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teaching children mathematics

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Viral criticisms of Common Core mathematics

SAMUEL OTTEN AND ZANDRA DE ARAUJO

IN MY OPINION

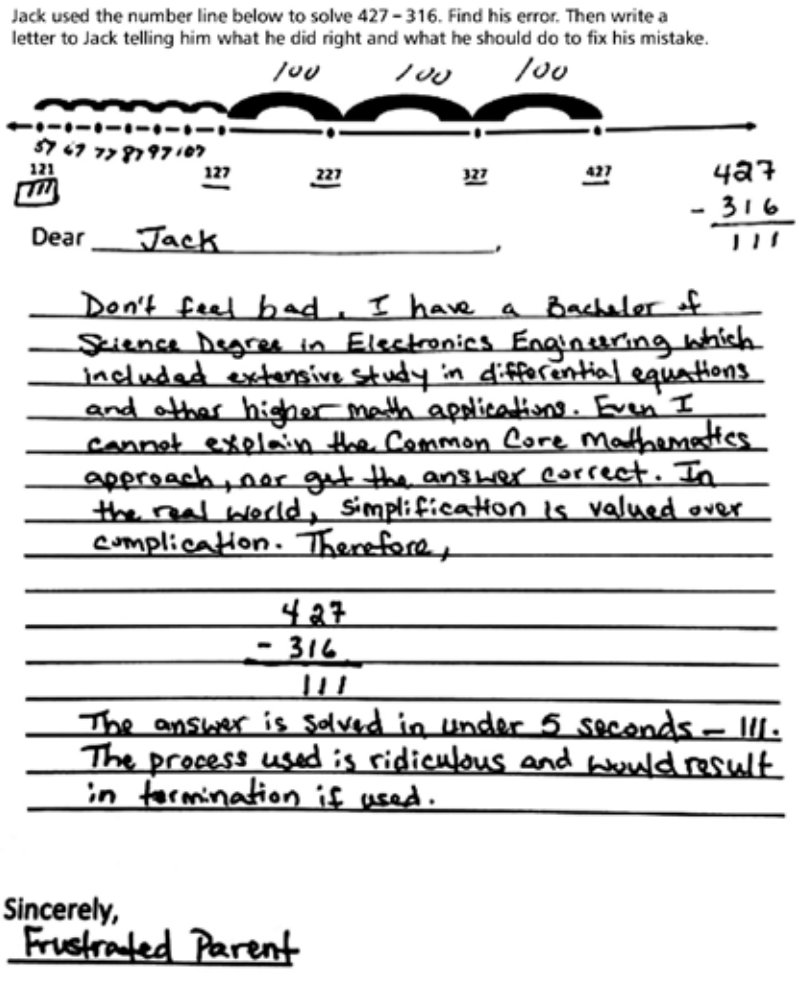
Although many reasons exist to support the Common Core State Standards for Mathematics (CCSSI 2010), different groups have found many reasons to oppose them. One particular criticism circulating on social media is an attack on problems embodying the Standards for Mathematical Practice (SMPs), which are mathematical ways of thinking, or habits of mind, that "mathematics educators at all levels should seek to develop in their students" (p. 6). This criticism rests on invalid reasoning and a desire for back-to-basics mathematics teaching that has been discredited in research literature (e.g., Fennema et al. 1996; Slavin and Lake 2008). In our opinion, the mathematics education community must raise a unified voice in response.

Because many Americans, and therefore many parents, were taught mathematics in a procedure-focused manner (Davis 2009; Stigler and Hiebert 1999), there is danger of a disconnect between parents' expectations for mathematics education and the recommendations of the Common Core. These differing expectations may lead parents to think that their children should learn the same procedures they learned rather than the conceptual underpinnings they may have never grasped themselves.

