Catalyzing Change in High School Mathematics: Initiating Critical Conversations

Matt Larson
NCTM President
# Thank You to the Writing Team!

<table>
<thead>
<tr>
<th>Name</th>
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<tr>
<td>Karen J. Graham, Chair</td>
<td>University of New Hampshire</td>
</tr>
<tr>
<td>Gail Burrill</td>
<td>Michigan State University</td>
</tr>
<tr>
<td>Ed Dickey</td>
<td>University of South Carolina</td>
</tr>
<tr>
<td>Christine Franklin</td>
<td>American Statistical Association</td>
</tr>
<tr>
<td>Jennifer Curtis</td>
<td>NC Math Alliance Supporting Teachers, Durham, North Carolina</td>
</tr>
<tr>
<td>Kanita DuCloux</td>
<td>Western Kentucky University</td>
</tr>
<tr>
<td>Damarrio C. Holloway</td>
<td>Discovery High School</td>
</tr>
</tbody>
</table>

(Durham, New Hampshire)  
(East Lansing, Michigan)  
(Columbia, South Carolina)  
(Athens, Georgia)  
(Bowling Green, Kentucky)  
(Lawrenceville, GA)
# Thank You to the Writing Team!

<table>
<thead>
<tr>
<th>Author</th>
<th>Institution</th>
<th>Location</th>
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<tr>
<td>Paul Kelley</td>
<td>Anoka High School</td>
<td>Anoka, Minnesota</td>
</tr>
<tr>
<td>Yvonne Lai</td>
<td>University of Nebraska-Lincoln</td>
<td>Lincoln, Nebraska</td>
</tr>
<tr>
<td>Max Ray-Riek</td>
<td>NCTM</td>
<td>Reston, Virginia</td>
</tr>
<tr>
<td>John W. Staley</td>
<td>Baltimore County Public Schools</td>
<td>Towson, Maryland</td>
</tr>
<tr>
<td>Francis Su</td>
<td>Harvey Mudd College</td>
<td>Claremont, CA</td>
</tr>
<tr>
<td>Daniel J. Teague</td>
<td>North Carolina School of Science &amp; Math</td>
<td>Durham, North Carolina</td>
</tr>
<tr>
<td>Gwendolyn Zimmermann</td>
<td>Adlai E. Stevenson High School</td>
<td>Lincolnshire, Illinois</td>
</tr>
<tr>
<td>David Barnes, Staff Liaison</td>
<td>NCTM</td>
<td>Reston, Virginia</td>
</tr>
<tr>
<td>Matt Larson, Contributing Author</td>
<td>Lincoln Public Schools</td>
<td>University of Nebraska-Lincoln</td>
</tr>
<tr>
<td>Robert Q. Berry III, Contributing Author</td>
<td>University of Virginia</td>
<td>Charlottesville, Virginia</td>
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The Last Three Decades Have Seen Significant Progress in the Teaching and Learning of Mathematics
We Have Seen Improved Mathematics Learning at the Elementary and Middle Levels

Trend in fourth-grade NAEP mathematics average scores

- Advanced
- Proficient
- Basic

Scale Score

ASSESSMENT YEAR
Why Catalyzing Change in High School Mathematics?

Why now?
The steady improvement in mathematics learning at the elementary and middle levels has not been shared to the same degree at the high school level.

NAEP 10 Year Trend Line (Math)

Grade 12

An implementation gap persists at the high school level between the calls for change and the comprehensive actions needed to support each and every high school student.

High School Mathematics Learning Progress

• The percentage of high school students enrolling in upper-level mathematics courses over the last three decades has increased.

• Advanced Placement participation rates in mathematics have steadily increased, reaching an all-time high in the 2016-2017 school year.


College Board, 2017
U.S. high school students trail their international peers in mathematical literacy with respect to their “capacity to formulate, employ, and interpret mathematics in a variety of contexts”

OECD, 2016
U.S. young adults lack the numeracy and problem-solving skills necessary for “meaningful participation in our democratic institutions.”

Goodman, Sands, & Coley, 2015
Most critically, large numbers of high school students do not have access to the mathematics they need either for their personal or for their professional lives.

Many mathematics standards at the high school level lack the focus of mathematics standards at the K-8 level.

In most high schools, teachers find it difficult to teach at the desired level of rigor, given the sheer amount of content expected.

To Be Clear: This is Not About Blame or Shame!

