



# DIVISION QUILTS: A Measurement Model

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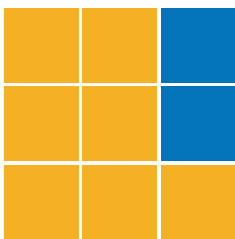
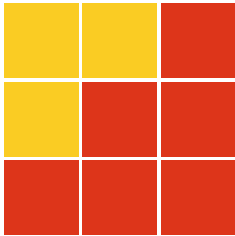
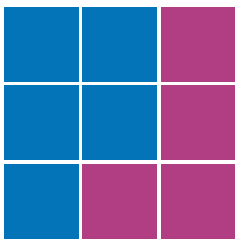
Students can create an image of the measurement model of division as an alternative to the long-division algorithm.

**W**hen division is taught as an algorithm in isolation, student understanding of it has limited flexibility. Current expectations in mathematics education include that students should be able to identify and use relationships between operations (NCTM 2000, Common Core State Standards for Mathematics [CCSSM] CCSSI 2011). As teachers seek activities to assist students in understanding division as more than just the algorithm, they find many examples of division as fair sharing. In actuality, Fischbein and his colleagues (1985) defined two types of whole-number division: partitive (fair sharing) and quotative (measurement). Students are able to engage in activities that draw on their intuitive sense of what it means to share fairly; however, teachers have few activities to engage students in a quotative (measurement) model of division. Division Quilts is one such activity that allows students to connect to their understandings of quotative division, thereby enhancing their understandings of the operation of division. Furthermore, engaging in both forms of division models gives students opportunities to connect in rich ways when presented with the division algorithm.

Partitive and quotative division have clear distinctions. For example, the problem  $25 \div 7$  can be modeled using partitive/fair sharing

by assembling twenty-five stars and equally distributing them to seven circles, leaving four remaining (see **fig. 1**). What students should see with the manipulatives is that twenty-five is reduced by seven to eighteen, then eighteen is reduced by seven to eleven, then eleven is reduced by seven to four (leaving not enough for every circle to receive another star). This model is easily accessible for students to connect to everyday experiences, can be demonstrated using manipulatives, and also relates to their understanding that each group must receive the same amount. This model shows division as repeated subtraction, which connects as an inverse operation to multiplication as a recursive process (Confrey et al. 2009). However, just as moving students beyond understanding multiplication as only repeated addition (Devlin 2008, Wheatley and Reynolds 2007) is important, moving students beyond understanding division as only fair sharing is also important. What we propose is a quotative model of whole-number division that is accessible for all learners and allows for a direct correlation to the formal division algorithm. Included in this model is an image creation, and imaging is an important activity to foster deep, rich understandings of mathematical concepts (Richardson, Pratt, and Kurtts 2010; Wheatley 1998).



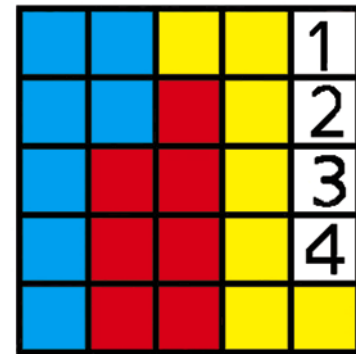


## Division quilts

This activity displays a quotative model by a simple process of showing the dividend, coloring sets of the divisor until the dividend has no more sets the size of the divisor, then determining the quotient by counting how many sets were created and any white spaces as the remainder (see **fig. 2**). Comparing and contrasting **figure 1** to **figure 2** show similarities as well as differences. In **figure 1**, the partitive model includes starting with the dividend, then distributing one unit in each set and repeating this process until no more distributions can be shared fairly. The resulting image resembles the division quilt. However, the process by which the image is created is the significant difference that leads to a unique understanding of division. The process of Division Quilts is bounding an area defined as the dividend, then designating one set the size of the divisor and repeating the designation of another set the size of the divisor until no more sets can be created. The shift from equal distribution (partitive) to creating sets of the divisor (quotative) allows for a different form of division that transitions easily to the partial quotient model of division (sometimes known as the *Big 7 strategy* because the division symbol is made to look like a big number 7) and then to the formal division algorithm. (See Russell et al. 2008 for more details about this strategy; also see **lesson 2**, “**Explain**” in **table 1** in the lesson plan appended to this article **online**.) Allowing students to first imagine a quotative model gives them the chance to make sense of the division

**FIGURE 2**

A quotative model of  $25 \div 7 = 3 \text{ R}4$  has students color sets of the divisor until the dividend has no more sets the size of the divisor. Students count the sets to determine the quotient, and then count white spaces to find the remainder.



algorithm. Limiting them to division activities that show only the partitive model requires them to make a greater cognitive leap to the partial quotient model of division and ultimately to the formal division algorithm.

