trend Mathematics Assessments.

An **implementation gap persists** at the high school level between the calls for change and the comprehensive actions needed to support each and every high school student.



NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

High School Mathematics Learning Progress

- The percentage of high school students enrolling in upper-level mathematics courses over the last three decades has increased.
- Advanced Placement participation rates in mathematics have steadily increased, reaching an all-time high in the 2016-2017 school year.

Dossey, J. A., McCrone, S. S., & Halvorsen, K. T. (2016). *Mathematics education in the United States 2016.* Reston, VA: NCTM.

College Board, 2017

High School Mathematics Learning Stagnation

U.S. high school students trail their international peers in mathematical literacy with respect to their “capacity to formulate, employ, and interpret mathematics in a variety of contexts”

OECD, 2016

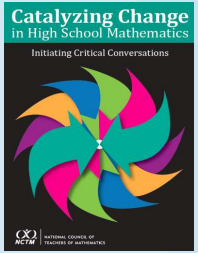
High School Mathematics Learning Stagnation

U.S. young adults lack the numeracy and problem-solving skills necessary for “meaningful participation in our democratic institutions.”

Goodman, Sands, & Coley, 2015

High School Mathematics Learning Challenges

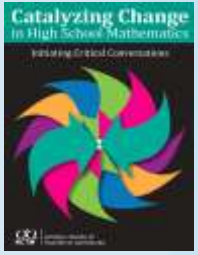
Most critically, large numbers of high school students do not have access to the mathematics they need either for their personal or for their professional lives.



NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

High School Mathematics Learning Challenges

- Many mathematics standards at the high school level lack the focus of mathematics standards at the K-8 level.
- In most high schools, teachers find it difficult to teach at the desired level of rigor, given the sheer amount of content expected.



NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

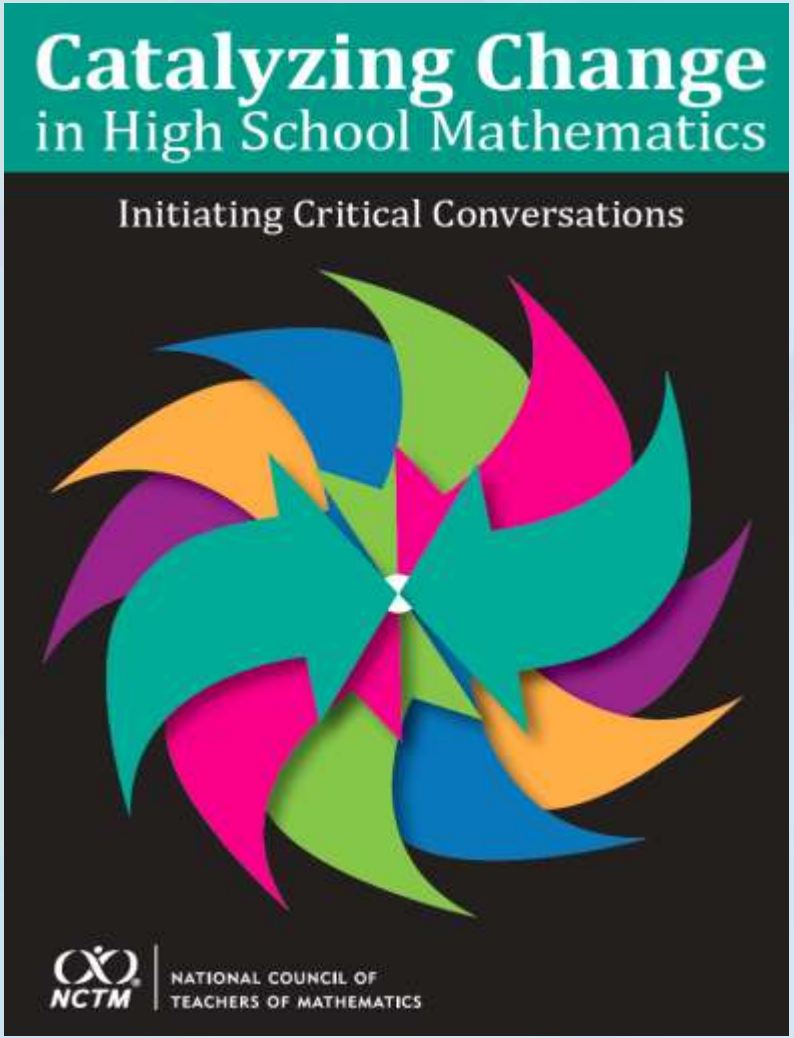
To Be Clear: This is Not About Blame or Shame!

Mathematics education at the high school level is part of a **complex system** of policies, traditions, and societal expectations.

Catalyzing Change is written **to engage all individuals** with a stake in high school mathematics in the **critical conversations** that must take place to bring about and give support to **necessary changes** in high school mathematics.

NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

To Initiate Critical Conversations, *Catalyzing Change* makes four key recommendations.



Four Key Recommendations

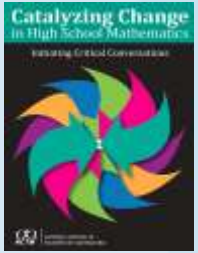
- The purpose of learning mathematics and Essential Concepts
- Equitable Structures
- Equitable Instruction
- A Common Essential Concepts Pathway

NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Key Recommendation

Each and every student should learn the Essential Concepts in order to

- ***expand professional opportunities,***
- ***understand and critique the world, and***
- ***experience the joy, wonder, and beauty of mathematics.***



NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Why Learn Mathematics?

This is a simple question, but worth considerable reflection. Because how you answer this question will strongly determine ***who* you think** should be doing mathematics, and ***how* you will teach** it.

Su, F. (2017). *Mathematics for Human Flourishing.* Presidential Address, AMS-MAA Joint Math Meetings, Atlanta, January 6, 2017.

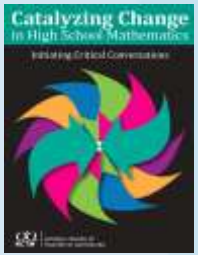
Traditionally Math Education Reform Has Been Driven by Economic Concerns

Traditionally, mathematics education has been connected to issues of national economic survival, rather than to the development of democratic citizenship through critical thinking in mathematics.

Tate, W. F. (2013). Race, retrenchment, and the reform of school mathematics. In E. Gutstein & B. Peterson (Eds.), *Rethinking mathematics: Teaching social justice by the numbers, second edition* (pp. 42-51). Milwaukee, WI: Rethinking Schools.

Broadening the Aims of Mathematics Education

Never have the broader aims of mathematics education been more important than they are today when **mathematics underlies much of the fabric of society**, from polling and data mining in politics, to algorithms targeting advertising to groups on social media, to complex mathematical models of financial instruments.



NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Expand Professional Opportunities

- The economy is expected to create a large number of STEM jobs over the next decade.
- Beginning salaries for STEM majors will outpace those of other college majors.
- Salary growth in STEM careers will outpace that in other careers.

National Association of Colleges and Employers, 2016

PCAST, 2012

Vilorio, 2014

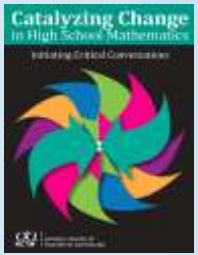
Expand Professional Opportunities

High school mathematics is increasingly essential today for access to and success **in a wide variety of careers and the social sciences that were not historically seen as quantitative in nature**.

NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Understand and Critique the World

Mathematics is deeply embedded in many aspects of life, both seen and unseen – communication systems, transportation systems, medicine, manufacturing, finance, security, science, engineering, technology, and “big data.”



NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

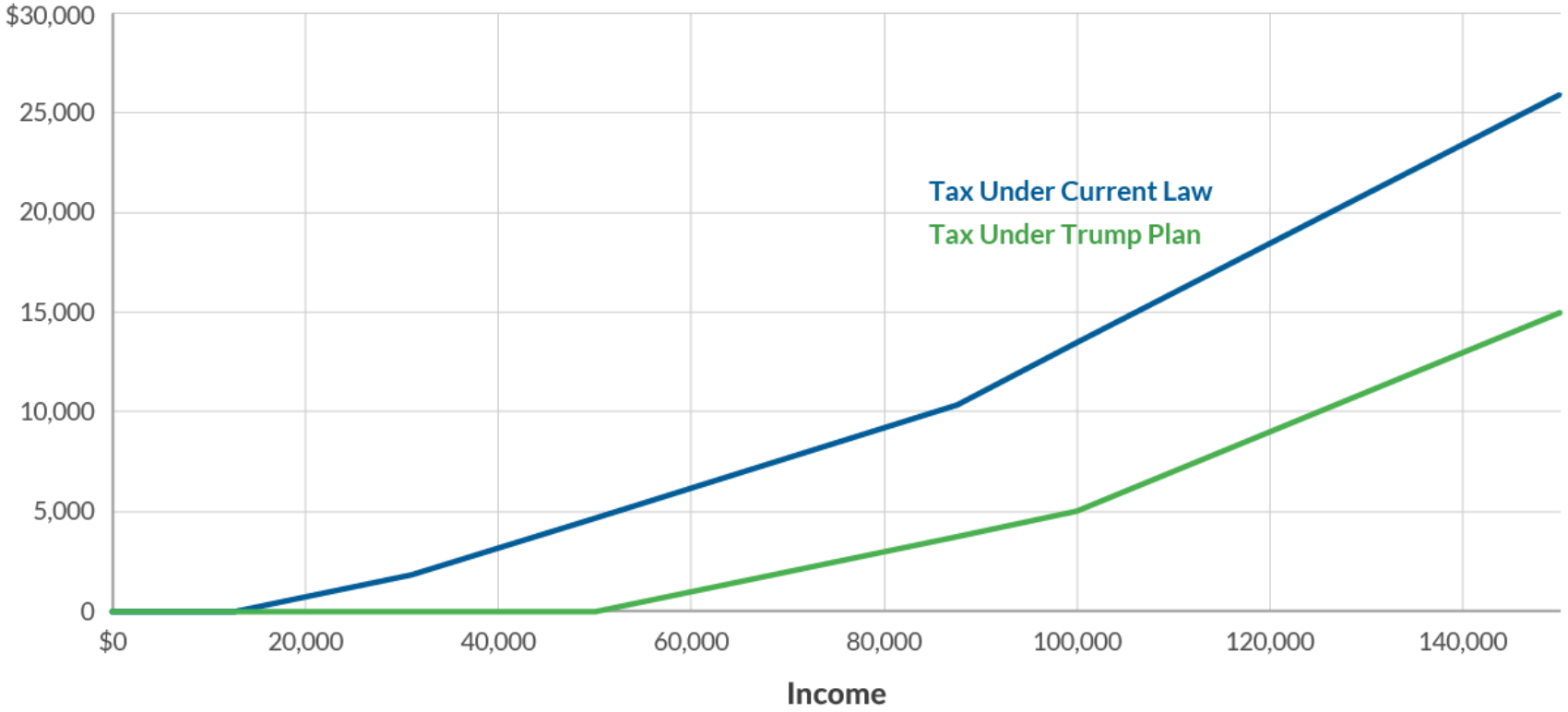
Understand and Critique the World

Students should be able to identify, interpret, evaluate, and critique the mathematics embedded in social, scientific, commercial, and political systems, as well as the claims made in the private and public sectors and in public interest group pronouncements.

Ernest, 2010

The Trump Plan Is A Large Tax Cut For The Middle Class

Taxes for a Married Couple Under the Trump Plan and Current Law



Source: IRS, donaldjtrump.com
Note: This comparison includes only the brackets and the standard deduction. All income assumed to be wage income.

In Trump's plan, the top 1% of earners would pay a smaller share of total federal taxes than it does currently, while shares would rise for groups in the middle of the income spectrum.