A widely circulated Internet post that reveals the schism between parents' expectations and the SMPs originated on "The Patriot's Post" Facebook page (see **fig. 1**). This problem moves beyond simple computational procedures, requiring students to engage with a conceptual representation of subtraction (i.e., the number line) and critique

FIGURE 1

A viral Internet post purportedly critiqued Common Core mathematics (retrieved from https://patriotpost.us/posts/24251).



another's reasoning, that of the fictional character Jack. Jack solved 427 – 316 by first plotting 427 on a number line. He then went down incrementally by hundreds. Jack should have then subtracted 1 ten before subtracting 6 ones; however, he forgot the ten and arrived at 121 instead of 111. The focus is not on simply calculating the correct answer but on the process of thinking through subtraction using knowledge of place value and presenting a well-reasoned assessment of Jack's thinking. A written explanation is also required, promoting communication skills and literacy, which aids learning (Chapin and O'Connor 2004) and aligns with the SMPs (e.g., construct arguments, attend to precision of language). Frustrated Parent, who is the child's father, gives us an opportunity to diagnose expectations so that we may improve our own communication about the goals of mathematics education. He seems to want to compute the answer of 111 as quickly as possible ("in under 5 seconds"), revealing a traditional emphasis on speed and correctness rather than understanding and reasoning. He also may not be accustomed to error-analysis problems, as he was unable to diagnose Jack's error. You can see that he tried to follow the number line by subtracting 3 hundreds (he wrote in the 100s over the larger jumps) and then 6 tens (he wrote in 107, 97, 87, 77, 67, 57, 47 over the smaller jumps), not realizing that Jack had actually subtracted 6 ones not 6 tens. His remark that it was a "complicated" and "ridiculous" process for computing the answer shows that the parent did not see the purpose of the problem. Moreover, he did not seem to realize that it was incorrect on purpose, underscoring the parent's expectation for straightforward procedural problems rather than problems involving reasoning, writing, and analyzing mathematical mistakes—all of which are aspects of the SMPs. The parent's appeal to personal history and the work environment are not compelling arguments, and his claim that Jack's erroneous work is the "Common Core approach" is incorrect. In fact, the Common Core explicitly requires the standard subtraction algorithm in fourth grade (4.NBT.4).

The general use of the number line, however, fits with the Common Core approach to such operations as subtraction. Common Core takes into consideration children's learning progressions. For example, starting in second grade, students are exposed to models of subtraction and are expected to use place value to explain why subtraction strategies work, much like Jack's attempt in the problem. In third grade, students continue using strategies and invented algorithms for subtraction; and then in fourth grade, they are expected to have a deep understanding and be fluent with the standard algorithm. In Frustrated Parent's response, he used the standard algorithm to solve the subtraction problem and stated that this was the proper way to solve the problem. Although this is a valid approach, it may be inappropriate for his child's place in the learning progression. Carpenter and his colleagues' (1997) longitudinal study of children's understanding of number concepts and operations found that about 90 percent of the children used invented strategies. Further, those children who used such strategies before learning the standard algorithm had stronger understanding and greater flexibility with number concepts than those who learned the standard algorithm first. Studies such as this provided the basis for the Common Core's goals for students to gain flexibility with a number of strategies, such as number lines, *prior* to learning the standard algorithm.

FUSE/THINKSTOCK

The criticisms of Common Core that are embedded in **figure 1** are not unique to that particular Web post; the examples in **figure 2** raise similar issues. These problems move beyond

FIGURE 2

Problems posted on Twitter by @LouisCK and @Hollaatme_baby, respectively, purportedly document "painfully awful" Common Core mathematics.