The lesson plan on **page 106** outlines how the Division Quilts activity can be taught in a classroom. The 5E model is used as a format. (In general terms, the 5E model is a lesson cycle that includes the elements of engage, explore, explain, elaborate, and evaluate. These elements are not necessarily linear; the overall goal of implementing a 5E lesson is to stimulate student interest and questioning before addressing a concept. For more description, see Bybee et al. 2006.) The lesson plan shows two activities: The first teaches Division Quilts; the second lesson moves students from the quilts to the partial division strategy to the formal division algorithm. In the Division Quilts lesson, students have the opportunity to interpret division through a quotative model, then compare it to a partitive model. This allows for richer connections in division as well as to inverse operations of multiplication.

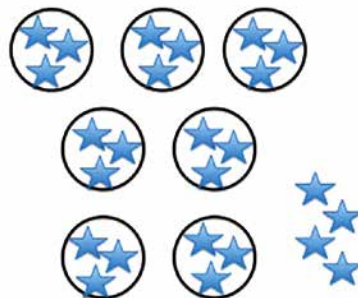
## Powerful results

The data were collected from a Title I school in the southern part of the United States with more than 80 percent of the students

**FIGURE 1**

The partitive (fair sharing) model of  $25 \div 7 = 3 \text{ R}4$  shows division as repeated subtraction, which connects as an inverse operation to multiplication as a recursive process.

Partitive (fair sharing)  
 $25 \div 7$



considered as living at or below the poverty level. The demographics of students in this fifth-grade classroom consisted of approximately 13–15 boys, 10–12 girls, 5–8 English as a second language learners, 2–4 exceptional children (one classified with behavioral and emotional disorder, two classified with attention deficit hyperactivity disorder, one classified with other health impairments), and 2–4 academically gifted students.

Student samples provided for this Division Quilts activity show how different students solved the same problem (see fig. 3) and how the activity can be used as a strategy to meet the needs of such diverse learners. By adapting the dividend and the divisor, a teacher can adjust the difficulty of the task. This imaging activity addresses misconceptions that are associated with division, and students can engage in interpreting and making sense of division.

### Misconceptions addressed using division quilts

With the use of Division Quilts, students are exposed to the terminology of the long-division algorithm; understanding the vocabulary associated with division helps students in problem solving with division. Division Quilts creates an image for the dividend (the area of the quilt).

#### What are we dividing?

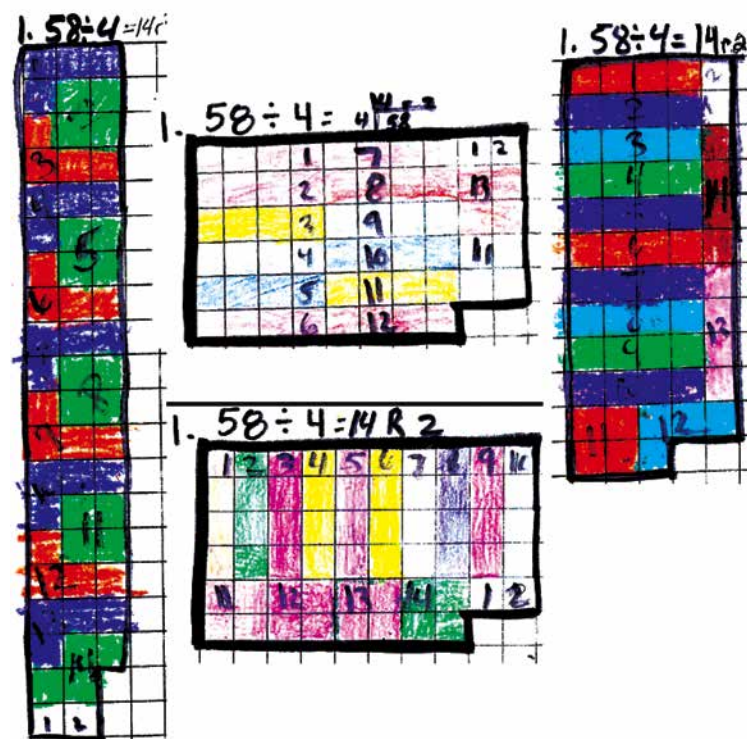
A common misconception for students is what they are dividing; students can start division by physically seeing what they are dividing, which can lead to an increase in success in interpreting division because what they understand is more concrete than abstract.