Mathematics education at the high school level is part of a complex system of policies, traditions, and societal expectations.

*Catalyzing Change* is written to engage all individuals with a stake in high school mathematics in the critical conversations that must take place to bring about and give support to necessary changes in high school mathematics.

To Initiate Critical Conversations, *Catalyzing Change* makes four key recommendations.
Four Key Recommendations

- The purpose of learning mathematics and Essential Concepts
- Equitable Structures
- Equitable Instruction
- A Common Essential Concepts Pathway
Each and every student should learn the Essential Concepts in order to

• *expand professional opportunities*,
• *understand and critique the world, and*
• *experience the joy, wonder, and beauty of mathematics.*

Why Learn Mathematics?

This is a simple question, but worth considerable reflection. Because how you answer this question will strongly determine who you think should be doing mathematics, and how you will teach it.

Traditionally, mathematics education has been connected to issues of national economic survival, rather than to the development of democratic citizenship through critical thinking in mathematics.

Never have the broader aims of mathematics education been more important than they are today when mathematics underlies much of the fabric of society, from polling and data mining in politics, to algorithms targeting advertising to groups on social media, to complex mathematical models of financial instruments.

Expand Professional Opportunities

• The economy is expected to create a large number of STEM jobs over the next decade.

• Beginning salaries for STEM majors will outpace those of other college majors.

• Salary growth in STEM careers will outpace that in other careers.

National Association of Colleges and Employers, 2016
PCAST, 2012
Vilorio, 2014
High school mathematics is increasingly essential today for access to and success in a wide variety of careers and the social sciences that were not historically seen as quantitative in nature.
Understand and Critique the World

Mathematics is deeply embedded in many aspects of life, both seen and unseen – communication systems, transportation systems, medicine, manufacturing, finance, security, science, engineering, technology, and “big data.”

Understand and Critique the World

Students should be able to identify, interpret, evaluate, and critique the mathematics embedded in social, scientific, commercial, and political systems, as well as the claims made in the private and public sectors and in public interest group pronouncements.

Ernest, 2010
The Trump Plan Is A Large Tax Cut For The Middle Class

Taxes for a Married Couple Under the Trump Plan and Current Law

Source: IRS, donaldjtrump.com
Note: This comparison includes only the brackets and the standard deduction. All income assumed to be wage income.

TAX FOUNDATION
In Trump’s plan, the top 1% of earners would pay a smaller share of total federal taxes than it does currently, while shares would rise for groups in the middle of the income spectrum.

Percentage share of all federal taxes

- **Current law**
- **Trump proposal**

### Income Quintiles

- **Below $24,800**
  - 0–20%
- **$24,800–48,400**
  - 20–40%
- **$48,400–83,300**
  - 40–60%
- **$83,300–143,100**
  - 60–80%
- **$143,100–699,000**
  - TOP 20%
- **$699,000 and above**
  - 25.0%
Trump's campaign tax plan was skewed toward the top

The top 1 percent got half the tax cut

Healthy young child goes to doctor, gets pumped with massive shot of many vaccines, doesn't feel good and changes - AUTISM. Many such cases!
Mathematical Illiteracy

The plural of anecdote is neither “proof” nor is it “evidence.”
Do vaccines cause autism? The evidence

**AUTISM IS NOT CAUSED BY VACCINES**

A study of more than **1.25 million people**, combining results from multiple countries, multiple scientists, and multiple funding sources proves that **vaccines do not cause autism**.


**VACCINES DO NOT CAUSE AUTISM**

THE EVIDENCE IS CLEAR

**VACCINES LINKED WITH AUTISM**

1998 study of **12 children**.
3 of the children did not have autism at all, and 5 had developmental concerns before their vaccination.
Study funded by lawyers planning to sue a vaccine manufacturer.
Researcher struck off the medical register for **serious professional misconduct**.
Researcher described by the General Medical Council as being dishonest and irresponsible, having acted with callous disregard for the children studied.
Research formally retracted.
Measles Cases Increased Dramatically in 2014

U.S. Measles cases by year*

*Provisional data from 27 states reported to CDC's National Center for Immunization and Respiratory Diseases (NCIRD)

Source: CDC
All high school graduates will, as members of society, be presented with data-based claims throughout their lives. Therefore, they must be able to examine these claims and be intelligent consumers of studies, capable of reasoning critically and questioning the implementation of the statistical investigation process in those studies.
• Increasing disagreement about facts and analytical interpretation of facts and data.