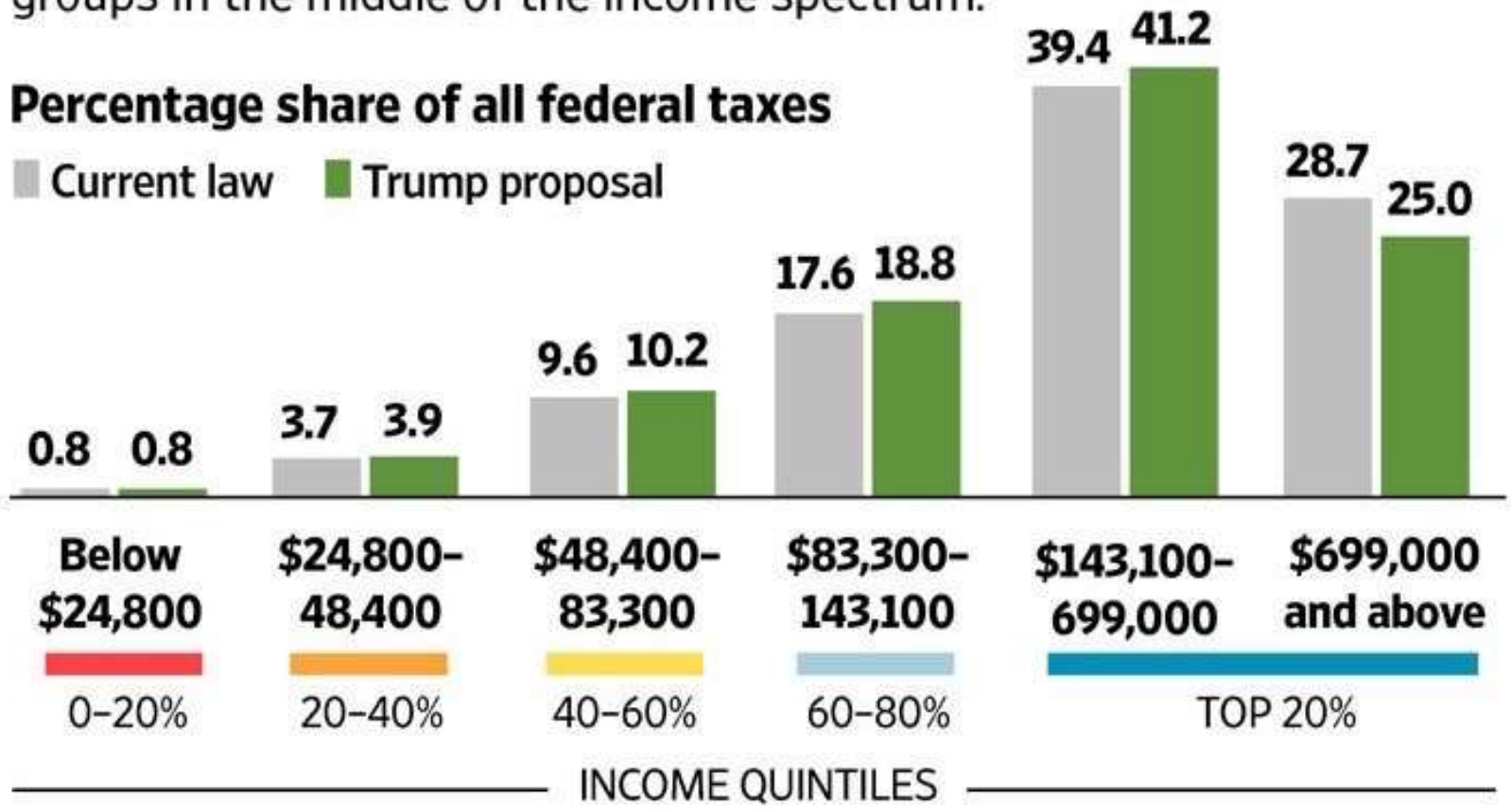
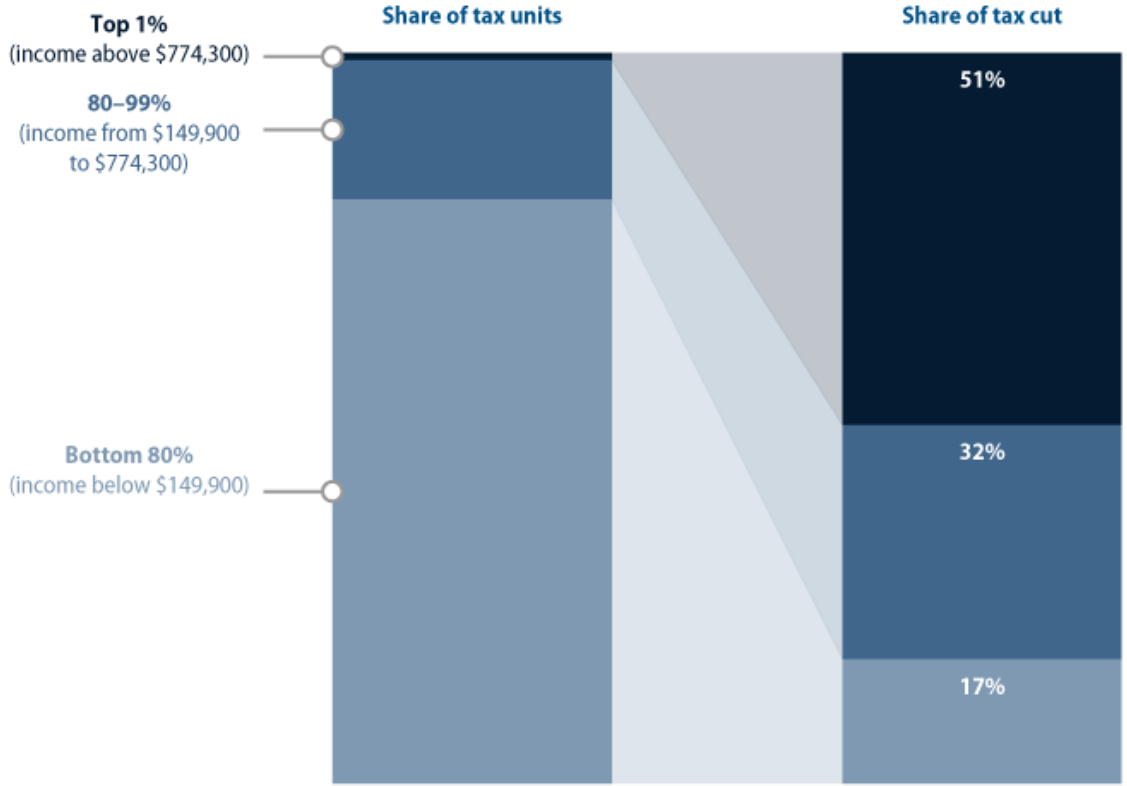


FIGURE 1

Trump's campaign tax plan was skewed toward the top

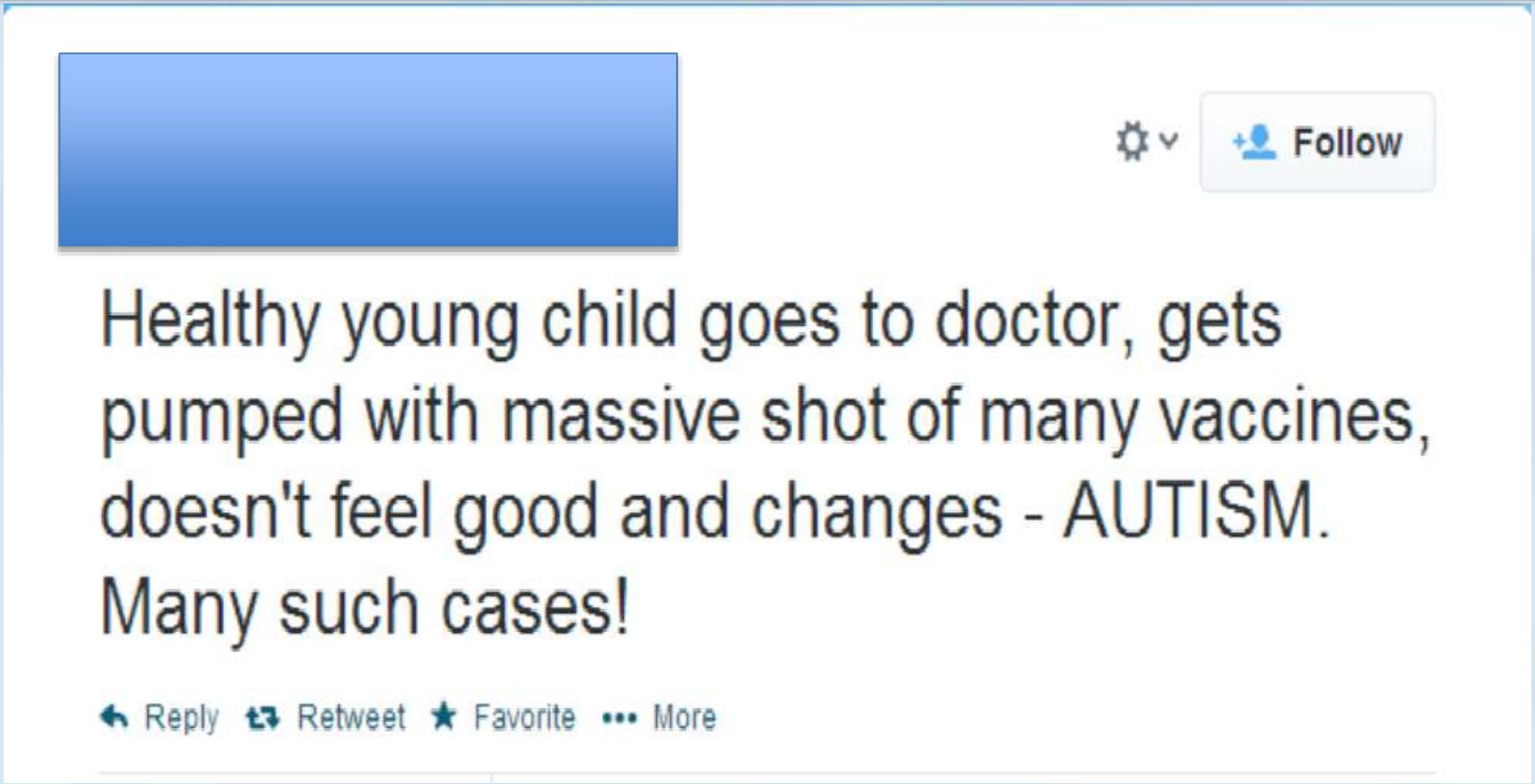
The top 1 percent got half the tax cut



Source: Tax Policy Center, "T16-0214 - Donald Trump's Revised Tax Plan, Distribution of Federal Tax Change by Expanded Cash Income Percentile, 2025" (2016), available at http://www.taxpolicycenter.org/model-estimates/donald-trumps-revised-tax-plan-oct-2016/t16-0214-donald-trumps-revised-tax-plan.