#### What is a remainder?

Another common misconception for students is the importance of a remainder. Division Quilts supply a useful way for students to visualize a remainder, as well as how the value of the remainder may or may not change the value of the quotient, depending on the situation and need for the remainder. Some students believe that the remainder shows how many groups are left instead of how many individual items are left. Some students' misconceptions of remainders stem from inexperience with manipulating a problem. Many times, students will go through the mathematical process of division and know

FIGURE 3

Different student solutions to  $58 \div 4 = 14 \text{ R}2$  show that the Division Quilts activity can be used as a strategy to meet the needs of diverse learners.



**“BY ADAPTING THE DIVIDEND AND THE DIVISOR, A TEACHER CAN ADJUST THE TASK’S DIFFICULTY.”**

that there is a remainder but not know what it means. Students can recognize that the white space in the quilt is part of the dividend but not part of the divisor or the quotient.

Division Quilts can help students make connections to multiplication. Prompting students by asking questions that relate the image to multiplication as well as to division will generate

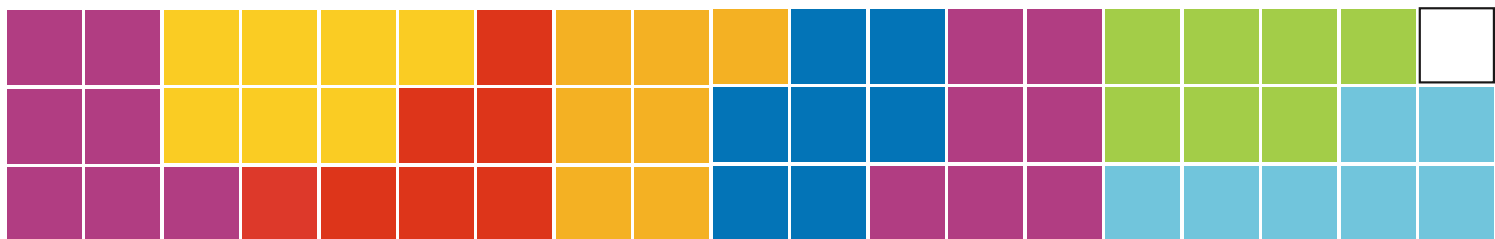
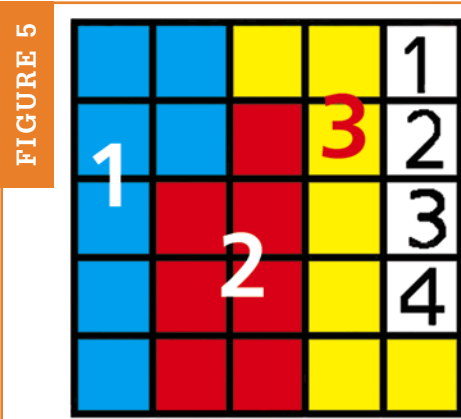
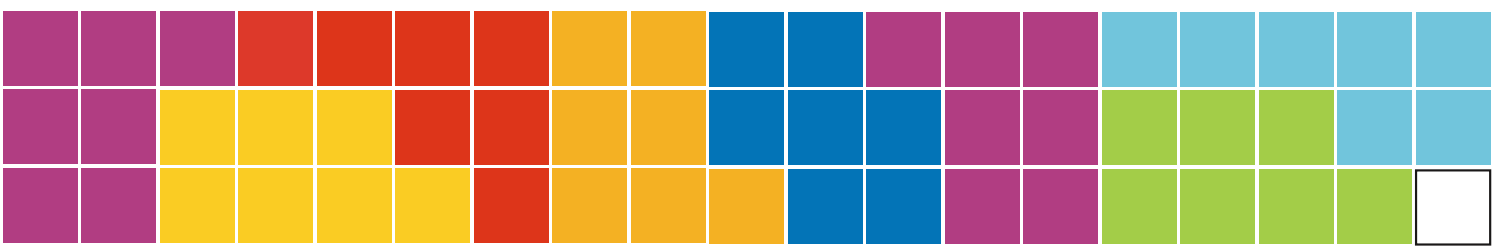


