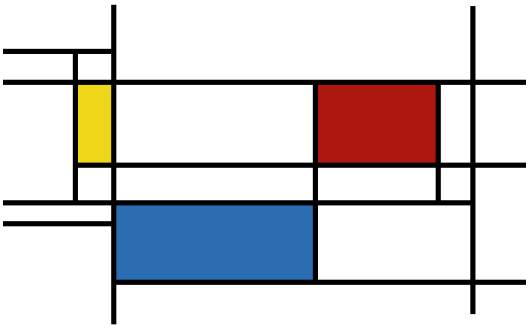
Math is beautiful

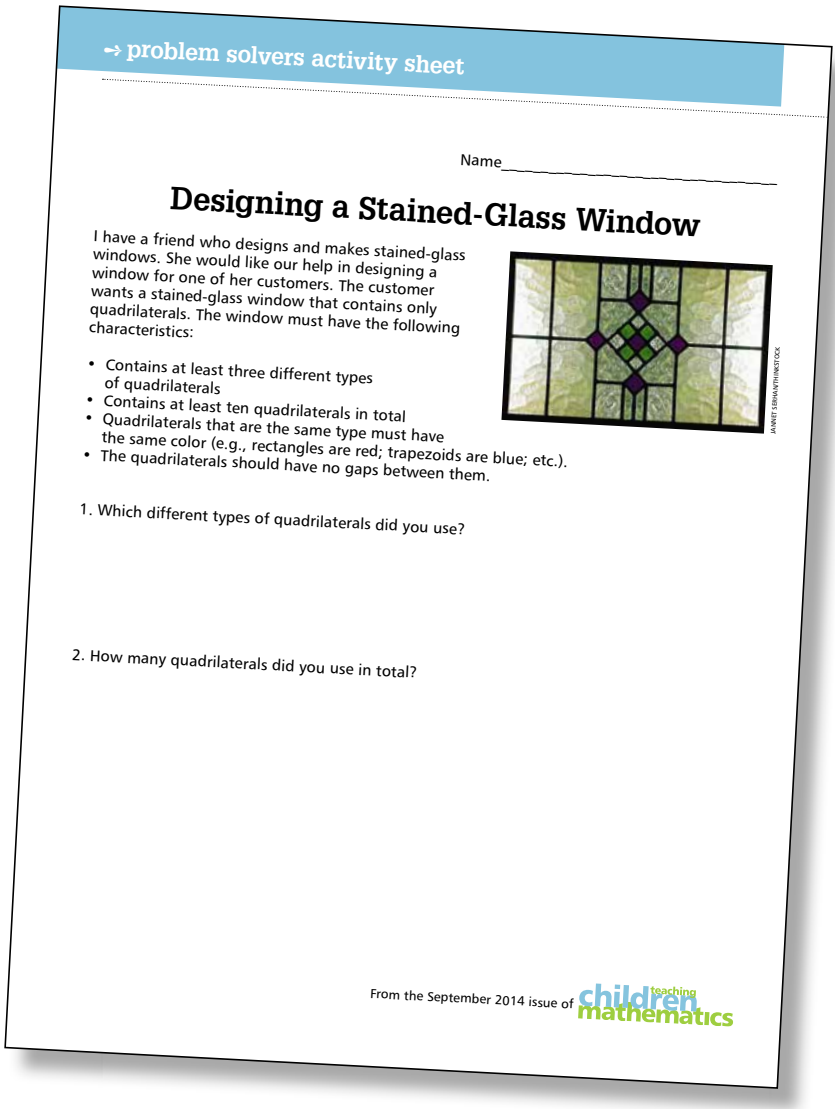


Two different solution contexts for the Math Is Beautiful investigation are described here: Ali Duncan's third-grade class at Prueter Public School in Kitchener, Ontario, solved the problem during a mathematics class; and Cynthia Hockman-Chupp's son in the second grade explored the investigation while on a plane trip during a family vacation. In both cases, students were asked to design a stained-glass window using specific criteria related to quadrilaterals. To meet the criteria, students had to consider how many shapes would fill a designated space, how different shapes can fit together to tile a surface, and the properties and attributes of different types of quadrilaterals.

