


# The *Language* of Mathematics

*Students may excel in computation, but their ability to apply their skills will suffer if they do not understand the math vocabulary used in instructions and story problems. This action research project examined two methods for strengthening students' ability to communicate mathematically.*



**L**earning math is like learning a new language. All languages have their own vocabulary, and mathematics is no exception. Unlike common English, which students hear, see, and use daily in reading, watching television, and conversing, the language of mathematics is limited largely to school (Thompson and Rubenstein 2000). *Principles and Standards for School Mathematics* (NCTM 2000, p. 60) states that students who have opportunities, encouragement, and support for speaking, writing, reading, and listening in math classes benefit because “they communicate to learn mathematics, and they learn to communicate mathematically,” which helps children be successful in math class.

By  
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Some students used dry erase boards to draw their adaptation of a Frayer model.



FIGURE 1

Every day for five weeks, students in the class that used the modified Frayer model analyzed a fourth-grade math vocabulary word using one of their thirty visual vocabulary activity sheets to define the word, write examples and nonexamples, and then draw a picture that would help them connect to the correct definition.

Visual Vocabulary	
<p>Word: <u>Addition</u></p> <p>Definition</p> <p>a mathematical operation in which the sum of two numbers or more is calculated usually a plus sign(+)</p>	<p>Example</p> <p><math>9+3</math></p>
	<p>Nonexample</p> <p><math>9-3</math></p>
<p>Picture</p>	

Vocabulary instruction is as important to math comprehension as it is to reading comprehension, especially because so much of today's math instruction and testing are administered using story problems. No longer are students expected to calculate a numerical problem only. First, they have to figure out which operation to use by making sense of the structure of the word problem. Part of that sense making involves understanding the vocabulary in the word problems. Although students may excel in computation, their ability to apply their math skills will suffer if they do not understand the vocabulary of mathematics used in instructions, story problems, and problems that use math vocabulary. Therefore, effectively teaching mathematics vocabulary as an essential part of their everyday instruction is imperative for teachers.

Many authors have suggested general strategies to promote vocabulary development. These include encouraging cooperative learning, using journal writing, and having students develop personal glossaries (Rubenstein and Thompson 2002). Two graduate students who teach fourth grade at the same elementary school completed an action research project using different methods to teach math vocabulary. One used journal writing and peer discussion about the math vocabulary word of the day. The other teacher employed a modified Frayer model with a place for the student to draw a picture illustrating the vocabulary word of the day. At the end of the action research project, both teachers reported students' increase in the recall of definitions in a vocabulary survey and test scores of word problems incorporating the math vocabulary.

### Method 1: Journal writing and peer discussion

The first strategy involves students writing in their math journals to explain their thought processes, which allows students time to think about the meanings of the words and explain them in their own words with the connections they have made. Teachers can read students' journals or have students orally share their writing in the classroom to check for accuracy (Wilcox and Monroe 2011). By bringing writing to math class, Carter (2009) found that she could determine what her students were thinking mathematically. Children have an

easier time figuring out the meanings of words through personal interaction rather than reading word definitions from a book or paper. Therefore, creating a classroom environment of learning and acceptance is necessary for oral communication strategies to be effective.

## Method 2: The modified Frayer model

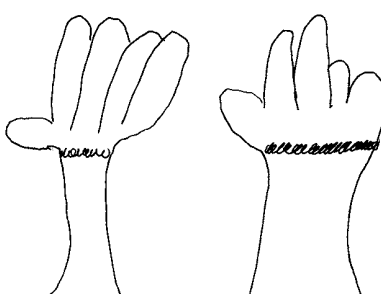
When teaching vocabulary, the Frayer model is a popular graphic organizer to implement. It includes a box for the word being defined, its definition, characteristics, examples, and non-examples (see **figs. 1 and 2**). Frayer and her colleagues at the University of Wisconsin designed this graphic organizer to provide for a thorough understanding of new words (Frayer, Frederick, and Klausmeier 1969). The original Frayer model used words to break down unfamiliar vocabulary, whereas this study model included a step whereby students drew a picture of their connection with the word to help them make a visual memory for later use on math tests with word problems. Foster (2007) found that having students employ the reading strategy of visualization and drawing pictures of the math vocabulary enabled them to connect with the text; pictures helped them show their understanding. When a teacher uses verbal and visual strategies at the same time, students who are oral or visual learners benefit (Thompson and Rubenstein).