TABLE 1

# Division Quilts—5 E Lesson Plan—Lesson 1: Introducing Division Quilts

Essential question	What does it mean to divide, and how can I model division in different ways?	
Engage	<p>Students use base-ten blocks to manipulate <math>25 \div 7</math>. Starting with 2 tens rods and 5 units, they determine what needs to happen to “break apart” the blocks and group them (see <b>fig. 4</b>). After students have manipulated the blocks (exchanged the 2 tens for 20 units) and arrived at an answer (3 sets of 7 with 4 left), clear the blocks and have them take out the materials needed for the division quilts.</p>	<p><b>FIGURE 4</b></p>
Explain Making a division quilt	<p>Distribute grid paper; a dark, bold marker; and colored pencils.</p> <p>Students will learn how to model <math>25 \div 7</math> using Division Quilts.</p> <ol style="list-style-type: none"> <li>Count 25 adjacent units in the grid paper, then outline the 25 squares with a dark, bold marker. (It may help to start with a pencil, check that there are in fact 25 units, then trace the outline using the dark, bold marker.)</li> <li>Select one color from the colored pencils. Count and color 7 adjacent squares from within the 25.</li> <li>Select a different color and color 7 adjacent squares from within the remaining 18.</li> <li>Select a different color and color 7 adjacent squares from within the remaining 11.</li> <li>Note that there are only 4 now remaining, so these are left as white squares.</li> <li>Using the dark, bold marker, consecutively number each different-color set, starting at 1.</li> <li>The remaining white squares should be consecutively numbered as well, starting at 1.</li> <li>The solution is written above the model, <math>25 \div 7 = 3 \text{ R}4</math>. (See <b>fig 5</b>.)</li> </ol> <p><b>Notes:</b></p> <ul style="list-style-type: none"> <li>The dividend can be created using any set of adjacent units. The teacher should begin to encourage students to use a rectangular representation of the dividend.</li> <li>Students can count the squares vertically, horizontally, or in a pattern that makes sense, as long as the 7 units are bound together.</li> <li>Discourage students from doing “checkerboard” style coloring.</li> <li>Remind students that the divisor is a group, so when coloring, the group is colored and numbered as one set.</li> </ul>	





Essential question	What does it mean to divide, and how can I model division in different ways?
<b>Evaluate</b>	<p>Walk around while students are coloring their own patterns of <math>25 \div 7</math>.</p> <p>Engage in asking questions:</p> <ul style="list-style-type: none"> <li>• What is the reason for counting 25 squares?</li> <li>• If the dividend is 25, which number is the divisor?</li> <li>• Why count to 7?</li> <li>• Why was it important to change colors when starting a new group?</li> <li>• Why are there 4 left?</li> <li>• What is the quotient, and how do you see it in your quilt?</li> </ul>
<b>Explain</b>	<ol style="list-style-type: none"> <li>1. Have students display their quilts on the wall.</li> <li>2. Conduct a gallery walk so students can see that the quilts may appear different but show the same quotient.</li> <li>3. Ask questions as they walk around: <ul style="list-style-type: none"> <li>• How is this one different from yours?</li> <li>• How is this one similar to yours?</li> <li>• What is the quotient?</li> </ul> </li> <li>4. Facilitate a class discussion during which a selected number of students explain what they did.</li> <li>5. Connect back to the Engage activity by comparing and contrasting the processes and the quotients.</li> </ol>
<b>Evaluate</b>	<p>Ask students to show the connection between multiplication and division.</p> <p>Ask students to explain what they see as the connection between the algorithm and the quilt.</p>
<b>Elaborate</b> <i>Interpreting the remainder</i>	<p>In small groups, using the created Division Quilt, determine the answer to each scenario:</p> <p>Scenario 1: There are 25 pieces of candy in a party bag. Each person receives 7 pieces of candy. How many people are sharing? (How much, if any, is left?)</p> <p>Scenario 2: There are 25 students going on a field trip. Seven people can fit in each car. How many cars are needed?</p>
<b>Evaluate</b>	<p>While students are discussing, walk around to monitor and prompt their thinking.</p> <p>Ask what would happen with the 4 units that are left.</p>
<b>Explain</b>	<ol style="list-style-type: none"> <li>1. Select one group to present their explanation of scenario 1 to the class.</li> <li>2. Select a different group to present their explanation of scenario 2 to the class.</li> <li>3. Ask students to justify why they agree or disagree with each group's interpretation.</li> </ol>
<b>Evaluate</b>	<p>Pass out a pair of notecards to each individual student, with a division problem on each card. (Accommodate students' abilities by varying the difficulty of the problems.)</p> <p>Have students replace their display of the <math>25 \div 7 = 3 \text{ R}4</math> with a new problem that they create.</p>