• A blurring of the line between opinion and fact.

• The increasing relative volume, and resulting influence, of opinion and personal experience over fact.

• Declining trust in formerly respected sources of factual information.

Understand and Critique the World

When high school mathematics courses integrate tasks that, for example, address income distributions, sustainability, mortality rates, taxing structures, etc., students have “access to rich, rigorous mathematics that offers opportunities and self-empowerment for them to understand and use mathematics in their world.”

Experience Wonder, Joy and Beauty

High school mathematics can potentially cultivate in students a sense of wonder, beauty and joy – and doing so is an important, but often neglected, purpose for learning mathematics.

For what percent of U.S. adults was their last formal mathematics learning experience negative?

Do you think this percentage is higher or lower for history, art, or literature?
For what percent of U.S. adults was their last formal mathematics learning experience negative?

Do you think this percentage is higher or lower for history, art, or literature?
For what percent of U.S. adults was their last formal mathematics learning experience negative?

Do you think this percentage is higher or lower for history, art, or literature?

98.5%
Great ideas of mathematics are as beautiful as great works of art, and just as in the study of art, students can learn to see mathematics as expressions of beauty fashioned by drama and struggle.

Experience Wonder, Joy and Beauty

Mathematical ideas are a triumph of human creativity and framing mathematics in this way may support students in coming to see mathematics as a creative endeavor.

Lockhart and Devlin, 2009
Experience Wonder, Joy and Beauty

Rehumanizing mathematics ... recouples it with connection, joy, and belonging. When people are encouraged to express themselves through the practice of mathematics, they are more likely to draw upon an innate sense of aesthetics and intuition.

Experience Wonder, Joy and Beauty

Mathematics has been shaped by cultures across the globe in which it was developed, and it is important to see the role of mathematics in history and society.

Ernest, 2010
Experience Wonder, Joy and Beauty

The mathematician does not study pure mathematics because it is useful; he studies it because he delights in it and he delights in it because it is beautiful.

Henri Poincare, 19th Century
Experience Wonder, Joy and Beauty

The mathematician does not study pure mathematics because it is useful; he studies it because he delights in it and he delights in it because it is beautiful.

Henri Poincare, 19th Century
High school mathematics should discontinue the practice of tracking teachers as well as the practice of tracking students into qualitatively different or dead-end course pathways.

Mathematics education will likely always involve significant tensions.

There is a longstanding, thoroughly documented, and seemingly intractable problem in mathematics education: inequity.

Children of certain racial, ethnic, language, gender, ability, and socio-economic backgrounds experience mathematics education in school differently and many are disaffected by their mathematics education experience.

We Must Face Hard Truths

Mathematics education often reinforces, rather than moderates, inequalities in education.

Creating Equitable Structures

Equity in mathematics education will not be achieved until it is no longer possible “to predict mathematics achievement and participation based solely on student characteristics such as race, class, ethnicity, sex, beliefs, and proficiency in the dominant language.”

Gutiérrez, 2002
Creating Equitable Structures

Current reform efforts that focus largely on standards, with some attention to improved instructional practice, are unlikely to address and alleviate equity concerns unless they also **address and dismantle the conditions and system structures** that stand as barriers to the creation of positive mathematical experiences for students.

Three Significant Structural Barriers Within Educators’ Sphere of Influence

• Tracking students into course pathways that do not prepare students for **continued study** of mathematics.

• Tracking teachers in ways that deny certain students **access** to high-quality instruction.

• Providing **inadequate instructional supports** both before and during high school.

A pathway is a course progression for a student through high school mathematics. Pathways include tracks – fixed sequences of courses that are often determined in middle school or earlier. Courses in a track often place students in different levels of the same course.

Student Tracking

Tracking is insidious because it places some students into qualitatively different or lower levels of a mathematics course and, in some cases, puts students into terminal mathematics pathways that are not mathematically meaningful and do not prepare them for any continued study of fundamental mathematics.

Student Tracking

Too often placement into different tracks is based on a variety of nonacademic factors, such as perceived (but not potential) academic ability, race, socioeconomic status, gender, language, or other expectations ascribed to students by adults.

Stiff and Johnson, 2011
Student Tracking Often Begins Early

Although tracking is often viewed as a secondary concern, the reality is that “tracks” in mathematics are often established as early as the primary grades when students who struggled in kindergarten are placed in a “low-slow” mathematics group in first grade.