## Conducting the action research study

Both teachers involved in this action research were graduate students completing their capstone project. Each taught two classes of fourth graders and used the same vocabulary list of thirty words based on the school's adopted math curriculum. A math vocabulary survey (see **online appendix A**) was administered to all the students to determine their previous knowledge of particular math vocabulary words. Students were to read the word and decide if they knew it. If they did not know the word at all, they were instructed to place a check mark in the column labeled "I do not know this word at all." If they had seen the word before but could not give the definition, they were instructed to place a check mark in the appropriate column. If students knew the vocabulary word well, they

FIGURE 2

The Graphic Organizer task took students no longer than 20 minutes each day to work on a new word and review the past words. This included time for the teacher to check student work and identify misconceptions.

Visual Vocabulary	
<p>Word: <u>Subtraction</u></p> <p>Definition: <u>The operation or process of finding the difference between two numbers, using the (-) minus sign</u></p>	<p>Example</p> <p><u><math>10 - 0 = 0</math></u></p>
	<p>Nonexample</p> <p><u>not Right</u> <u><math>10 + 2</math></u></p>
<p>Picture</p> <p><u><math>50 - 3 = 2</math></u></p> 	

were to write the definition in the last column. The teachers checked students' work for correct definitions because students sometimes thought they knew the meaning of a word when they did not. For example, one student wrote, "Addend means I 'addend' school everyday" instead of *attend school every day*.

A fourth-grade math pretest and posttest was constructed that incorporated all the mathematical vocabulary words that were taught in the classroom during the data collection period. The test consisted of twenty-four problems using math vocabulary (see **online appendix B** for nineteen of those questions).

### Method 1 procedure

The following procedure was applied by the teacher, using the journal writing and peer discussion method.

1. Each day for five weeks, students were explicitly taught one or two mathematics vocabulary words from the vocabulary list (see **online appendix A** for an example).
2. The teacher wrote the word and definition for students to copy into their math journal. Students were asked each day to write in their math journals what they knew about the word and how it was used in mathematics, based on information given by the teacher. If a student was having difficulty journaling, the teacher would ask leading questions to help scaffold students' ideas.
3. Students were also asked to "turn and teach" to their partner the vocabulary word or phrase of the day and how it is used mathematically.
4. This turn-and-teach strategy happened at random times throughout the day. The teacher would listen in on the conversations to be sure students were on task and were explaining the vocabulary correctly. If something was being said incorrectly, the teacher would redirect the conversation.
5. The teacher reviewed previously learned vocabulary throughout the data collection period. The teacher also reviewed the math journals to be sure information was being recorded and explained.

### Method 2 procedure

Students in the class that used the modified Frayer model had thirty visual vocabulary activity sheets enclosed in a folder for organization (see **online appendix C** for a sample sheet). This form had four sections: a definition box, an example box, a nonexample box, and a picture box for their drawings. Each day for five weeks, students analyzed a fourth-grade math vocabulary word by using the required form to define the word, write examples and nonexamples, and then draw a picture that would help them connect to the correct definition. The whole task did not take more than 15–20 minutes each day to work on a new word and review the past words.



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The daily process was as follows:

1. The teacher wrote the vocabulary word with the definition on the board.
2. The teacher read the word and definition while students copied it in their folder.
3. As a group, they discussed the word and came up with examples that they could use to describe or demonstrate the word of the day.
4. Students copied the information from the board, and then they drew their own examples of what the math word of the day meant to them.
5. The teacher circulated around the room to make sure students had copied the information correctly and had created their own example.
6. When folders had been collected, the teacher asked for oral definitions for each of the words that had been given to students up to that day.



These girls are using the turn-and-teach strategy for the math vocabulary word *perimeter*.

7. If a student gave part of the definition, the teacher prompted him or her to finish so that the entire class would benefit from hearing the definition again.