Fill in the blanks below with whole numbers greater than 1 that will make the number sentences true. $7 \times (\_\_\_ \times \_\_\_) = 21 \times 7$	Use number bonds to help you skip-count by seven by making ten or adding to the ones. $7 + 7 = \underline{10} + \underline{4} = \_\_\_$ 3 4

Parents and educators share a common goal: an improved mathematics education system.

memorization of answers and toward a conceptual understanding of the mathematics involved. Rather than simply asking students to compute an answer, the first problem expects students to think about the mathematical structure of multiplication (i.e., that it is commutative) and the structure of 21 (i.e., its factors) to write a true statement. This also pushes for a conceptual understanding of the equal sign "=" as a relationship between quantities rather than as a signal meaning to compute (Knuth et al. 2006). The second problem asks students to decompose numbers in ways that help them reach a solution. This use of number sense is an aspect of the SMPs ("Reason quantitatively," "Look for and use mathematical structure") and is an important outcome of mathematics education overall, yet online critics of such problems seem to expect only procedural-type items on their children's homework, even though research shows that such an approach leads directly to poor mathematics performance overall and a large portion of students disinterested in mathematics (Kilpatrick, Martin, and Schifter 2003).

How should we mathematics educators respond to such criticisms? We think that such criticisms as these are rooted in differing expectations for what it looks like to be in the process of learning mathematics, rather than disagreement about the end goal of an improved mathematics education system. As such, it is unproductive to explain to Common Core critics that their expectations for what mathematics learning should look like are "wrong." Instead, Common Core supporters can work to articulate why our expectations are what they are. We can justify the value of conceptual understanding (Hiebert 1986), the benefits of engaging students in the SMPs (Koestler et al. 2013), and the strengths of learning progressions (Schmidt and Houang 2012). Reforms involving conceptual understanding and critical reasoning are difficult because they involve a kind of mathematics education to which many parents (as well as teachers and students) are unaccustomed, as the examples above illustrate. Nevertheless, the Common Core is a worthwhile cause, and its principles are worth defending. Hopefully, through the enactment of the Common Core and the SMPs, the viral criticisms of future generations will involve well-reasoned arguments and empirically supported claims for how best to meet everyone's underlying goal of quality mathematics education for all children.

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COACHES' CORNER

Are students in the zone to learn today's concept?

BY ROBYN SILBEY, PD AND CAMPUS CONSULTANT

Picture this: One of your teachers is working through the direct-instruction part of his lesson, and students appear to be engaged. When he assigns the independent practice and begins to circulate, the teacher finds that most students are struggling. How could a teacher avoid this scenario?

Encourage teachers to open their lessons by informally assessing students' readiness for the new content. Following these steps will help teachers think about and plan a successful, data-rich lesson opener:

1. **Goal:** Identify the intended goal for the day.
2. **Prerequisite skills:** Analyze the goal and identify key prerequisite skills.
3. **Informal assessment:** Construct a brief informal assessment of the prerequisite skills. The best assessments involve partner or group talk, a demonstration, or whiteboard work—not "Number your paper 1–5" items that have to be graded at a later date. Data must be collected and analyzed immediately.
4. **Data-driven instruction:** Use the lesson opener as an informal assessment before starting direct instruction. Depending on students' responses, the teacher (a) forges ahead as planned or (b) reviews one or more of the goal's prerequisite skills.

The following is an example.

1. **Goal:** To understand and make sense of subtraction with regrouping
2. **Prerequisite skills:** Understanding of base-ten place value, the meaning of subtraction, and the relationship between addition and subtraction
3. **Lesson opener:** Students work in pairs. They are asked to (1) describe what they know about place value, (2) define subtraction and its relationship to addition, and (3) determine whether regrouping is needed to find the sums of 15 + 32 and 15 + 37 and justify their thinking. The teacher circulates to hear the range of responses.
4. **Data-driven instruction:** The teacher moves forward with instruction accordingly. If most of the students have significant learning gaps in any of the prerequisite skills, the teacher closes those gaps before moving on with the lesson. If most students demonstrate reasonable understanding of all the skills, the teacher moves forward knowing that his students have the ability to succeed.

Help your teachers see lesson openers as the starting point for their instruction based on students' needs. In so doing, they will have the best chance for executing the successful lesson they have planned.

Questions? Comments? Contact **robyn@robynsilbey.com**.