Different Opportunities for Different Students

The learning opportunities provided for low-ability, average-ability, and high ability-grouped classrooms are hierarchically different.

While marginalized students tend to learn simple facts and figures and are exposed to simple applied problems, their privileged counterparts experience mathematics instruction that help them think like a mathematician, develop deep conceptual understanding and advanced mathematical reasoning skills.

Low expectations often result in self-fulfilling prophecies. Once placed in the low tracks, it is very difficult for students to move to a higher track.

Student Tracking Exists in Various Forms

• Different lengths of courses.
• Different versions of an Algebra or Algebra 2 course.


There Are Differential Outcomes For “Different” Algebras

Students placed into less rigorous versions of Algebra 1 ultimately have lower math attainment in high school even if their performance in the less rigorous version of algebra is greater than that of students in more rigorous versions.

Double-period versions of a course do not represent lower-level versions of a course. Double-period versions of a course do not constitute tracking if the double-period version of the course has the same instructional objectives and expectations and uses the same core instructional materials and assessments.

Acceleration may be appropriate if a student has demonstrated deep understanding of grade-level or course-based mathematics standards beyond his or her current level.
Distinguishing Tracking from Appropriate Acceleration

Appropriate acceleration ensures that ...

• Opportunities are available to each and every prepared student and no critical concepts are rushed or skipped.

• Is along a single common shared pathway that provides each student an opportunity to learn the same Essential Concepts.

Distinguishing Tracking from Appropriate Acceleration

Appropriate acceleration ensures that ...

- **Opportunities are open to a wide range of students** who express a higher degree of interest in mathematics, not just those who are identified through traditional assessment instruments.

We Must Put an End to “Educede”

There is no good reason to tell a student she doesn’t belong in math [your class] ... you see a snapshot of her progress, but you don’t see her trajectory. You can’t know how she will grow and flourish in the future. But you can help get her there.

Francis Su, Past President of the Mathematical Association of America, 2017
Teacher Tracking

Like mathematics students, mathematics teachers themselves are often tracked, with the most experienced teachers, or those who are perceived to be most effective, assigned to upper-level mathematics courses and the least experienced assigned to entry-level mathematics courses.

Darling-Hammond, 2007
Strutchens, Quander, and Gutiérrez, 2011
Whenever possible, high school mathematics teachers in the same department should have teaching assignments that are balanced to include both upper-level and entry-level mathematics courses.

Balanced Teaching Assignments

Balancing teaching assignments deepens teachers’ knowledge of the overall curriculum expectations, can reduce burnout among new teachers, can populate collaborative teams with experienced teachers, and can develop among teachers a collective sense of responsibility for all students.

Gutiérrez, 2002
Strutchens, Quander, and Gutiérrez, 2011
A Personal and Societal Loss

We deny society potential solutions to many of the problems we face, and we slow the advancement of mathematics itself, when we systematically ignore vast human potential.
Student and Teacher Tracking are Not Just

“We have to recognize that even if people are just, even they desire to be just, a society may not be just if its structures and practices are not also just.”

Su, 2017
Classroom instruction should be consistent with research-informed and equitable teaching practices.

Providing high school students with more rigorous instruction requires improvements in teaching practices. These improvements must not only consider ways to make mathematics more accessible to students but also support students in seeing themselves as knowers and doers of mathematics.

Equitable Mathematics Teaching Practices

Such improvements in teaching practices must consider **mathematical identity and agency** as essential constructs for raising achievement and mathematical dispositions for each and every student.

Implementing equitable instructional practices is an action teachers can undertake immediately to improve the experiences and learning outcomes of students.

Equitable Mathematics Teaching Practices

The eight Mathematics Teaching Practices articulated in *Principles to Actions* provide a framework for making connections between the high-leverage teaching practices and the development of identity, agency, and competence.

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<th>Mathematics Teaching Practices: Supporting Equitable Mathematics Teaching</th>
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<td><strong>Mathematics Teaching Practices</strong></td>
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<td>Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.</td>
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</tbody>
</table>
Pose purposeful questions.

Effective teaching of mathematics uses purposeful questions to assess and advance student reasoning and sense making about important mathematical ideas and relationships.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.
Promoting Equity By Posing Purposeful Questions

“Teacher questioning and positioning of students influences how students view themselves as members of the mathematics learning community in the classroom.”