## Results and observations

The findings of this study indicate that both methods, journal writing with peer discussion and the modified Frayer model, had a positive effect; students increased their scores on both the posttest and the vocabulary survey. The teachers observed that while attempting the pretest, many students were flustered and frustrated, displaying little work because the vocabulary in the problems was so unfamiliar to them. However, during the posttest, many students finished the test quickly. The teacher observed that students who had used the Frayer method were drawing visual clues on their test to remind themselves what the questions were asking them to do, and then they were answering the problems correctly.

All eighty-four students using both methods increased the number of correct answers on the posttest, with the exception of five students who

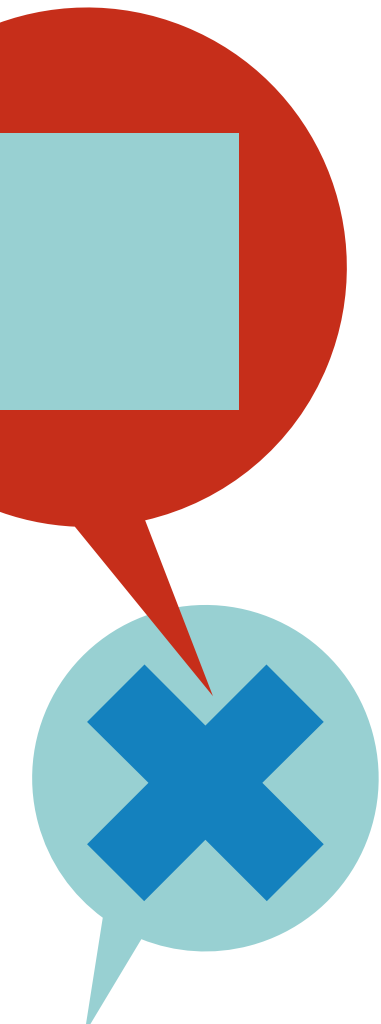
used journal writing and peer discussion, two of whom were English language learners (ELLs). The teacher concluded that journal writing and peer discussion did not prove to be effective for ELLs because of their difficulty with writing and speaking. Conversely, a visual strategy would have been a more advantageous strategy for them to demonstrate their understanding of the words.

Because many of the words were new to the students, the vocabulary survey had been administered to students anonymously to avoid anxiety. For the journal writing and peer discussion group in the presurvey, students' knowledge of the words increased by 26 percent on the vocabulary postsurvey, and students using the Frayer model increased their knowledge of the words by 17 percent.

The teachers observed that student motivation was a limitation for both methods. For the journal writing and peer discussion method, the teacher noticed that students enjoyed the turn-and-teach part of the lesson; therefore, she would ask them to do this at any time throughout the day, motivating them to remember the meanings of the words. Journal writing gave the teacher an opportunity to read what a student was thinking. As an example definition of the vocabulary word *compatible*, a student wrote, "numbers that are easily added or divided." The student showed understanding by writing, "When adding compatible numbers, the numbers that will make a ten. In division, compatible numbers divide evenly and there are no remainders." The student related the word to *compatible* in real life, meaning "to get along together."

For the Frayer model strategy, the teacher observed that in the first couple of weeks, students showed interest in the process of the activity. Some showed great detail in their illustrations of a classroom (see **fig. 1**), but the student who drew the example in **figure 2**





exhibited deep conceptual understanding of subtraction with the drawing of two hands and fewer fingers shown on one of the hands. However, after students had acquired a significant number of words, they would struggle with remembering the definitions. The Frayer model strategy sometimes seemed rushed, and time constraints did not always allow for slower-paced students to complete the task. After the action research project was over, the teacher assigned a vocabulary word to a student for homework using the Frayer model method, and the student taught it to the class the next day. This adaptation allowed this student more time to complete the work, which is always a concern in the classroom.

### Recommendations for vocabulary instruction

The teachers learned that they are going to explicitly teach math vocabulary in the future using both methods because students developed conceptual understanding of the math vocabulary. The teachers found that using a visual strategy can be helpful with the comprehension of problems using math vocabulary. Too often, teachers forget that teaching their lesson verbally without using some visual cues may not help the visual learners grasp the required math concept.