CAP

Understand and Critique the World



Mathematical Illiteracy

The plural of anecdote is neither “proof” nor is it “evidence.”

Do vaccines cause autism? The evidence

AUTISM IS NOT CAUSED BY VACCINES

A study of more than **1.25 million people**, combining results from multiple countries, multiple scientists, and multiple funding sources proves that **vaccines do not cause autism**.

Taylor, L. E., Swerdfeger, A. L., & Eslick, G. D. (2014). Vaccines are not associated with autism: An evidence-based meta-analysis of case-control and cohort studies. *Vaccine*.

VACCINES DO NOT CAUSE AUTISM

THE EVIDENCE IS CLEAR

VACCINES LINKED WITH AUTISM

1998 study of **12 children**.

3 of the children did not have autism at all, and 5 had developmental concerns before their vaccination.

Study funded by lawyers planning to sue a vaccine manufacturer.

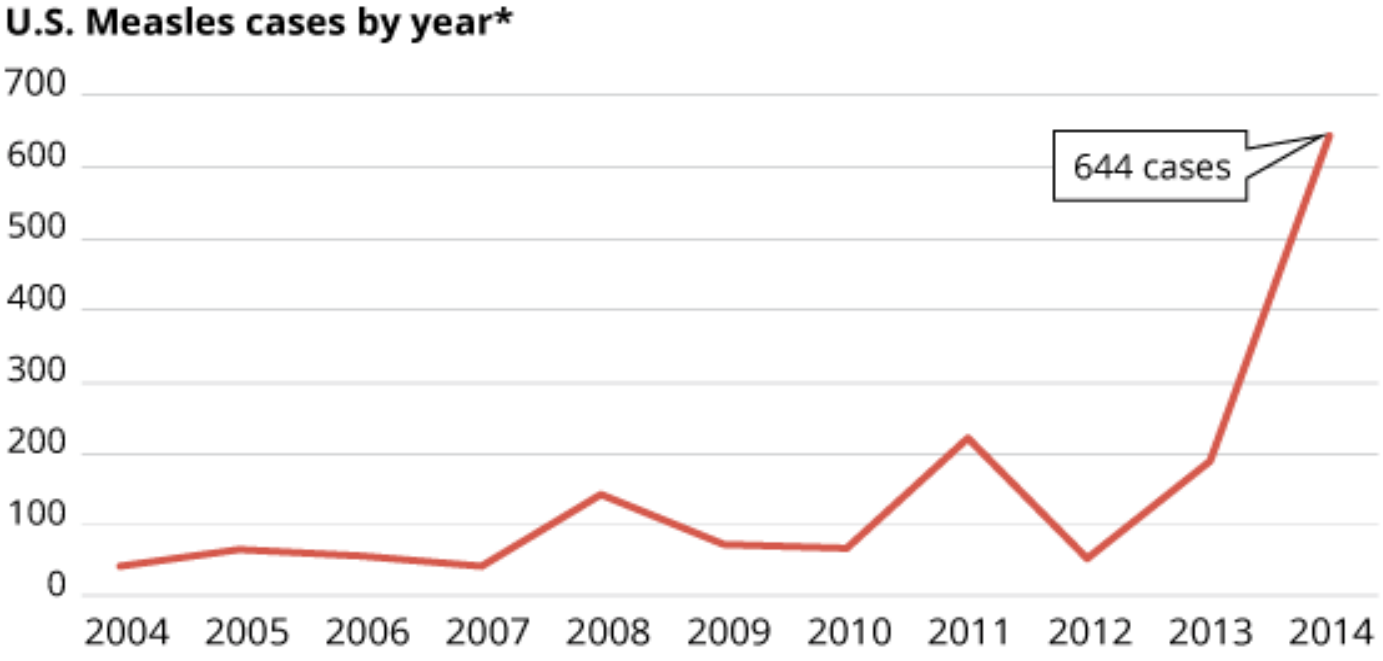
Researcher struck off the medical register for **serious professional misconduct**.

Researcher described by the General Medical Council as being dishonest and irresponsible, having acted with callous disregard for the children studied.

Research formally retracted.

Deer, B. (2011). How the case against the MMR vaccine was fixed. *BMJ, 342*.

Measles Cases Increased Dramatically In 2014



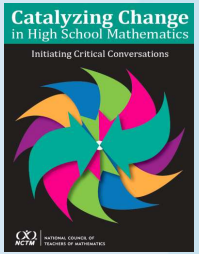
*Provisional data from 27 states reported to CDC's National Center for Immunization and Respiratory Diseases (NCIRD)

Source: CDC

THE HUFFINGTON POST

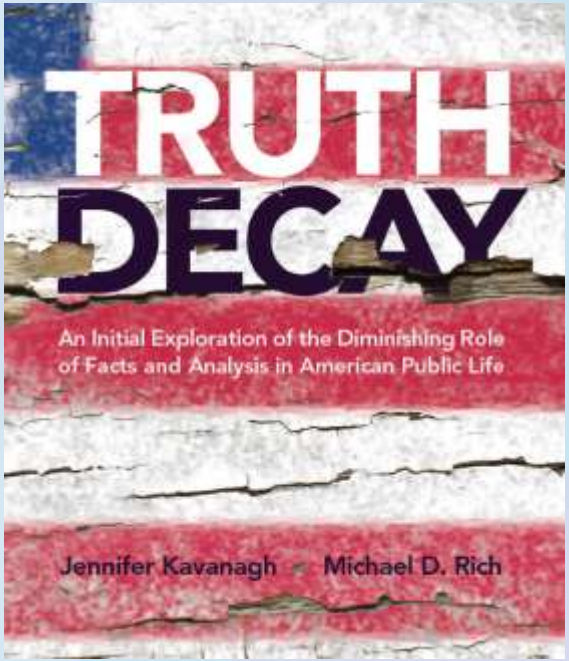
Understand and Critique the World

All high school graduates will, as members of society, be presented with data-based claims throughout their lives. Therefore, they must be able to examine these claims and be intelligent consumers of studies, capable of reasoning critically and questioning the implementation of the statistical investigation process in those studies.



NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Effective Mathematics Education Can Help Combat Truth Decay



- Increasing disagreement about facts and analytical interpretation of facts and data.
- A blurring of the line between opinion and fact.
- The increasing relative volume, and resulting influence, of opinion and personal experience over fact.
- Declining trust in formerly respected sources of factual information.

Kavanagh, J., & Rich, M. D. (2018). *Truth decay: An initial exploration of the diminishing role of facts and analysis in American public life.* Santa Monica, CA: RAND Corporation.

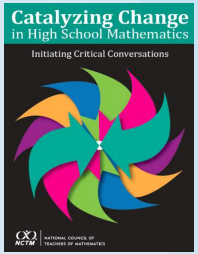
Understand and Critique the World

When high school mathematics courses integrate tasks that, for example, address income distributions, sustainability, mortality rates, taxing structures, etc., students have "access to rich, rigorous mathematics that **offers opportunities and self-empowerment** for them to understand and use mathematics in their world."

Wager, A. A., & Stinson, D. W. (Eds.). (2012). *Teaching mathematics for social justice: Conversations with educators.* Reston, VA: NCTM.