Getting started

Duncan introduced the investigation to her third graders by showing some sample works of art by Mondrian, Delaunay, and Kandinsky and inviting students to identify the geometric shapes they could see in the samples. Duncan connected this art to a picture of a stained glass window (see the introductory **sidebar** on this page) as she shared the problem with her students. Duncan recorded the criteria on the board for reference. She asked her students to describe the investigation to an elbow partner to ensure that everyone had understood the task. Duncan organized students into pairs, provided them with paper and pencil crayons, and then asked them to find a place to work on the investigation.

The September 2014 problem offers students an opportunity to design their own stained-glass windows by tiling a surface with a variety of quadrilaterals. The investigation focuses on student understanding of the properties of different quadrilaterals and the ability to develop and defend their thinking. To access the full-size activity sheet, go to **http://www.nctm.org/publications/teaching-children-mathematics/** and select All Issues.



problem solvers activity sheet

Name_____

Designing a Stained-Glass Window

I have a friend who designs and makes stained-glass windows. She would like our help in designing a window for one of her customers. The customer wants a stained-glass window that contains only quadrilaterals. The window must have the following characteristics:

- Contains at least three different types of quadrilaterals
- Contains at least ten quadrilaterals in total
- Quadrilaterals that are the same type must have the same color (e.g., rectangles are red; trapezoids are blue; etc.).
- The quadrilaterals should have no gaps between them.

JANNET ERIKA/ISTOCK

1. Which different types of quadrilaterals did you use?

2. How many quadrilaterals did you use in total?

From the September 2014 issue of teaching children mathematics

Hockman-Chupp was going on a family vacation. Her son's second-grade teacher had asked that he review the topic of quadrilaterals while he was away. Hockmann-Chupp had been reading the Math Is Beautiful investigation and thought it presented a perfect opportunity to explore quadrilaterals. While traveling on the plane, her son used a free iPad® app called Pattern Shapes to explore the investigation.

Working on it

As the pairs of third graders chose a place to work in the classroom and settled in, Duncan circulated around the room, observing initial strategies and ensuring that all students understood the task. She occasionally asked questions of her students, to clarify the task or provoke their thinking:

- What are quadrilaterals?
- How will you make sure each group member gets to draw part of the design?
- How many shapes do you plan to use in total?
- How many would be on half of the page?

Duncan noticed that some students experienced difficulty in tiling the shapes in such a way that the quadrilaterals had no gaps between them. These students often drew individual quadrilaterals in different areas of the page and then tried to fill in the gaps. Other students had more success in tiling by continually using one side of a previously drawn quadrilateral as the starting point for drawing the next quadrilateral. All students were able to successfully include some quadrilaterals and identify some of the same type using the same color. Some students had difficulty completely tiling the paper with quadrilaterals; others included shapes that were not quadrilaterals; and some students incorrectly identified some of the quadrilaterals as the same or different. As her students worked, Duncan recorded some assessment data, took digital photos of student work, and selected several solutions to highlight at the end of the investigation (see **fig. 1**).

CYNTHIA HOCKMAN-CHUPP

A second-grade student designed his stained-glass window on an iPad while traveling on a plane during a family vacation.

On the airplane, Hockmann-Chupp noted that her son first identified all the Pattern Shape quadrilaterals in the iPad app, counting four available shapes with four sides. He then wanted to know what each was called, bringing mathematical vocabulary into the activity. To tile the window and leave no gaps between the shapes, he discovered that using the same shapes on both sides of his window was helpful, thus informally exploring reflective symmetry. When he completed the drawings on the iPad, he carefully reviewed each shape, examining the attributes and coloring those that were alike. Hockman-Chupp reports that even the flight attendant became intrigued with the activity.