Varying strategies with vocabulary is recommended, because some words lend themselves to being illustrated and therefore would work better with the modified Frayer model. Other words are better suited to be written about in students' journals and discussed with peers. For example, *addition* is much a easier vocabulary word to illustrate than *approximate*.

Both teachers plan to use the vocabulary teaching strategies to keep students' interest and attention and to reach all types of learners. Because it was action research, the teachers were required to use only one method, but to achieve greater student mathematical success, both plan to be more direct in teaching the language of mathematics by using both strategies.

### REFERENCES

Carter, Susan. 2009. "Connecting Writing and Mathematics Workshop: It's Kinda Like Ice Skating." *The Reading Teacher* 62 (April): 606–10. doi:<http://dx.doi.org/10.1598/RT.62.7.7>

- Foster, Shannon. 2007. "The Day Math and Reading Got Hitched." *Teaching Children Mathematics* 14 (November): 196–201.
- Frayer, Dorothy A., Wayne C. Frederick, and Herbert J. Klausmeier. 1969. *A Schema for Testing the Level of Cognitive Mastery*. Madison, WI: Wisconsin Center for Education Research.
- National Council of Teachers of Mathematics (NCTM). 2000. *Principles and Standards for School Mathematics*. Reston, VA: NCTM.
- Rubenstein, Rheta N., and Denisse R. Thompson. 2002. "Understanding and Supporting Children's Mathematical Language Development." *Teaching Children Mathematics* 9 (October): 107–12.
- Thompson, Denisse R., and Rheta N. Rubenstein. 2000. "Learning Mathematics Vocabulary: Potential Pitfalls and Instructional Strategies." *Mathematics Teacher* 93 (October): 568–74.
- Wilcox, Brad, and Eula Ewing Monroe. 2011. "Integrating Writing and Mathematics." *The Reading Teacher* 64 (April): 521–29. doi:<http://dx.doi.org/10.1598/RT.64.7.6>



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Appendix A, B, and C are attached to the end of this article online.

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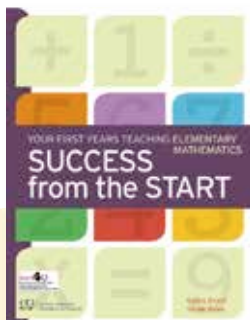
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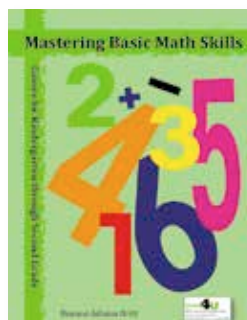
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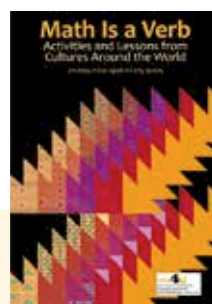


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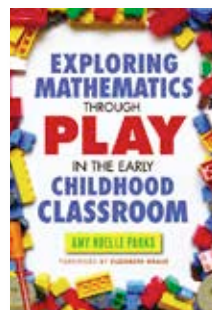
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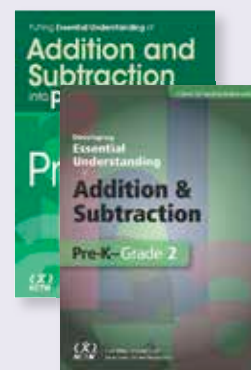


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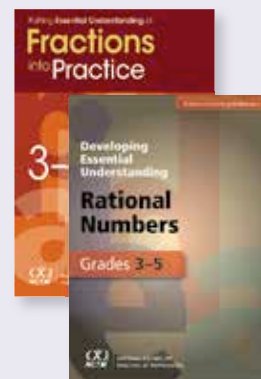
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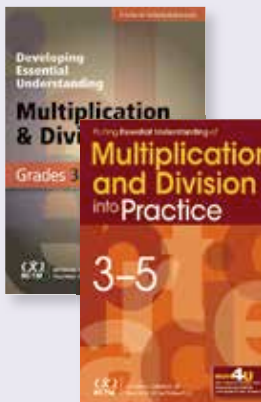
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