Experience Wonder, Joy and Beauty

High school mathematics can potentially cultivate in students a sense of wonder, beauty and joy – and doing so is an important, but often neglected, purpose for learning mathematics.



NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Turn to Your Neighbor …

For what percent of U.S. adults was their last formal mathematics learning experience negative?

Do you think this percentage is higher or lower for history, art, or literature?

Turn to Your Neighbor …

For what percent of U.S. adults was their last formal mathematics learning experience negative?

Do you think this percentage is higher or lower for history, art, or literature?

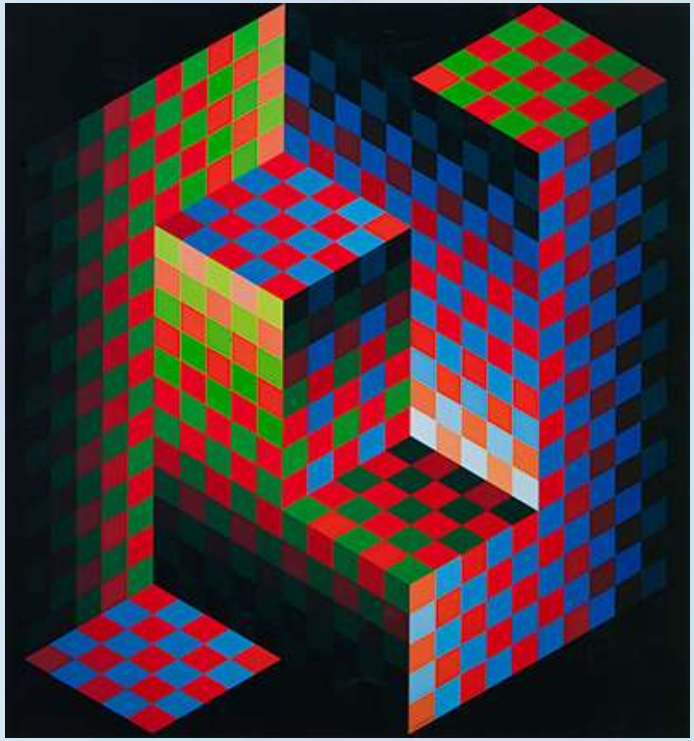
Turn to Your Neighbor …

For what percent of U.S. adults was their last formal mathematics learning experience negative?

98.5%

Do you think this percentage is higher or lower for history, art, or literature?

Experience Wonder, Joy and Beauty



Great ideas of mathematics are as beautiful as great works of art, and just as in the study of art, students can learn to see mathematics as expressions of beauty fashioned by drama and struggle.

NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Experience Wonder, Joy and Beauty

Mathematical ideas are a triumph of human creativity and framing mathematics in this way may support students in coming to see mathematics as a creative endeavor.

Lockhart and Devlin, 2009

Experience Wonder, Joy and Beauty

Rehumanizing mathematics … recouples it with connection, joy, and belonging. When people are encouraged to express themselves through the practice of mathematics, they are more likely to draw upon an innate sense of aesthetics and intuition.

Gutiérrez, R. (2018). Introduction: The need to rehumanize mathematics. In R. Gutiérrez, I. Goffney, & M. Boston (Eds.), *Rehumanizing mathematics for Black, Indigenous, and Latinx students* (pp. 1-9). Reston, VA: NCTM.

Experience Wonder, Joy and Beauty

Mathematics has been shaped by cultures across the globe in which it was developed, and it is important to see the role of mathematics in history and society.

Ernest, 2010

Experience Wonder, Joy and Beauty

The mathematician does not study pure mathematics because it is useful; he studies it because he delights in it and he delights in it because it is beautiful.

Henri Poincare, 19th Century

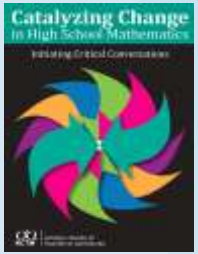
Experience Wonder, Joy and Beauty

The mathematician does not study pure mathematics because it is useful; ⚧ studies it because ⚧ delights in it and ⚧ delights in it because it is beautiful.

Henri Poincare, 19th Century

Key Recommendation

High school mathematics should discontinue the practice of tracking teachers as well as the practice of tracking students into qualitatively different or dead-end course pathways.



NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Mathematics education will likely always involve significant tensions.

Lawler, B. R., & Meyer, B. (2017, Nov.). *Tensions of teaching for positive mathematics identity.* National Council of Teachers of Mathematics 2017 Innov8 Conference, Las Vegas.

Our Challenge

There is a longstanding, thoroughly documented, and seemingly intractable problem in mathematics education: inequity.

Children of certain racial, ethnic, language, gender, ability, and socio-economic backgrounds **experience mathematics education in school differently** and many are disaffected by their mathematics education experience.

Aguirre, J., Herbel-Eisenmann, B., Celedon-Pattichis, S., Civil, M. Wilkerson, T., Stephan, M., Pape, S., & Clements, D. H. (2017). Equity within mathematics education research as a political act: Moving from choice to intentional collective professional responsibility. *Journal for Research in Mathematics Education, 48(2),* 124-147.

We Must Face Hard Truths

Mathematics education often reinforces, rather than moderates, inequalities in education.

OECD. (2016). *Equations and inequalities: Making mathematics accessible to all.* Paris: PISA OECD Publishing. Downloaded at http://dx.doi.org/10.1787/9789264258495-en.

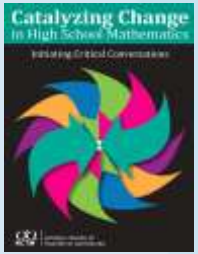
Creating Equitable Structures

Equity in mathematics education will not be achieved until it is no longer possible “to predict mathematics achievement and participation based solely on student characteristics such as race, class, ethnicity, sex, beliefs, and proficiency in the dominant language.”

Gutiérrez , 2002

Creating Equitable Structures

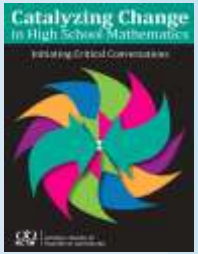
Current reform efforts that focus largely on standards, with some attention to improved instructional practice, are unlikely to address and alleviate equity concerns unless they also **address and dismantle the conditions and system structures** that stand as barriers to the creation of positive mathematical experiences for students.



NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Three Significant Structural Barriers Within Educators' Sphere of Influence

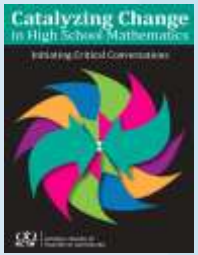
- Tracking students into course pathways that do not prepare students for **continued study** of mathematics.
- Tracking teachers in ways that deny certain students **access** to high-quality instruction.
- Providing **inadequate instructional supports** both before and during high school.



NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Student Tracking

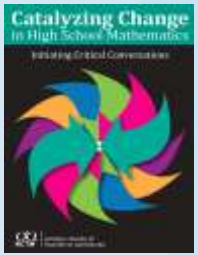
A pathway is a course progression for a student through high school mathematics. Pathways include tracks – fixed sequences of courses that are often determined in middle school or earlier. Courses in a track often place students in different levels of the same course.



NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Student Tracking

Tracking is insidious because it places some students into qualitatively different or lower levels of a mathematics course and, in some cases, puts students into terminal mathematics pathways that are not mathematically meaningful and do not prepare them for any continued study of fundamental mathematics.



NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Student Tracking

Too often placement into different tracks is based on a variety of nonacademic factors, such as perceived (but not potential) academic ability, race, socioeconomic status, gender, language, or other expectations ascribed to students by adults.

Stiff and Johnson, 2011

Student Tracking Often Begins Early

Although tracking is often viewed as a secondary concern, the reality is that “tracks” in mathematics are often established as early as the primary grades when students who struggled in kindergarten are placed in a “low-slow” mathematics group in first grade.

Flores, A. (2008). The opportunity gap. *TODOS research monograph. Promoting high participation and success in mathematics by Hispanic students: Examining opportunities and probing promising practices,* 1(1), 1-18.

Different Opportunities for Different Students

The learning opportunities provided for low-ability, average-ability, and high ability-grouped classrooms are hierarchically different.

Boaler, J., Wiliam, D., & Brown, M. (2000). Students' experiences of ability grouping – disaffection, polarisation and the construction of failure. *British Educational Research Journa*l, 26(5), 631-648.

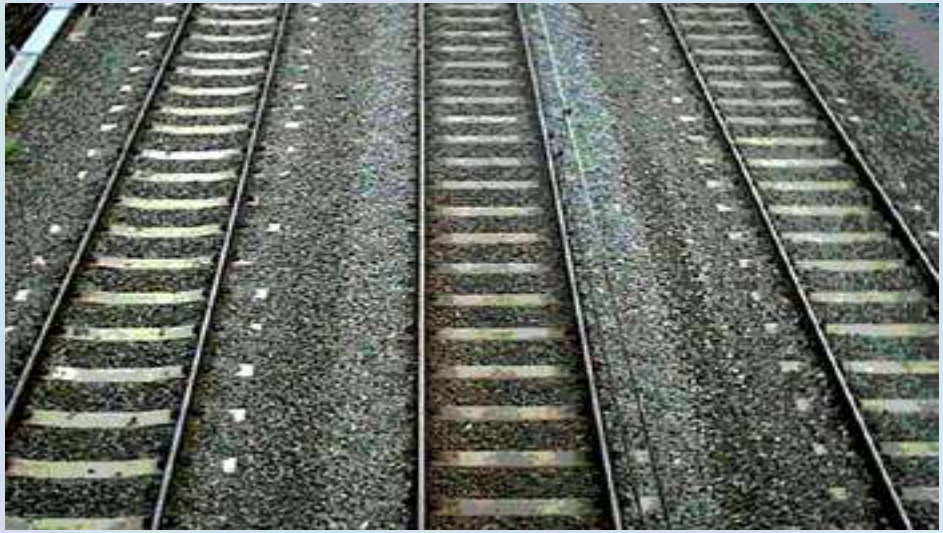
Type and Quality of Instruction Matters

While marginalized students tend to learn simple facts and figures and are exposed to simple applied problems, their privileged counterparts experience mathematics instruction that help them think like a mathematician, develop deep conceptual understanding and advanced mathematical reasoning skills.

OECD. (2016). *Equations and inequalities: Making mathematics accessible to all.* Paris: PISA OECD Publishing. Downloaded at http://dx.doi.org/10.1787/9789264258495-en.

“Educide” Via Tracking

Low expectations often result in self-fulfilling prophecies. Once placed in the low tracks, it is very difficult for students to move to a higher track.



Flores, A. (2008). The opportunity gap. *TODOS Research Monograph: Promoting High Participation and Success in Mathematics by Hispanic Students: Examining Opportunities and Probing Promising Practices*, 1(1), 1-18.

Student Tracking Exists in Various Forms

- Different lengths of courses.
- Different versions of an Algebra or Algebra 2 course.

Darling-Hammond, L. (2007). The flat earth and education: How America's commitment to equity will determine our future. *Educational Researcher*, 36(6), 318-334.

AERA. (2006). Do the math: Cognitive demand makes a difference. *Research Points: Essential Information for Education Policy*, 4(2).

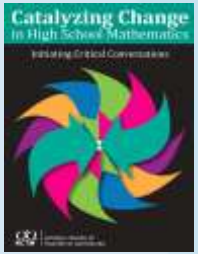
There Are Differential Outcomes For "Different" Algebras

Students placed into less rigorous versions of Algebra 1 ultimately have lower math attainment in high school even if their performance in the less rigorous version of algebra is greater than that of students in more rigorous versions.

Tyson, W., & Roksa, J. (2017). Importance of grades and placement for math attainment. *Educational Researcher,* 46(3), 140-142.

Supporting Student Success in High School Mathematics

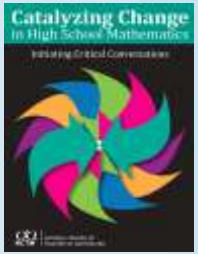
Double-period versions of a course **do not** represent lower-level versions of a course. Double-period versions of a course do not constitute tracking **if** the double-period version of the course has the same instructional objectives and expectations and uses the same core instructional materials and assessments.



NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Distinguishing Tracking from Appropriate Acceleration

Acceleration may be appropriate **if a student has demonstrated deep understanding** of grade-level or course-based mathematics standards beyond his or her current level.

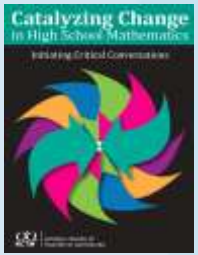


NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Distinguishing Tracking from Appropriate Acceleration

Appropriate acceleration ensures that …

- Opportunities are available to each and every prepared student and **no critical concepts are rushed or skipped**.
- Is along a single common shared pathway that provides each student an opportunity to learn the same Essential Concepts.

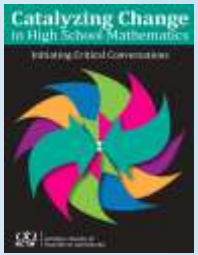


NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Distinguishing Tracking from Appropriate Acceleration

Appropriate acceleration ensures that …

- **Opportunities are open to a wide range of students** who express a higher degree of interest in mathematics, not just those who are identified through traditional assessment instruments.



NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

We Must Put an End to “Educide”

There is no good reason to tell a student she doesn’t belong in math [your class] … you see a snapshot of her progress, but you don’t see her trajectory. You can’t know how she will grow and flourish in the future. **But you can help get her there.**

Francis Su, Past President of the Mathematical Association of America, 2017

Teacher Tracking

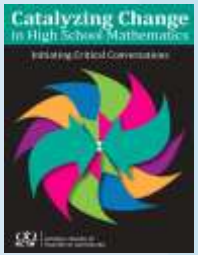
Like mathematics students, mathematics teachers themselves are often tracked, with the most experienced teachers, or those who are perceived to be most effective, assigned to upper-level mathematics courses and the least experienced assigned to entry-level mathematics courses.

Darling-Hammond, 2007

Strutchens, Quander, and Gutiérrez, 2011

Teacher Tracking

Whenever possible, high school mathematics teachers in the same department should have teaching assignments that are balanced to include both upper-level and entry-level mathematics courses.



NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

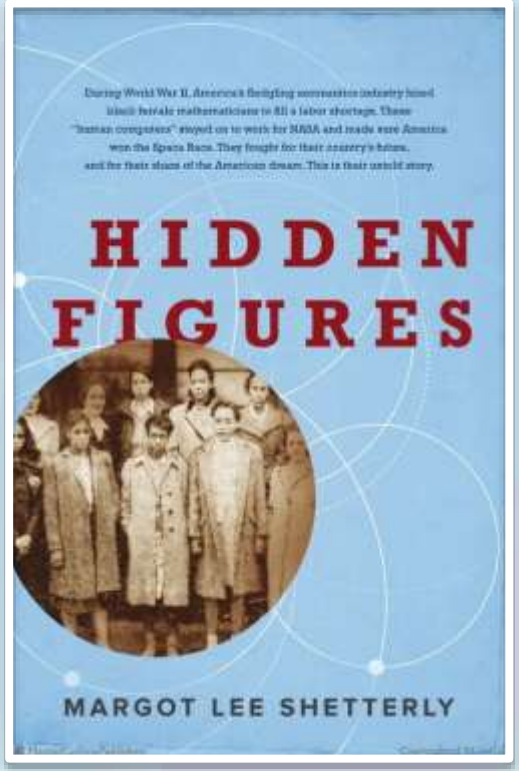
Balanced Teaching Assignments

Balancing teaching assignments deepens teachers' knowledge of the overall curriculum expectations, can reduce burnout among new teachers, can populate **collaborative teams with experienced teachers**, and can develop among teachers a **collective sense of responsibility for all students**.

Gutiérrez, 2002

Strutchens, Quander, and Gutiérrez, 2011

A Personal and Societal Loss



We deny society potential solutions to many of the problems we face, and we slow the advancement of mathematics itself, when we systematically ignore vast human potential.

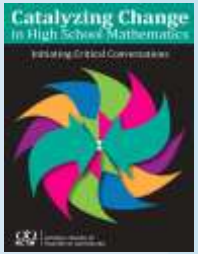
Student and Teacher Tracking are Not Just

"We have to recognize that even if people are just, even they desire to be just, a society may not be just if its structures and practices are not also just."

Su, 2017

Key Recommendation

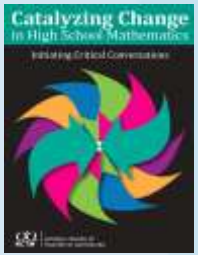
Classroom instruction should be consistent with research-informed ***and equitable*** teaching practices.



NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Equitable Mathematics Teaching Practices

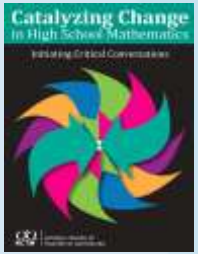
Providing high school students with more rigorous instruction requires improvements in teaching practices. These improvements must not only consider ways to make mathematics more accessible to students but also **support students in seeing themselves as knowers and doers of mathematics**.



NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Equitable Mathematics Teaching Practices

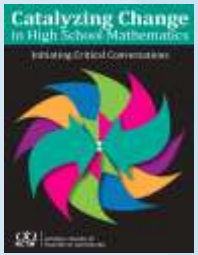
Such improvements in teaching practices must consider **mathematical identity and agency** as essential constructs for raising achievement and mathematical dispositions for each and every student.



NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Equitable Mathematics Teaching Practices

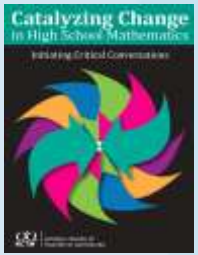
Implementing equitable instructional practices is an action teachers can undertake immediately ***to improve the experiences*** and learning outcomes of students.



NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Equitable Mathematics Teaching Practices

The eight Mathematics Teaching Practices articulated in *Principles to Actions* provide a framework for making connections between the high-leverage teaching practices and the development of identity, agency, and competence.



NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Equitable Mathematics Teaching Practices

Mathematics Teaching Practices: Supporting Equitable Mathematics Teaching	
Mathematics Teaching Practices	**Equitable Teaching**
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.	• Establish learning progressions that build students' mathematical understanding, increase their confidence, and support their mathematical identities as doers of mathematics. • Establish high expectations to ensure that each and every student has the opportunity to meet the mathematical goals. • Establish classroom norms for participation that position each and every student as a competent mathematics thinker. • Establish classroom environments that promote learning mathematics as just, equitable, and inclusive.

NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

An Example of the Connection Between Research-Informed Instructional and Equitable Practices

Pose purposeful questions.

Effective teaching of mathematics uses purposeful questions to assess and advance student reasoning and sense making about important mathematical ideas and relationships.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

Promoting Equity By Posing Purposeful Questions

"Teacher questioning and positioning of students influences how students view themselves as members of the mathematics learning community in the classroom."

Boston, M., Dillon, F., Smith, M. S., & Miller, S. (2017). *Taking action: Implementing effective mathematics teaching practices grades 9-12.* Reston, VA: NCTM.

Promoting Equity By Posing Purposeful Questions

- Are all students students' ideas and questions heard, valued, and pursued in the mathematics classroom?
- Who does the teacher call on to answer questions?
- What mathematical ideas does the class examine and discuss?
- Whose thinking does the teacher select for further inquiry, and whose thinking does the teacher disregard during small-group and whole-class discussion?"

Boston, M., Dillon, F., Smith, M. S., & Miller, S. (2017). *Taking action: Implementing effective mathematics teaching practices grades 9-12.* Reston, VA: NCTM.

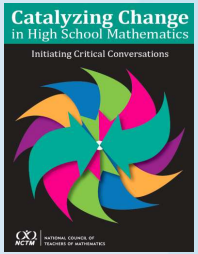
Supporting Identity

How teachers speak, listen, see, and interact with young people matters. They shape the environment and interactions that shape students' sense of themselves and their competence. **Teachers can reproduce marginalization or disrupt it.**

Goffney, I. (2018). Concluding thoughts. The need to rehumanize mathematics. In R. Gutiérrez, I. Goffney, & M. Boston (Eds.), *Rehumanizing mathematics for Black, Indigenous, and Latinx students*. Reston, VA: NCTM.

Equitable Mathematics Teaching Practices

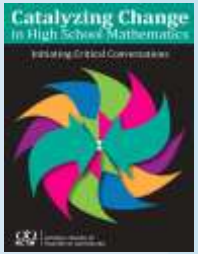
Mathematics teaching involves not only helping students learn concepts and develop skills and understanding but also empowering students **to see themselves as capable of participating in and being doers of mathematics**.



NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Key Recommendation

Each and every student should learn the Essential Concepts in order to expand professional opportunities, understand and critique the world, and experience the joy, wonder, and beauty of mathematics.



NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Essential Concepts

Essential Concepts **do not represent yet another set of standards** or a list of disjoint topics to be covered.

The Essential Concepts represent a distillation of the critical concepts and skills that, regardless of a state's, province's, or district's standards, students should acquire.

NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Essential Concepts

Essential Concepts represent the **most critical content** from the content domains – the deep understandings that are important for students to remember long after they have forgotten how to carry out specific techniques or apply particular formulas.

NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Essential Concepts

Catalyzing Change identifies a set of Essential Concepts from the content domains of …

- number,
- algebra and functions,
- statistics and probability, and
- geometry and measurement.

NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Essential Concepts

The roles of technology, reasoning and proof, and modeling, as well as connections among the content areas, are highlighted.