Reflecting and connecting

Duncan decided to begin the whole-group discussion by sharing one of the completed stained-glass windows (see **fig. 1**) and posting it next to the list of criteria. She asked her students if this solution contained all the

FIGURE 1

This is the first solution that was shared for the third-grade class to consider and discuss. Some students considered the blue shape near the center a hexagon; others noticed that it was two trapezoids.

criteria that were listed. After a brief discussion in pairs, one student focused on the blue shape near the center of the window and suggested that his classmates had mistakenly included a hexagon, which is not a quadrilateral. Another student argued that the hexagon was actually composed of two joined trapezoids. Most students agreed that the blue section was composed of two trapezoids, but one student asked, "What is a trapezoid?" Duncan invited the class to answer the question and began to list on the board any ideas that students agreed on.

Instead of providing the class with the definition of a trapezoid, Duncan chose had students develop their own definition. Different students in the class made suggestions that the rest of the class considered and discussed before the students decided as a group whether each was true of all trapezoids and should be included on the chart. As a beginning definition, the class concluded that a trapezoid has four vertices, two parallel lines, and can be described as "a triangle without the top" (see **fig. 2**). Duncan continued to extend students' thinking by asking such questions as the following:

- Do you see any other trapezoids in this window that should be blue?
- How do you know?
- Is a square a trapezoid? Why not?

The class discussion continued as students debated which shapes in the window were trapezoids and which were not by referring to the class chart.

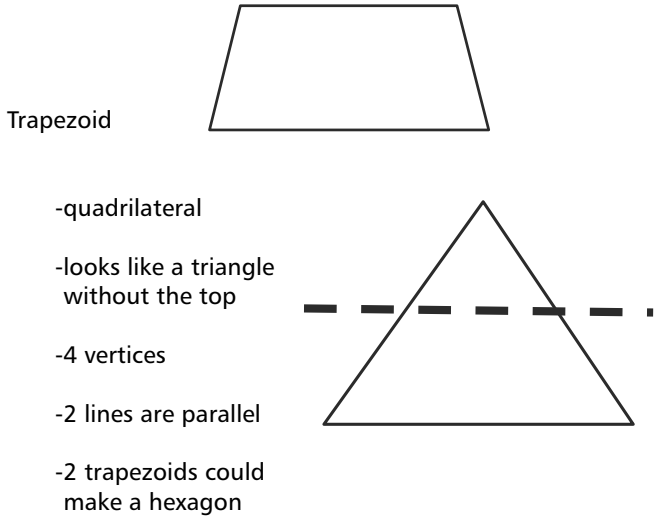
Independent practice and next steps

After the class discussion, Duncan presented the class with an independent task. Students were to draw and describe two different quadrilaterals that are not trapezoids.

Duncan feels that her class had a better understanding of the definition of a trapezoid because students had created the definition as a class by putting forth their ideas, testing them, and coming to an agreement. She senses that students will have a deeper understanding of the properties this way instead of memorizing a list provided by their teacher. Duncan also feels that even if a student forgets the properties, he or she will be able to describe them through analysis and comparison because of this experience. Duncan will continue to build the class chart of quadrilaterals as students build their definitions. They will need continued practice in examining and identifying quadrilaterals to consolidate their understanding.

FIGURE 2

The teacher created a chart to record the class definition of a trapezoid using criteria developed and agreed on by the class.



Edited by **Ed Enns**, **ed_enns@wrdsb.on.ca**, who works as an elementary learning services consultant with the Waterloo Region District School Board in Kitchener, Ontario, Canada. In his work with elementary school mathematics teachers, he emphasizes conceptual understanding and is exploring effective strategies for teaching mathematics through problem solving. Each month, this section of the Problem Solvers department discusses the classroom results of using problems presented in previous issues of *TCM*. Go to **http://www.nctm.org/tcmdepartments** to find detailed submission guidelines for all departments.