FIGURE 6

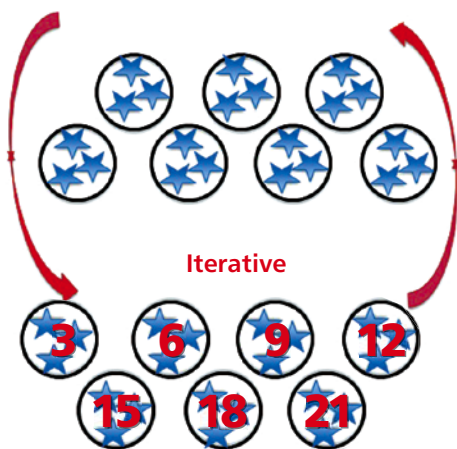
Connecting the iterative model of multiplication to the partitive model of division, and the recursive model of multiplication to the quotative model of division, supports students in imagining division in ways that strengthen connections to real-life scenarios.

### Connecting division models to multiplication

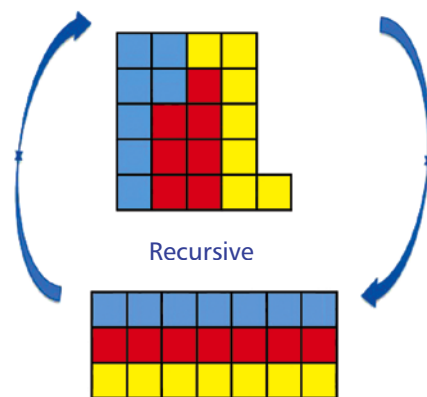
$$21 \div 7 = 3 \text{ (dividend} \div \text{divisor} = \text{quotient)}$$

**Partitive (fair sharing)**

**Quotative (measurement)**



**Iterative**



**Recursive**

Repeating a value for a certain number of times  
(7 sets of size 3)

Replacing a single fair share/part of size  $m$ , by  
 $n$  times as many of that part, to produce the  
original collection or whole (the product)  $mn$ .

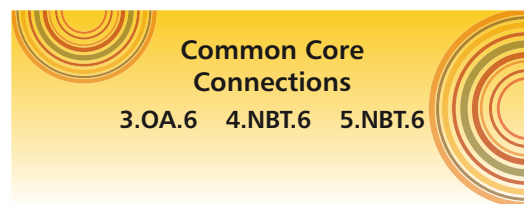
$$7 \cdot 3 = 21 \text{ (multiplier} \cdot \text{multiplicand} = \text{product)}$$

opportunities for them to recognize this relationship and will help them build their understanding between multiplication and division. Students also become comfortable using the vocabulary associated with division; when they are introduced to the formal division algorithm, they can connect it to an image, which provides a meaningful context to interpret division.

### Modeling connections

The Division Quilts activity offers a connection to the inverse operation of multiplication as a recursive process. By demonstrating the quotative model through Division Quilts, we build on Confrey and colleagues' (2009) argument by modeling distinct connections between each model of multiplication and division (see fig. 6). We believe that Division Quilts is an effective

strategy to foster opportunities for students to imagine how division is an inverse operation to multiplication. By connecting the iterative model of multiplication to the partitive model of division, as well as the recursive model of multiplication to the quotative model of division, students can imagine division in a variety of ways that strengthen connections to real-life scenarios. Division Quilts are an effective way to model quotative division; they grant







opportunities for students to make sense of division in a way other than fair sharing and to directly connect an image to the division algorithm.

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