Promoting Equity By Posing Purposeful Questions

• Are all students' ideas and questions heard, valued, and pursued in the mathematics classroom?
• Who does the teacher call on to answer questions?
• What mathematical ideas does the class examine and discuss?
• Whose thinking does the teacher select for further inquiry, and whose thinking does the teacher disregard during small-group and whole-class discussion?”

Supporting Identity

How teachers speak, listen, see, and interact with young people matters. They shape the environment and interactions that shape students’ sense of themselves and their competence. **Teachers can reproduce marginalization or disrupt it.**

Equitable Mathematics Teaching Practices

Mathematics teaching involves not only helping students learn concepts and develop skills and understanding but also empowering students to see themselves as capable of participating in and being doers of mathematics.

Each and every student should learn the Essential Concepts in order to expand professional opportunities, understand and critique the world, and experience the joy, wonder, and beauty of mathematics.

Essential Concepts do not represent yet another set of standards or a list of disjoint topics to be covered.

The Essential Concepts represent a distillation of the critical concepts and skills that, regardless of a state’s, province’s, or district’s standards, students should acquire.

Essential Concepts

Essential Concepts represent the most critical content from the content domains – the deep understandings that are important for students to remember long after they have forgotten how to carry out specific techniques or apply particular formulas.

Essential Concepts

*Catalyzing Change* identifies a set of Essential Concepts from the content domains of...

- number,
- algebra and functions,
- statistics and probability, and
- geometry and measurement.

Essential Concepts

The roles of technology, reasoning and proof, and modeling, as well as connections among the content areas, are highlighted.

Additional examples illustrating the Essential Concepts are presented explicitly in More4U resources.

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<th>Number of Essential Concepts</th>
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High schools should offer continuous four-year mathematics pathways with **all students studying mathematics each year**, including two to three years of mathematics in a common shared pathway focusing on the Essential Concepts.

A High Expectation

- A total of **four years of demanding high school mathematics** for each and every student.

- A **shared content experience** of Essential Concepts that would make up 2-3 years of the four year experience.

- A 1-2 year demanding mathematics experience [non-terminal] tailored to the interests and aspirations of each student.

Catalyzing Change envisions high school mathematics programs in which courses addressing the Essential Concepts will represent a *common shared content pathway* of at least two, but no more than three years, of a student’s four-year high school mathematics experience.

An educated, enlightened and informed population is one of the surest ways of promoting the health of a democracy.

~Nelson Mandela
The expectation in *Catalyzing Change* is that a single curricular model would deliver the common pathway offered to all students in a single school setting to avoid the creation of separate and unequal tracks.

Continued study of mathematics beyond the Essential Concepts must be based on each student’s desire to pursue the future the student imagines for himself or herself rather than on any difference in mathematical ability perceived by anyone else.

What Counts as a Mathematics Course?

Only courses that address mathematical standards (including statistics) and that are **mathematically demanding** should count toward high school mathematics graduation requirements.

Mathematically Demanding Courses

- Require clarity and precision in reasoning
- Have focused and significant mathematics learning standards
- Maintain the integrity of the mathematical standards
- Are part of a coherent mathematical learning progression (not dead-end/terminal courses)
- Approach mathematics in an instructionally balanced way

Next Steps

_Catalyzing Change in High School Mathematics_ is designed to **open critical conversations and sustained efforts on multiple levels to engage all stakeholders in** the system of high school mathematics education in the work of improving the learning experiences and outcomes of each and every student.

To Initiate Critical Conversations

*Catalyzing Change* offers initial actions for ...

- Teachers, Schools, and Districts
- Policymakers
- Postsecondary Educators

Actions for Teachers, Schools, and Districts

Conversations can be initiated by ...

• Analyzing and evaluating systemic policies, practices, and procedures that restrict student access to and success in mathematics.

• Analyzing teachers’ assignments to develop balanced and supportive assignments.

• Consistently implementing and linking research-informed and equity-based instructional practices.

Catalyzing Change can serve as the basis for negotiating, implementing, and supporting these necessary conversations about policies, teaching practices, Essential Concepts, courses, pathways, and other supportive structures. These critical conversations will not be easy, since the challenges are real and longstanding.
NCTM encourages the larger mathematics education community to take on this work not because it is easy, but because it is hard, long overdue, and the just thing to do.

Paraphrasing John F. Kennedy, 1962
We owe this effort not only to our students but also to ourselves as we work together to create and nurture the society we wish to inhabit.