Additional examples illustrating the Essential Concepts are presented explicitly in More4U resources.

more4U

More resources available online
www.nctm.org/more4u
Look inside for your access code

NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Content Area and Focus	Number of Essential Concepts
Number	2
Algebra and Functions	
Focus 1: Algebra	4
Focus 2: Connecting Algebra to Functions	2
Focus 3: Functions	4
Statistics and Probability	
Focus 1: Quantitative Literacy	2
Focus 2: Visualizing and Summarizing Data	6
Focus 3: Statistical Inference	7
Focus 4: Probability	2
Geometry and Measurement	
Focus 1: Measurement	3
Focus 2: Transformations	4
Focus 3: Geometric Arguments, Reasoning, and Proof	3
Focus 4: Solving Applied Problems and Modeling in Geometry	2

Catalyzing Change Sessions – Page 15

69 Critical Conversations to Catalyze Change in High School Mathematics

194 Rethinking What Each and Every High School Student Needs Related to Algebra and Functions

246 The Wonders and Joys of Mathematics and Statistics

317 Creating Equitable Structures to Support Success in High School Mathematics

420 Embracing Quantitative Literacy and Statistical Thinking for All High School Students

439 President Elect Address: *Catalyzing Change*: Identity, Agency, Positioning, and Equitable Instructional Practices

544 Pathways through High School Mathematics: Let's Start the Conversation

569 Modeling for Motivation and Proof for Power

592 Transforming High School Geometry

626 Why Ask Why? Proof & Inquiry in High-School Mathematics

664 Mathematics for Human Flourishing

Key Recommendation

High schools should offer continuous four-year mathematics pathways with **all students studying mathematics each year**, including two to three years of mathematics in a common shared pathway focusing on the Essential Concepts.

NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

A High Expectation

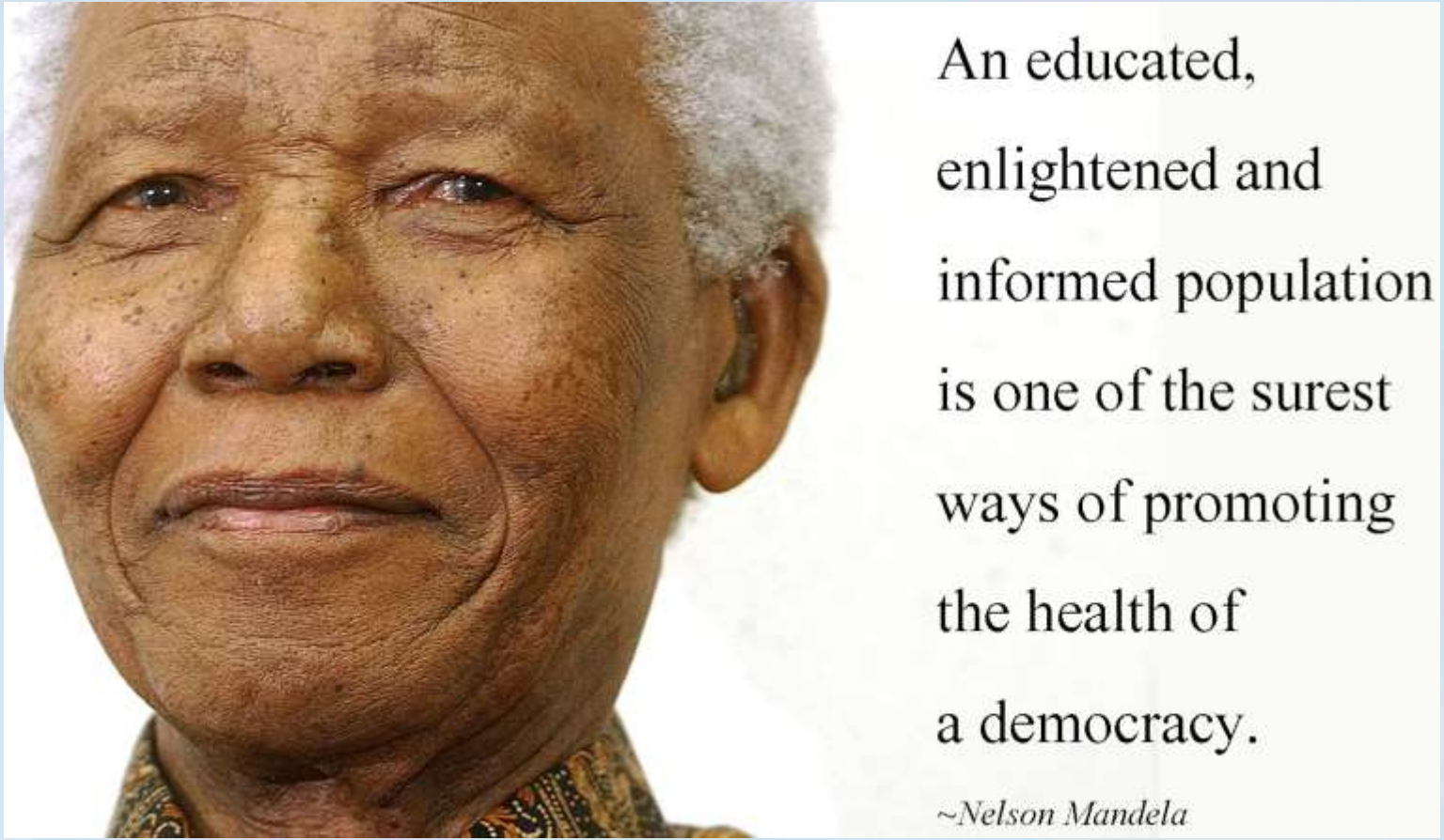
- A total of **four years of <u>demanding</u> high school mathematics** for each and every student.
- A **shared content experience** of Essential Concepts that would make up 2-3 years of the four year experience.
- A 1-2 year demanding mathematics experience [non-terminal] **tailored to the interests and aspirations** of each student.

NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Catalyzing Change envisions high school mathematics programs in which courses addressing the Essential Concepts will represent a ***common shared content pathway*** of at least two, but no more than three years, of a student's four-year high school mathematics experience.

NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

A Shared Content Experience is Critical



The expectation in *Catalyzing Change* is that a single curricular model would deliver the common pathway offered to all students in a single school setting to avoid the creation of separate and unequal tracks.

NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Following the Essential Concepts

Continued study of mathematics beyond the Essential Concepts must be based on each student's desire to **pursue the future the student imagines for himself or herself** rather than on any difference in mathematical ability perceived by anyone else.

NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

What Counts as a Mathematics Course?

Only courses that address mathematical standards (including statistics) and that are **mathematically demanding** should count toward high school mathematics graduation requirements.

NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Mathematically Demanding Courses

- Require clarity and precision in reasoning
- Have focused and significant mathematics learning standards
- Maintain the integrity of the mathematical standards
- Are part of a coherent mathematical learning progression (not dead-end/terminal courses)
- Approach mathematics in an instructionally balanced way

NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Next Steps

Catalyzing Change in High School Mathematics is designed to **open critical conversations and sustained efforts on multiple levels to engage all stakeholders** in the system of high school mathematics education in the work of improving the learning experiences and outcomes of each and every student.

NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

To Initiate Critical Conversations

Catalyzing Change offers initial actions for …

- Teachers, Schools, and Districts
- Policymakers
- Postsecondary Educators

NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Actions for Teachers, Schools, and Districts

Conversations can be initiated by …

- Analyzing and evaluating systemic policies, practices, and procedures that restrict student access to and success in mathematics.
- Analyzing teachers' assignments to develop balanced and supportive assignments.
- Consistently implementing and linking research-informed and equity-based instructional practices.

NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Catalyzing Change can serve as the basis for negotiating, implementing, and supporting these necessary conversations about policies, teaching practices, Essential Concepts, courses, pathways, and other supportive structures. **These critical conversations will not be easy, since the challenges are real and longstanding**.

NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

NCTM encourages the larger mathematics education community to take on this work not because it is easy, but because it is hard, long overdue, and the just thing to do.

Paraphrasing John F. Kennedy, 1962

We owe this effort not only to our students but also to ourselves as we work together to create and **nurture the society we wish to inhabit.**

NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.